Traffic assignment models in large-scale applications

PhD Thesis

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TRAFFIC ASSIGNMENT MODELS IN LARGE-SCALE APPLICATIONS

PhD thesis

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Preface

This PhD thesis entitled 'Traffic assignment models in large-scale applications' is submitted to fulfil the requirements for obtaining a PhD degree at the Department of Transport, Technical University of Denmark. The PhD study has been supervised by Professor Otto Anker Nielsen, Professor Carlo Giacomo Prato and Rasmus Dyhr Frederiksen from Rapidis ApS.

The thesis consists of the papers listed below:

Rasmussen et al. (2014a): Rasmussen, T.K., Anderson, M.K., Prato, C.G., Nielsen, O.A., 2014. Timetable-based simulation method for choice set generation in large-scale public transport networks (under resubmission after second round of review to *Transportmetrica A: Transport Science*).

Rasmussen et al. (2014b): Rasmussen, T.K., Ingvardson, J.B., Halldórsdóttir, K., Nielsen, O.A., 2014. Improved methods to deduct trip legs and mode from travel surveys using wearable GPS devices: A case study from the Greater Copenhagen area (under resubmission after first round of review to *Computers, Environment and Urban Systems*).

Prato et al. (2014): Prato, C.G., Rasmussen, T.K., Nielsen, O.A., 2014. Estimating Value of Congestion and of Reliability from Observation of Route Choice Behavior of Car Drivers. *Transportation Research Record: Journal of the Transportation Research Board*, 2412, 20-27.
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Watling et al. (2014): Watling, D.P., Rasmussen, T.K., Prato, C.G., Nielsen, O.A., 2014. Stochastic User Equilibrium with Equilibrated Choice Sets: Part I – Model Formulations under Alternative Distributions and Restrictions (paper submitted for second round of review to *Transportation Research Part B: Methodological*).

Rasmussen et al. (2014c): Rasmussen, T.K., Watling, D.P., Prato, C.G., Nielsen, O.A., 2014. Stochastic User Equilibrium with Equilibrated Choice Sets: Part II – Solving the Restricted SUE for the Logit Family (paper submitted for second round of review to *Transportation Research Part B: Methodological*).

Rasmussen et al. (2014d): Rasmussen, T.K., Watling, D.P., Prato, C.G., Nielsen, O.A., 2014. Stochastic User Equilibrium with Equilibrated Choice Sets: Part III – Model reformulation to include Thresholds on costs and large-scale application (working paper, DTU Transport and ITS Leeds).

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Finally, I would like to thank my family and friends for their continuous support throughout the years. I owe the greatest thanks to my beloved wife Stine for her constant encouragement and understanding, and to our 6 months old daughter Laura.

Summary

Transport models are becoming more and more disaggregate to facilitate a realistic representation of individuals and their travel patterns. In line with this development, the PhD study focuses on facilitating the deployment of traffic assignment models in fully disaggregate activity-based model frameworks. In the correct integration, such frameworks allow realistic representation of individual-specific household interactions, time-space constraints and preference structures. Individual trips can also be evaluated on a detailed address-to-address level and aggregation biases are avoided. The study focuses on large-scale applications and contributes with methods to actualise the true potential of disaggregate models. To achieve this target, contributions are given to several components of traffic assignment modelling, by (i) enabling the utilisation of the increasingly available data sources on individual behaviour in the model specification, (ii) proposing a method to use disaggregate Revealed Preference (RP) data to estimate utility functions and provide evidence on the value of congestion and the value of reliability, (iii) providing a method to account for individual mis-perceptions in the choice set generation for complex multi-modal networks, and (iv) addressing the difficulty of choice set generation by making available a theoretical framework, and corresponding operational solution methods, which consistently distinguishes between used and unused paths.

The availability of *data* is essential in the development and validation of realistic models for large-scale applications. Nowadays, modern technology facilitates easy access to RP data and allows large-scale surveys. The resulting datasets are, however, usually very large and hence data processing is necessary to extract the pieces of information relevant to the analysis at hand. Manual processing of the datasets are typically not possible, and it is therefore necessary to have methods available which in some automated ways clean and prepare the datasets for the desired use. The present study proposes a fully automatic post-processing procedure that combines fuzzy logic- and GIS-based methods to process raw individual-based GPS data with no additional information required from the respondent. The method categorises trips and trip legs and associates the trip legs with the most probable mode of transport used. The method was validated through the application to a dataset consisting of raw individual-based GPS logs collected among 183 respondents living in the Greater Copenhagen area. Through the use of a control-questionnaire, the study found that the proposed method (i)

identified corresponding trip legs for 82% of the reported trip legs, (ii) avoided classifying non-trips such as scatter around activities as trip legs, and (iii) identified the correct mode of transport for more than 90% of the trip legs. These results are very promising, especially when compared to results generated by existing algorithms. The results highlight the potential of the method proposed and the possibility to use individual-based GPS units for travel surveys in real-life large-scale multi-modal networks.

Congestion is known to highly influence the way we act in the transportation network (and organise our lives), because of longer travel times, but the *reliability* of the travel time also has a large impact on our travel choices. Consequently, in order to improve the realism of transport models, correct understanding and representation of two values that are related to the value of time (VoT) are essential: (i) the *value of congestion* (VoC), as the VoT varies with traffic conditions and hence congestion multipliers reflect the complexity of driving conditions when more vehicles are present on the road, and (ii) the *value of reliability* (VoR), as the VoT relates to the predictability of travel time and the repeatability of the travel experience. Congestion and reliability highly influence each other, but so far only studies based on Stated Preference (SP) data considered concurrently congestion and reliability variables.

The PhD study contributes to the state-of-the-art by presenting a new approach to estimate the VoR and VoC based on RP data. The approach applies a mean-variance model that considers congestion and reliability concurrently. The model was applied to GPS data and it successfully estimated mixed Path Size Logit models, using a sample of 5,759 observations in the peak period and a sample of 7,964 observations in the off-peak period. Results illustrated that the value of the different time components (free-flow, congestion, and reliability) and the congestion multiplier were significantly higher in the peak period. This seems reasonable because of possible higher penalties for being late and, as a consequence, possible higher time pressure. Results also showed that the marginal rate of substitution between travel time reliability and the *total* travel time, considering the average congestion level, did not vary across time periods and traffic conditions. The study highlights the potential of exploiting the growing availability of observations of actual behaviour to obtain estimates of the (monetary) value of different travel time components, thereby increasing the behavioural realism of large-scale models.

The *generation of choice sets* is a vital component in route choice models. This is, however, not a straight-forward task in real-life applications, as: (i) there are almost infinitely many alternatives, but large choice sets are computationally demanding or even unfeasible; (ii) congestion effects need to be considered; (iii) the choice sets should contain all relevant alternatives, including the observed route if one such is available, while leaving out non-reasonable and redundant routes; and (iv) the attributes of the alternatives should vary enough to facilitate consistent parameter estimates if the choice sets are to be used for choice model estimation.

The PhD study contributes to the state-of-the-art by proposing and validating a simulation-based choice set generation method for general networks. The validation used 5,131 observed route choices collected on the highly complex large-scale Greater Copenhagen area public transport network. By evaluating alternative ways to specify the stochasticity and the level of this, it was found that the level of stochasticity should be high to induce high coverage and statistically efficient parameter estimates when the choice sets are used for estimation. The level of stochasticity should, however, be introduced with parsimony, as significant increases translate into generating redundant and counter-intuitive paths with no considerable improvement in coverage. Adding heterogeneity across travellers improved the results considerably, and induced coverage levels up to a very high 98.8% at an 80% overlap threshold. This shows the potential of the method proposed as well as the importance of accounting for as much individual heterogeneity as possible as models become more disaggregate.

A revisit to the original conditions underlying the Stochastic User Equilibrium (SUE) has led to the realisation that the difficulty of specifying the choice set is related to the assumption on the distribution of the mis-perceptions. It is the commonly adopted assumption that the distributed elements follow unbounded distributions which induces the need to enumerate all paths in the SUE, no matter how unattractive they might be. The Deterministic User Equilibrium (DUE), on the other hand, has a built-in criterion distinguishing definitely unused from potentially used routes, but the cut-off in terms of cost differences is strict. Based on this, two *new model frameworks* and corresponding equilibrium formulations are introduced. Both models combine the strengths of the SUE and DUE by permitting the *consistent* combination of (i) *equilibrated* non-universal choice sets and (ii) flow distribution according to random utility maximisation theory. One model allows distinction between used and unused routes based on the distribution of the random error terms, while the other model allows this distinction by posing restrictions on the costs of used/unused routes.

Generic path-based solution algorithms and convergence measures are introduced for the model which seemed the most straightforward to apply given its connection to existing RUM-based models (the one adding restrictions). Different variants of the algorithms were validated for the MultiNomial Logit and Path Size Logit choice models on the Sioux Falls as well as the large-scale Zealand network. A novel consistent convergence measure verified extremely fast and well-behaved convergence to an equilibrated solution on non-universal choice sets (across different congestion levels, scale parameters and step-sizes). The composition of the choice sets were validated by comparison to real-life route choices of 16,618 individual trips on the Zealand network. The applications were also very successful in reproducing observed link counts. The solution algorithms are thus computationally attractive, and the solutions and the underlying framework are behaviourally realistic. This causes the new framework and solution algorithms to be highly attractive to apply as models become more disaggregate.

Summarising, the PhD study has given contributions to several of the components that concern the estimation and solution of traffic assignment models in large-scale applications. Through this, the PhD study has successfully facilitated the consistent integration at the disaggregate level across traffic model parts. This means that the true potential of the activity-based models can be actualised.

Dansk Resumé

Trafikmodeller bliver mere og mere detaljerede, for derved bedre at kunne beskrive individer og deres rejsemønstre. I overensstemmelse med denne udvikling, fokuserer dette Ph.D. projekt på at muliggøre brugen af rutevalgsmodeller i fuldt disaggregerede aktivitets-baserede modeller. Sådanne modeller tillader, i den rette integrering mellem komponenterne, realistisk repræsentation af individspecifikke præferencer, hensyntagen til begrænsninger forbundet med hjemlige pligter samt sikrer at tid-sted sammenhængen mellem en pågældende dags ture er realistisk. Derudover kan individuelle ture modelleres på et mere realistisk adresse-til-adresse niveau, hvorved der ikke genereres usikkerheder pga. aggregering. Projektet fokuserer på storskalamodeller og præsenterer metoder der muliggør at disaggregerede modellers sande potentiale kan realiseres. For at nå disse mål har projektet bidraget til flere af de komponenter, der udgør rutevalgsmodellerne ved at (i) muliggøre brugen af de mere hyppigt tilgængelige datakilder, indeholdende information om individers rejsemønstre i specifikationen af modellen, (ii) foreslå en metode til at udnytte data om observerede præferencer i virkelige situationer (Revealed Preference, RP) til at give indsigt i værdien af trængselstid og værdien af påligeligheden af rejsetid, (iii) bidrage med en metode, der formår at tage hensyn til individers ukomplette information ved generering af valgsæt i komplekse multi-modale netværk, (iv) adressere vanskelighederne i forbindelse med valgsætgenereringen, ved at foreslå en ny teoretisk ramme med tilhørende løsningsalgoritmer, som på konsistent vis kan separere anvendte og ikke-anvendte ruter.

I forbindelse med udvikling og validering af realistiske storskalamodeller er det essentielt, at der er *data* tilgængeligt. Moderne teknologi gør det muligt at få nem adgang til RP data samt gennemføre store dataindsamlinger. De resulterende datasæt er imidlertid meget store, og der skal derfor typisk foretages en bearbejdning af data, før disse kan tages i anvendelse i egentlige analyser. Datasættenes størrelse umuliggør dog manuel bearbejdning, hvorfor det er nødvendigt, at der er metoder tilgængelige, der automatisk bearbejder og konverterer data til det rigtige format. Ph.D. projektet foreslår en fuldautomatisk metode til at forarbejde sådanne datasæt. Metoden bearbejder de 'rå' person-baserede GPS data ved at kombinere fuzzy logic- og GIS-baserede metoder og identificerer ture, delture samt det mest sandsynlige transportmiddel anvendt på hver deltur. Metoden blev valideret ved brug af et

datasæt bestående af GPS logs fra 183 beboere i københavnsområdet. Et spørgeskema blev brugt til at validere resultaterne, der viste, at den foreslåede metode (i) identificerede tilsvarende delture i GPS data for 82 % af de rapporterede delture, (ii) undgik at klassificere ikke-ture (fx 'støj' omkring aktivitetspunkter) som delture og (iii) identificerede det korrekte transportmiddel for mere end 90% af delturene i GPS data. Disse resultater synes meget lovende, særligt i sammenligning med resultater genereret af eksisterende algoritmer. Resultaterne understreger den foreslåede metodes potentiale samt muligheden for at bruge person-baserede GPS enheder som dataindsamlingsmetode i store multi-modale transportnetværk.

Det er velkendt at længere rejsetider pga. *trængsel* påvirker vores transportadfærd (samt hvordan vi tilrettelægger vores dagligdag), men også rejsetidens *pålidelighed* har stor indflydelse på vores adfærd. For at øge trafikmodellens realisme er det derfor afgørende at forstå og repræsentere to værdier, der er relateret til tidsværdien (VoT) på korrekt vis: (i) værdien af trængsel (VoC), idet VoT varierer med kørselsforholdene, og trængselsmultiplikatoren repræsenterer derfor den øgede kompleksitet når mængden af biler på vejene øges; (ii) værdien af pålidelighed (VoR), idet VoT også relaterer sig til forudsigeligheden af rejsetiden og muligheden for gentagelse af rejseoplevelsen. Trængsel og pålidelighed har stor indflydelse på hinanden, men hidtil er det kun studier baseret på erklærede præferencer i hypotetiske situationer (Stated Preference, SP), der samtidigt har inkluderet variable relateret til *begge* disse komponenter.

Ph.D. projektet bidrager til den nyeste forskning ved at foreslå en ny metode, der anvender RP data til at estimere VoR og VoC i én samlet model. Metoden estimerer en middel-varians model, der således tager hensyn til både trængsel og pålidelighed i samme model. Der blev anvendt GPS data til at estimere to mixed Path Size Logit modeller – en model blev estimeret på 7,964 observationer uden for myldretid, mens en anden model blev estimeret på 5,759 observationer inden for myldretid. Resultaterne viste, at værdien af de forskellige tidskomponenter (fri rejsetid, trængselstid samt pålideligheden af rejsetiden) samt trængselsmultiplikatoren var signifikant højere i myldretiderne i forhold til uden for myldretiderne. Dette synes plausibelt, idet rejsende i myldretiderne ofte oplever større sanktioner ved forsinkelser og derved oplever større tidspres. Den marginale substitution mellem pålideligheden af rejsetiden samt den *totale* rejsetid (når der korrigeres for det gennemsnitlige trængselsniveau) varierede ikke mellem tidsperioder eller trafikforhold. Projektet understreger potentialet for at anvende (de mere og mere tilgængelige) data omkring rejsendes faktiske adfærd, til at bestemme den monetære værdi af de enkelte rejsetidskomponenter – for derved at øge realismen af storskalamodeller.

Valgsætgenerering er en vital komponent i rutevalgsmodeller. Det er imidlertid ikke en nem opgave at generere valgsæt i virkelige netværk, idet: (i) der nærmest er uendeligt mange alternativer, men store valgsæt kræver megen beregningskapacitet, (ii) der skal tages hensyn til effekten af trængsel, (iii) valgsættene bør indeholde alle relevante alternativer (også de observerede ruter, hvis sådanne er tilgængelige) og udelade irrelevante og overflødige ruter og (iv) attributterne skal variere tilstrækkeligt mellem alternativerne, hvis valgsættene skal bruges til modelestimering.

Ph.D. projektet bidrager til den nyeste forskning ved at foreslå og validere en ny metode til valgsætgenerering. Metoden er baseret på simulation og kan benyttes på generelle netværk. Valideringen benyttede 5,131 observerede ruter for ture foretaget med offentlig transport i københavnsområdet. Dette netværk er kendetegnet ved at være meget stort og komplekst. Forskellige konfigurationer samt størrelser (varians) af stokastikken blev undersøgt. Resultaterne viste, at variansen af stokastikken skal være høj, for at de observerede ruter er repræsenterede i de genererede valgsæt (dækningsgrad). Tilmed skal variansen af stokastikken også være høj for at statistisk stabile parametre genereres, hvis valgsættene bruges til modelestimering. Variansen skal imidlertid øges med forsigtighed, idet for store forøgelser resulterer i, at der bliver genereret ulogiske og overflødige ruter uden stigning af dækningsgraden. Hensyntagen til at individer har forskellige præferencer, gav store forbedringer af resultaterne, og der blev genereret dækningsgrader helt op til 98.8 % ved 80 % tærskel for overlappet. Dette understreger potentialet af den foreslåede metode samt vigtigheden af hensyntagen til så mange individuelle præferencer som muligt, når modellerne udvikles til at blive mere og mere detaljerede.

Ph.D. projektet foretog en grundig analyse af den oprindelige formulering, der ligger til grund for modellen baseret på stokastisk brugerligevægt (Stochastic User Equilibrium, SUE).

Denne analyse ledte frem til en konstatering af, at det velkendte problem med at specificere valgsættene i SUE modeller kan relateres til de underliggende distributioner, der typisk anvendes til repræsentation af de rejsendes unøjagtige information/opfattelse af netværket. Disse distributioner er ikke-begrænsede, hvilket medfører et behov for at fastlægge og tilskrive trafik til alle alternativer, uanset hvor uattraktive de er. En anden hyppigt anvendt model, nemlig den deterministiske brugerligevægt (Deterministic User Equilibrium, DUE), har derimod en indbygget mekanisme, der adskiller potentielt anvendte ruter fra ruter, der med sikkerhed ikke anvendes. Adskillelsen er imidlertid strikt, således at kun ruten/ruterne med præcis den mindste omkostning tillades anvendt. Disse styrker og svagheder ved SUE og DUE har ledt Ph.D. projektet frem til at foreslå to *nye modeltyper* samt tilhørende ligevægtsformuleringer. Begge nye modeltyper kombinerer styrkerne af SUE og DUE ved at tillade *konsistent* kombination af (i) valgsæt, der er i ligevægt men ikke er universelle samt (ii) fordeling af trafikken ifølge teorien omkring nyttemaksimering (Random Utility Maximisation, RUM). Den ene modeltype adskiller anvendte fra ikke-anvendte ruter via de underliggende distributioner. Den anden modeltype foretager denne adskillelse via funktioner, der definerer grænser for omkostningerne på benyttede/ikke-benyttede ruter.

Den sidstnævnte modeltype synes mest ligetil at anvende (modeltypen hvor funktioner definerer omkostningsgrænser) og Ph.D. projektet har foreslået generiske rute-baserede løsningsalgoritmer og konvergensmål for denne. Projektet testede og validerede forskellige varianter af løsningsalgoritmerne under brug af MultiNomial Logit og Path Size Logit valgmodellerne. Dette blev gjort på Sioux Falls netværket samt Sjællandsnetværket, som er et storskalanetværk. Et nyt konvergensmål verificerede ekstremt hurtig og stabil konvergens til en løsning i ligevægt, hvor der anvendes konsistente ikke-universelle valgsæt (på tværs af efterspørgselsniveauer, skalaparametre samt step-size strategier). Sammensætningen af valgsættene blev valideret ved sammenligning med 16,618 observerede ruter indsamlet i Sjællandsnetværket. Valideringen fastlagde tilmed, at alle tests formåede at reproducere vejtællinger meget nøjagtigt. Løsningsalgoritmerne er meget hurtige og de fundne løsninger, samt den underliggende modeltype, er adfærdsmæssigt meget realistiske. Dette gør, at disse er særdeles attraktive at benytte, når modellerne bliver mere detaljerede.

Ph.D. projektet har således givet bidrag til flere af de komponenter, der vedrører estimering og løsning af rutevalgsmodeller i storskalamodeller. Gennem dette har Ph.D. projektet bidraget til, at der kan foretages en konsistent og fuldt disaggregeret integration af trafikmodellens komponenter. Dette muliggør, at det sande potentiale af aktivitetsbaserede modeller kan udnyttes.

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1 INTRODUCTION

Transport models are becoming more and more disaggregate to facilitate realistic representation of individuals and their travel patterns. The overall model framework however needs to be specified accordingly, so that all model components operate at a fully disaggregate and integrated level. Only thereby is the true potential of the developments exploited, allowing to account for individual-specific household interactions, time-space constraints and preference structures. Trips can also be evaluated at a detailed address-to-address level, and thereby aggregation biases can be avoided. In line with this development, the present study facilitates the deployment of traffic assignment models in fully disaggregate activity-based model frameworks. The study focuses on large-scale applications, and presents theoretical and empirical contributions to the state-of-the-art for different components of traffic assignment modelling. Specifically, attention is given to the generation of choice sets for individual trips as well as the use of disaggregate data in the estimation of utility functions in route choice models. Attention is also given to the difficulty of obtaining theoretical consistency of the choice sets and route choice model in the solutions. Finally, the identification of such solutions via algorithms that are operational at a fully disaggregate level is also addressed.

1.1 BACKGROUND

The transportation network has been shown to highly influence the way people organise their lives in terms of residential location, work location, daily activity-patterns, vehicle ownership, choices of mode of transport, route choices etc. (e.g., Cascetta, 2001; Ortúzar and Willumsen, 2001; Badoe and Miller, 2000; Chakraborty and Mishra, 2013). Altering the network thereby impacts many people and is often very expensive, and the decision process should be supported by a transport model. It is, however, not a trivial task to specify such a model due to the complexity of the system, consisting of individuals which make rational or non-rational decisions in an always changing and very large network. Among the main elements to consider in the model specification is the selection of the underlying *behavioural framework* and the *aggregation level* on which the model operates. Also, the potential *utilisation of data* and the need for *computational feasibility* influence the model specification and thus the behavioural realism.

The models are becoming more and more *disaggregate* both on the demand side and the supply side. This is done to better capture the fact that individuals behave differently, depending on their individual preferences and knowledge about the transportation network. The demand side derives the demand for travel, and on this side models are becoming more disaggregate in their representation of individuals. Such models include, for instance, the daily time constraints of individuals and their preferences towards travel time reliability and delays due to congestion etc. The supply side determines the route choice of travellers and computes the network response in accordance with the demand. On the supply side, traffic assignment models are becoming more disaggregate in their representation of elements such as the spatial and temporal formation and dissipation of congestion as well as the preferences of individuals when conducting the route choice. The advancements towards more disaggregate and accurate models have been pursued quite independently within each of the sides of supply and demand (Lin et al., 2008). However, to get the full benefit and synergy of the advancements on both sides, the state of the art in each of these has to be closely integrated into an overall framework.

The disaggregation towards more behaviourally realistic models has been made possible by (i) easier access to large computational power, (ii) more readily available disaggregate data in an easily accessible format, such as digital databases containing network information or the home and work addresses of inhabitants, and (iii) the development of efficient survey methods enabling the collection of route level Revealed Preference (RP) data among many respondents by using GPS units. Access to *larger computational power* combined with the development of more efficient solution algorithms facilitate that computation times remain reasonable when disaggregating models to account for e.g. additional variables, individual preferences and congestion at a far more detailed level. The *increased availability of data* at a very disaggregate (often individual) level also facilitates an increase in the behavioural realism of disaggregate models. The data sources are often RP data, collected by GPS devices, Travel Card technologies, or Smart Phones. Implemented correctly, the data facilitate not only large-scale validation and estimation of finished models. Rather, the data should be thought into the actual development process in close connection/integration with the theoretical considerations such as maintaining convergence of the solution methods. Thereby the full potential of these increasingly available data sources is utilised. However, one fundamental issue lies in the size and quality of the datasets, which may contain millions of passively collected observations such as GPS logs. It is essential that some efficient post-processing procedures are available, to make such data operational at all levels of the model development and validation process. These should process the raw data in an automated manner and convert the results into the proper data formats (e.g., Chen et al., 2010; Schüssler and Axhausen, 2009; Bolbol et al., 2012; Stopher et al., 2008).

The realism of the transport model output relies heavily on the assumptions of the *behavioural framework* underlying the traffic assignment model. Most traffic assignment models adopt either the Deterministic User Equilibrium (DUE, Wardrop, 1952) or the Stochastic User Equilibrium (SUE, Daganzo and Sheffi, 1977) framework. The DUE has been widely applied in large-scale applications. One reason for this is its computational attractiveness in that it implicitly distinguishes between potentially used routes and definitely unused routes, thereby circumventing the computationally intractable enumeration of the universal choice set. The drawback of the DUE is, however, that it is based on an assumption of perfect information of the travellers and the modeller. This non-realistic assumption is removed by the SUE as this allows the adaptation of Random Utility Maximisation (RUM) models. RUM models allow the behaviourally realistic representation of perception errors of the traveller as well as the modeller. Consistent adaptation and integration of state-of-the-art RUM models into practical large-scale traffic assignment models, however, poses some distinct challenges. A major issue related to this lies in the generation of the set of routes used. Under the commonly adopted assumptions on the perception errors, RUM models suffer from the theoretical need to enumerate and assign traffic to the universal choice set. This is intractable for large-scale traffic assignment problems as the universal choice set may contain millions of routes for each relation. Consequently, SUE is usually found among a subset of the universal choice set in real-life applications and this induces a theoretical inconsistency with the underlying framework. The generation of the subset is not trivial, as the SUE does not provide any conditions/requirements to help distinguishing between relevant and irrelevant routes. Rather, the issue of sampling the choice sets in such a way that they are composed of all relevant alternatives, while leaving out non-relevant alternatives, is left to the modeller.

1.2 AIMS AND MAIN CONTRIBUTIONS

Overall, the present study aims to facilitate increased behavioural realism of large-scale transport models. When compared to existing models, fully disaggregate models seem to have the potential to improve the realism through a more detailed representation of individuals and their travel patterns. This is however only true under the correct specification of and integration across model components. The study focuses on the *traffic assignment model* and aims to address the potentials and challenges that emerge for this when adopting a fully disaggregate approach.

Specifically, the study aims to:

1) Investigate actual path choices of public transport users through the proposal and assessment of a choice set generation method applicable in a large-scale multi-modal network;
2) Investigate the possibility to process GPS data into a format that enables its use in traffic assignment models;
3) Investigate the possibility to use RP data to provide evidence on the value of congestion and the value of reliability;
4) Explore the possibility to obtain theoretically consistent equilibrated SUE-like flow solutions among equilibrated non-universal choice sets for large-scale applications.

The following subsections clarify each of the four aims and state how and in which papers they are addressed.

1.2.1 CHOICE SET GENERATION METHOD

A lot of research is focusing on how to generate route choice sets that consist of all relevant alternatives, leaving out non-relevant and redundant alternatives (e.g., Bovy, 2009; Ben-Akiva et al., 1984; Prato and Bekhor, 2006; Frejinger, 2007). The aim of the present study is to contribute to this research by proposing a choice set generation method which is in line with the development towards disaggregate models, by accounting for taste heterogeneity and perception errors. The model should be applicable to general transport networks, but the aim is to validate it by using 5,131 observed route choices collected in the highly complex multi-modal Greater Copenhagen area public transport network.

Rasmussen et al. (2014a) address these aims. This work develops and validates a simulation-based choice set generation method that can account for taste heterogeneity and perception errors. By evaluating alternative ways of specifying the stochasticity and its level, it is found that the level of stochasticity should be high to provide statistically efficient parameter estimates when the choice sets are used for estimation. The level of stochasticity should, however, be introduced with parsimony; significant increases translate into the generation of redundant and counter-intuitive paths and no considerable improvement in coverage. Adding heterogeneity across travellers into the model improves the results considerably, and induced coverage levels up to 98.8% at an 80% overlap threshold. These results show the importance of accounting for as much individual heterogeneity as possible as models become more disaggregate.

1.2.2 Deducing detailed trip information from raw GPS data

The collection of individual-based GPS data is a valuable RP survey method to investigate the travel patterns and route choices of travellers. This is because it provides a cheap way of collecting very disaggregate information on actual route choices and travel patterns over long periods of time (e.g., Nielsen, 2004; Liu et al., 2010; Bierlaire et al., 2013). In order to exploit the potential of the data, it is, however, essential that automated methods are available to post-process the large raw datasets. The present study aims to propose such a method to identify trips, trip legs, and assign the most probable mode of transport. A GPS dataset collected as part of the 'Analysis of activity-based travel chains and sustainable mobility' (ACTUM) project should be used to validate the method. This dataset consists of individual-based 3-day GPS logs from 183 respondents, collected in the highly complex large-scale Greater Copenhagen area multi-modal network.

Rasmussen et al. (2014b) address these aims. A combined fuzzy logic- and GIS-based algorithm is developed to process raw individual-based GPS data with no additional information requested from the respondent. The validation showed very promising results, especially when compared to existing algorithms. The findings highlight the possibility to use individual-based GPS units for travel surveys in real-life large-scale multi-modal networks. The processed GPS data are used in the case studies of Prato et al. (2014) and Rasmussen et al. (2014d), thereby demonstrating its applicability to improve the behavioural realism of traffic assignment models.

1.2.3 Value of Congestion and Value of Reliability

Travel time due to congestion and travel time reliability are recognised as two major influences on choices related to travel. Several studies have addressed the issue of evaluating the Value of Congestion (VoC) and the Value of Reliability (VoR) (e.g., Train, 1976; Noland and Small 1995; Li et al., 2010; Wardman and Ibánez, 2012). Most studies, however, used Stated Preference (SP) survey methods for the evaluation (for an overview, see Wardman and Ibánez, 2012). The present study aims to contribute to the existing literature on the topic by proposing an approach that utilises the increasingly available disaggregate RP data to estimate the VoR and VoC concurrently. Additionally, the study aims to demonstrate the applicability of the approach and to provide evidence on the value of the VoR and VoC by estimating these using GPS data.

Prato et al. (2014) address these aims. The work proposes a mean-variance model based on RP data which considers the free-flow travel time, congestion travel time, and travel time reliability concurrently. The model is successfully applied to estimate mixed Path Size Logit (PSL, Ben-Akiva and Bierlaire, 1999) models using a sample of 5,759 observations in the peak period and a sample of 7,964 observations in the off-peak period. The results verify that congestion and reliability highly influence route choices. Therefore, disaggregating the models to account for this improves their behavioural realism. The results also show that the marginal rate of substitution between the travel time reliability and the total travel time does not vary across the peak and off-peak periods and that, as expected, the VoR and VoC are higher in peak periods than outside peak periods.

1.2.4 New Modelling Framework

SUE suffers from the need to enumerate the full universal choice set. This is not feasible for large-scale applications, and SUE is therefore often approximated by distributing flow among a pre-specified set of paths that are not trivial to specify (e.g., Prato and Bekhor, 2007; Bliemer and Bovy, 2008). The DUE allows unused routes at equilibrium, but fails to account for mis-perceptions of travellers. The study aims to remove these limitations of the SUE and DUE by seeking to propose an alternative modelling framework. This should facilitate flow distribution according to RUM on consistently equilibrated, but non-universal, choice sets. Furthermore, the study aims to contribute with the proposal and large-scale validation of a corresponding generic solution algorithm.

Watling et al. (2014) and Rasmussen et al. (2014cd) address these aims. Based on an analysis of the existing frameworks, two new model frameworks and corresponding equilibrium formulations are introduced. One model allows distinction between used and unused routes based on the distribution of the random error terms, while the other model allows this distinction by posing restrictions on the costs of used/unused routes. Both models fulfil the aim by allowing an equilibrated flow solution according to some RUM among a non-universal but *equilibrated* choice set. Generic path-based solution algorithms and convergence measures are proposed for the model that seems the most straightforward to apply. Different variants of the algorithms are validated on the large-scale Zealand case study. These are found to be highly computationally attractive and provide extremely fast convergence to an equilibrated solution with reasonable choice set sizes and composition. The choice set composition is evaluated using, among other, the GPS data collected as part of the ACTUM project.

1.3 OUTLINE

The remainder of the present thesis is structured as follows. Section 2 starts by giving an introduction to transport models, the demand and supply sides, and the importance of correct integration between these. Thereafter the section narrows the focus to the traffic assignment model by giving a short description of each of its components. Conclusions of the PhD study and future research possibilities are presented in section 3. The six papers that constitute the contributions of the PhD study can be found in appendices 1-6, and these should be read before reading section 3. Note that the working paper (Rasmussen et al., 2014d) is longer than a normal journal paper. This is because we have prioritised to report the results of a range of different tests, to demonstrate the applicability and attractiveness of the solution algorithms and model framework.

All the papers adopt (with minor variations) the same case study area and utilise some RP data source (except Watling et al. (2014) which do not conduct any large-scale applications). The RP data source is either diary data (Rasmussen et al., 2014a) or GPS data (Prato et al., 2014; Rasmussen et al., 2014bd). Appendix 7 provides additional details about the RP data sources. The appendix also describes the demand matrices and digital network representation that were used in the case studies. Some details regarding the preparation of the data used in Prato et al. (2014) are given in Appendix 8.

2 DISAGGREGATE TRANSPORT MODELS

This section gives a short introduction to transport models. Section 2.1 sets off with a description of the general transport model structure, followed by a brief introduction to the development towards disaggregate models on the demand side as well as the supply side. Hereafter, the section focuses on the vital need for close integration between the two sides. The thesis contributes to the traffic assignment model, which resides in the supply side. Section 2.2 is therefore dedicated to a general description of the elements of the traffic assignment model.

2.1 TRANSPORT MODEL STRUCTURE

2.1.1 MODEL STRUCTURE

A real-life transportation system is complex in multiple dimensions by consisting of individuals which make complex rational or non-rational behavioural decisions in an always changing and very large network. Transport models seek to represent this complex system as a mathematical problem constructed based on simplifying assumptions about the real-life system. Despite the simplifications, transport models often handle very complex mathematical problems that are not possible to solve by standard mathematical problem solvers (among others, Ben-Akiva and Lerman, 1985).

In practical problem solving, the highly complex problem is often split into several modelling parts. For example, the well-known four-stage model splits the problem into the elements of trip generation, trip distribution, mode choice and traffic assignment (see e.g., Ortúzar and Willumsen, 2001). An alternative way to look at the problem is from a demand-supply perspective, where the first three elements of the four-stage model would reside on the demand side and the traffic assignment on the supply side. In general, the demand side derives the demand for travel in terms of e.g. number of trips of a certain type and mode of transport between spatial locations, possibly at certain points in time. The supply side, constituted by a traffic assignment model, then applies a route choice model and derives the network performance in terms of link and route travel costs resulting from the demand. The two sides are mutually dependent, as the travel demand affects the network in terms of e.g. congestion in the network, and the network performance affects the travel demand by e.g. reducing demand as travel costs increase. Each of the two sides, therefore, constitutes problems that are 'conditional' on the other. To find a consistent solution, transport models often adopt a procedure which iterates between the two sides in some gradual way. In such an approach, the iteration scheme is continued until some convergence measure is fulfilled. This could e.g. specify that the derived travel demand should, at a certain threshold, be identical across two consecutive iterations.

2.1.2 Disaggregate models on the demand and supply side

In the development of transport models, there seems to be a clear tendency that the components of transport models are becoming more and more disaggregate. This is done to facilitate better representation of the real-life system and consequently improvement of the behavioural realism of the transport models. The development is undertaken on the demand side as well as the supply side.

On the demand side, the traditional *statistically*-oriented trip-based framework has been the most widely adopted approach because of its use in the four-stage model (Ortúzar and Willumsen, 2001). While many of the models that are developed today still use this approach, more and more models adopt the disaggregate and more *behaviour*-oriented activity-based model framework (e.g., Axhausen and Gärling, 1992; Jones et al., 1993; Bhat and Koppelman, 2003; Arentze and Timmermans, 2004; Bhat et al., 2004; Vovsha and Bradley, 2006; Lin et al., 2008; Hansen, 2014). This framework stems from a realisation that demand for travel is, possibly with the exception of sightseeing, derived from the more fundamental need to participate in activities. Such activities could be work, leisure, shopping etc. The locations of the activities are usually spatially distributed and the need to move between these locations induces the demand for travel.

Overall, the activity-based paradigm and models seem conceptually more appealing than the traditional trip- and tour-based methods for several reasons: (i) focus is on sequences and patterns of activities and travel rather than on individual trips; (ii) various activity-travel decisions are recognised as linked rather than as independent; (iii) emphasis is on individual-level travel patterns rather than on aggregate trips at a zone-to-zone level; (iv) intra-household interactions are typically incorporated and inter-personal and intra-personal consistency measures are considered within the model; (v) time is typically treated as a continuum or at least a detailed temporal dimension is accounted for within the model; (vi) space-time constraints on activities and travel are included within the model (Lin et al., 2008).

The activity-based paradigm fits well into a micro-simulation platform, as it is based on disaggregate complex structures and on behaviour of individuals and households. Several implementations in metropolitan areas have combined an activity-based framework with micro-simulation on the demand side. Among others Portland (see, e.g., Bowman et al., 1998), San Francisco (e.g., Bradley et al., 2001; Jonnalagadda et al., 2001), New York (e.g., Vovsha et al., 2002), Columbus (e.g., Vovsha et al., 2003; Vovsha et al., 2004), Dallas (e.g., Bhat et al., 2004; Pinjari et al., 2006), Toronto (Gao et al., 2010), Southern California (Goulias et al., 2011) and Tel-Aviv (Bekhor et al., 2011).

The activity-based models operate in continuous time (or with short time intervals) as well as at an individual and often address-to-address level. Ideally, the models would therefore be able to output very disaggregate trip information to the traffic assignment model, though not done very often. Moreover, this output would be address-to-address trip tables specifying departure time and mode of transport. The trip tables would also specify trip purpose and socioeconomic characteristics which, in the traffic assignment model, can be associated to specific route choice parameters of the individual trips.

On the supply side, the traditional approach for large-scale applications has been static zone-to-zone-based traffic assignment. This is also highly linked to its use in the widely adopted four-stage model. The approach typically models 'average' network conditions without considering the temporal dynamics with which congestion evolves over time. The traveller representation is also done at an aggregate zone level. In such a system each zone covers hundreds, thousands or even millions of inhabitants, dependent on the spatial extent of the study at hand. Travel preferences are also handled at an aggregate level, possibly by trip purpose, and in some applications the choice function contains only the travel time. Advances towards more disaggregate representations have been proposed, leading to an improvement in behavioural realism. The disaggregation has been conducted in multiple dimensions. Today, models exist that better capture the temporal dynamics (congestion built-up and dissipation e.g. around intersections) through more disaggregate time representation in dynamic or pseudo-dynamic models (e.g., Ben-Akiva et al., 2012; Florian et al., 2008; Balakrishna et al., 2012). The behavioural realism of the models has also been improved through the development of disaggregate choice models and choice set generation methods (section 2.2). These facilitate the use of advanced choice functions for multiple trip purposes and the representation of taste heterogeneity across individuals (e.g., Nielsen, 2000; Bovy and Fiorenzo-Catalano, 2007; Rasmussen et al., 2014a).

2.1.3 Integration of model components

Considerable advancements towards more disaggregate and realistic models have occurred on the supply side and the demand side. The progress in the two streams seems, however, to have been pursued quite independently, while less research has focused on how to consistently integrate state-of-the-art from both sides into a unified conceptual framework (Lin et al., 2008). Using only one of these would fail to exploit the true potential of either approach, and inconsistent results would most likely occur. On the one hand, using an activity-based paradigm with a static zone-based assignment algorithm, not considering temporal dynamics, would negate much of the advantages of predicting travel patterns at an individual level and in continuous time. On the other hand, using a trip-based approach that provides travel demand over a limited number of time periods and zones to develop the inputs for a dynamic traffic assignment model would cancel out the reasons for which dynamic traffic assignment models have been developed.

To obtain consistent results and to actualise the benefits of advancements on the demand and supply sides, the conceptual model framework must be designed accordingly. The ACTUM research project seeks to address this by developing a disaggregate transport model which combines state-of-the-art on disaggregate modelling on both sides into a consistent joint modelling framework (Hansen, 2014). The PhD study has been conducted within the ACTUM project, and several of the contributions of the PhD study are planned to be integrated directly into the ACTUM model framework. The Greater Copenhagen area is used as case study in the ACTUM project.

The framework of the ACTUM project uses an individual-level activity-based model to generate the demand and combines this with an individual-based dynamic or pseudo-dynamic approach on the assignment side. Such an approach that operates at a disaggregate level on both sides induces some distinct advantages. Primarily, consistency with no loss of information about the trips across the demand and assignment models can be obtained. This is possible as information regarding the individual trip and individual traveller is transferred directly at the disaggregate level between the components (no potential aggregation biases). Thereby, the explanation and prediction abilities of the model are increased. The remainder of this section focuses on the behavioural and computational advantages that arise on the traffic assignment side when model components are closely integrated at a fully disaggregate level.

From a behavioural perspective, the disaggregate approach allows the assignment framework to consider individual preference structures which depend on individual attributes such as value of time, income, age, trip purpose, time-of-day, etc. Taste heterogeneity needs not be considered in the choice set generation or the route choice, since neither the zones nor the population are to be considered in the traffic assignment component (no need to account for distributions of preferences across the population in a certain zone). Non-linear terms may also be considered in the utility functions, because there is no aggregation process that will affect the assignment results. This facilitates higher precision in the representation of preferences of individuals and thereby improved behavioural realism. Having no need to account for taste heterogeneity in the assignment also allows more flexibility in the selection of the choice model; it becomes more attractive to apply choice models for which closed-form expressions are available for the choice probabilities (such as the PSL model). This links very nicely to the solution algorithms proposed and tested in Rasmussen et al. (2014cd). The tested variants adopted the MultiNomial Logit (MNL) and PSL choice models. These do not account for taste heterogeneity, but allow (in the PSL implementation) for the important correction for path overlapping. Also the underlying RSUE/RSUET framework presented in Watling et al. (2014) and Rasmussen et al. (2014d) seem especially attractive to apply to disaggregate transport models. This is because they allow for a consistent adaption of advanced preference structures in the choice component as well as in the mechanism which, based on actual rather than perceived costs, distinguishes used and unused paths.

From a computational perspective, an individual-based approach does not necessarily induce higher calculation complexity in the assignment than a zone-based approach. This is especially the case when the assignment model operates at a disaggregate temporal level, as shall be explained in the following. The complexity of an individual-based approach in the assignment model is (roughly) independent of the temporal aggregation, but only depends on the number of trips performed. The number of trips performed remains the same when the temporal dimension is added to the model, and only some additional updating of the speed-flow or flow-density functions are required. The complexity of the zone-based approach is independent of the number of trips performed, but instead highly dependent on the temporal aggregation, in addition to the number of zones and user classes. The calculation complexity increases significantly when adding the time dimension to the matrix-based (zone-based) assignment; in a pseudo-dynamic assignment, complexity increases by the number of time intervals, whereas either a cell-based or a row-based approach is needed in the full dynamic case.

The road assignment component of the Danish National Transport Model (Landstrafikmodel, hereafter denoted as LTM) can serve as an illustrative example of this issue of calculation complexity (see details about the LTM in Rich et al. (2010)). The model is zone-based (907 zones), and the road assignment covers a total of 19 user classes. This results in 15.6 million relations to be investigated in the zone-based system (907x907x19). However, the model is pseudo-dynamic by dividing a 24h period into 10 time periods. This generates a total of 156 million relations to be investigated in the zone-based system, when no matrix thinning is done. However, the total number of trips conducted is 6.6 millions, which highlights that the complexity of an individual-based approach would be significantly lower in this example, even if the LTM had been a static model. Note that there exist path search algorithms that identify the shortest path from an origin to all destinations (e.g., Dijkstra, 1959). These potentially reduces the computational needs for the path-search, but still the flow allocation and network allocation mechanisms would have to consider all relations..

The use of an individual-based assignment model causes an issue to arise with respect to the re-evaluation of the demand in each iteration of the transport model. The re-evaluation on demand side uses level-of-service measures of chosen as well as non-chosen alternatives. This information is readily available in matrix-based assignment models, as the non-chosen alternatives are the cells other than the one corresponding to the chosen alternative. The level-of-service of non-chosen alternatives is, though, not readily available in the individual-based assignment model. It cannot be calculated as the average over trips either, because these other trips are performed by different persons with different preferences and hence different utilities.

This challenge could be solved using ghost travellers (Teklu et al., 2007). These are loaded onto the network along with the actual travellers, however they do not contribute to congestion. This may seem computationally demanding, but there is no need to load the ghost travellers in every iteration of the solution of the assignment model; each of the ghost travellers only needs to be loaded once (e.g., all-or-nothing assignment), namely when the network equilibrium has been found. The use of ghost travellers allows reproducing the individuals with their characteristics and preferences in order for the level-of-service of the alternatives to be accurately represented (i.e. no aggregation bias). The demand side may specify the ghost travellers, and thereby obtain the cost of travelling over relevant alternative destinations and with relevant alternative modes.

2.2 TRAFFIC ASSIGNMENT

There exists a number of different ways to specify a traffic assignment model, each of which leads to different theoretical properties as well as implications for real-life applications. Some specifications may have well-founded and proven theoretical properties but turn out to be computationally infeasible for large-scale applications. Other specifications may allow a solution to be found quickly for large-scale applications, but at the expense of compromising the theoretical consistency. In general, there are six overall issues to consider in the specification of a traffic assignment model; (i) route choice set generation, (ii) specification of the choice function, (iii) definition of equilibrium, (iv) solution approach, (v) level of network aggregation, and (vi) level of time aggregation. It is important to note that these elements are highly interrelated. For example, specifying a dynamic time representation requires the definition of the equilibrium also to have a dynamic formulation. The main contributions of this PhD study lies within the elements (i)-(iv), and the remainder of this section are dedicated to a brief introduction to these.

2.2.1 ROUTE CHOICE SET GENERATION

Route choice is often modelled using a discrete choice model (section 2.1). A discrete choice model requires a set of alternatives to be available and models the choice of an alternative within this set. All possible alternatives can be identified and distinguished clearly in most travel-related choice situations, such as e.g. in the choice of mode of transport. This is not as straightforward for route choice modelling; typically, in transport networks a very large number of alternatives exist which are hard to enumerate and distinguish, even in small networks. On top of this, literature has found that the size and composition of the choice sets influence estimation results and prediction abilities (Prato and Bekhor, 2007; Bekhor et al., 2008a; Bliemer and Bovy, 2008; Rasmussen et al., 2014). This makes the importance of the choice set generation component even more evident. Thus, the specification of the choice set is not trivial, and a literature review has not identified any formal or mathematically founded definition of how the choice set should be composed. Rather, the most commonly adapted statement about the choice set composition seems to be somewhat vague by stating (possibly in some alternative formulation) that 'it should consist of all relevant alternative routes, leaving out irrelevant routes' (Bovy, 2009). This statement lacks a formal and theoretically well-founded definition of the choice set composition, and this has motivated much of the work undertaken during the PhD study (Watling et al., 2014; Rasmussen et al., 2014cd).

Route choice set generation is generally conducted for one of three purposes; (i) generation of (unchosen) alternatives to use for the estimation of parameters in disaggregate route choice models; (ii) investigation of alternatives, in terms of the number of alternatives, their attributes, variety etc.; (iii) generation of a set of routes that are allocated traffic in a traffic assignment model (Prato, 2009). Algorithms for traffic assignment typically generate the choice sets either *explicitly* prior to the application of a flow allocation algorithm to find equilibrium, or *implicitly* as the algorithm iterates towards equilibrium.

Performing the choice set generation *explicitly* sets some strict requirements to the prior knowledge of network performance in its congested state (at equilibrium); near equilibrium link travel times should be used when generating the choice set. Using free-flow travel times to generate alternatives may not generate representative choice sets. The knowledge of congested travel times can stem from several sources, e.g. real-life link travel times/speeds obtained from GPS data. Such information is however not easy to observe on the full network or, especially, when models are used to forecast future situations or effects of policies.

Proponents of *explicit* choice set generation argue for the behavioural rationale of separating the generation of considered alternatives from the choice between these (e.g., Cascetta and Papola, 2001; Bovy, 2009). Bliemer and Taale (2006) highlight some advantages of *explicit* choice set generation, e.g. that such an approach allows a more flexible specification of the choice model for the allocation of flow. *Explicit* choice set generation may, however, involve some computational challenges, as all paths enumerated have to be stored in memory and continuously re-assigned flow (path-based solution algorithm required, see section 2.2.4). This can be avoided by using *implicit* choice set generation methods, which are very closely integrated with/into the mechanism allocating flow to solve for network equilibrium. *Implicit* choice set generation also has the advantage that paths are generated as the flow is assigned to the network. This means that the routes are generated while considering congestion (without the need of additional information from e.g. GPS devices).

Ideally, the same specification of the preferences should be used across the choice set generation component and the subsequent choice model (estimation or flow allocation) component. Only thereby does the 'hypothesis' about traveller preferences (used in the path generation) become consistent with the preferences that are actually estimated based on these. Some difficulties, however, arise for instance when using path-specific attributes in the choice model (see Rasmussen et al. (2014a) for a discussion of this), and it is only possible for cases where utility functions are used in the choice set generation process.

2.2.2 Specification of the choice function

The choice function (or utility function, or cost function) is a vital component of most traffic assignment models. The function expresses the trade-off between the attributes/characteristics that are assigned by the travellers when they evaluate the attractiveness of alternative routes and make their route decision. Numerous studies have investigated which elements the choice function should include, and how they should be weighed relative to each other. Studies have found that free-flow travel time, congested travel time, travel time reliability, distance covered, monetary travel costs, and other factors related to the level-of-service of alternative routes highly influence the route choice (e.g. Bekhor et al., 2006; Brownstone and Small, 2005; Prato et al., 2014; Rasmussen et al., 2014a; Rich and Nielsen, 2007; Wardman and Ibáñes, 2012; Anderson et al., 2014). The choice function may also consider socioeconomic and demographic characteristics such as income level, gender, and season ticket ownership (Vrtic et al 2010; Anderson, 2013). These and other characteristics may also be used to disaggregate the models to consider different driver categories and trip purposes (e.g., Dafermos, 1972; Mahmassani, 2001; Rich et al., 2010). Furthermore, Raveau et al. (2011) found that variables related to network topology, e.g. directness of routes, also have a significant impact on the route choice of metro users in Santiago, Chile.

The choice function may or may not consider stochastic elements, depending on the assumptions of the underlying theoretical framework. Leaving out consideration of stochastic elements typically leads to DUE, whereas SUE typically emerges by the inclusion of stochastic elements in the form of a distributed error-term or distributed parameters (section 2.2.3). The error-term represents the non-complete knowledge of the traveller (and modeller), i.e. that the route choice is based on what the travellers perceive the costs to be, rather than what they actually are.

Several studies have shown an interest in accounting for taste heterogeneity and heteroscedasticity in route choice models. This is done by letting the parameters of the choice function follow some distribution. Ben-Akiva et al. (1993) estimated a model with two alternatives, where the time coefficient is log-normally distributed. Dial (1997) formulated a traffic assignment algorithm, where drivers have different perceptions about travel times and costs because of habitual behaviour, taste differences, or information collection. Nielsen (2000) proposed a stochastic public transport traffic assignment model considering differences in the choice functions of passengers. The study argued that log-normal and gamma distributions are suitable to simulate the heterogeneous preferences of travellers. Han et al. (2001) used uniform and normal distributions for delay and travel time in high congestion conditions to model SP games of pairwise route choices and commented that the log-normal distribution does not produce satisfactory results. Jou (2001) estimated a model with a normally distributed travel time parameter to investigate the impact of pre-trip information on route choice behaviour. Nielsen et al. (2002) estimated a model for different driver categories and tested both normally and log-normally distributed coefficients for travel time and cost. Nielsen (2004) presented an SP and RP experiment about road pricing, where the RP data were collected by GPS units in vehicles. Models that accounted for heterogeneity in drivers' responses to pricing schemes were estimated. Anderson et al. (2014) collected RP data among public transport users and estimated various PSL models as well as a mixed PSL model to account for taste heterogeneity. The heterogeneity was introduced by letting some of the parameters, associated to travel time, follow a log-normal distribution. Prato et al. (2014) used RP data collected by GPS to estimate a mixed PSL model. Several different distributions were considered for parameters associated to cost, travel time components (free-flow, congestion, reliability), and turns. The parameters expressing taste heterogeneity for turns and costs were not significantly different from zero, and log-normal distribution of travel time components were found to give the best model fit.

Moving from zone-to-zone based approaches to an individual-based approach induces some distinct advantages in the specification of the choice function. In an individual-based approach, the question of distributed parameters is not an issue, because the choice function has a specific form and parameter-specification for each individual (as discussed in section 2.1.3). This function is 'transferred' from the demand side, and is defined for each individual and each trip for direct use in the assignment model. This facilitates the use of choice functions which do not require simulation of the choice probabilities. Not having to aggregate in the individual-based approach also allows exploring possible non-linearities in the choice function.

2.2.3 Definition of equilibrium

Equilibrium models are formulated based on the general assumption that travellers take their experiences into account when making decisions. Several types of equilibria are defined in literature, based on different assumptions on the behaviour and knowledge of travellers (and the modeller). The two most commonly applied equilibrium frameworks (DUE and SUE) will be introduced briefly below. An elaborate analysis of the two frameworks can be found in Watling et al. (2014).

The widely applied DUE defines a stable situation only when no traveller in the network can reduce his/her travel time (or in its general formulation: generalised cost) by unilaterally selecting an alternative route (Wardrop, 1952). The DUE is thus based on the assumption that each traveller has complete and accurate information on all paths and exactly knows their characteristics in terms of level-of-service. It is furthermore assumed that the modeller has perfect knowledge about the preferences of the travellers and the performance of the network. The assumption of perfect information of the travellers and the modeller may, however, not hold in reality because of e.g. the complexity of the network. This realisation has led to the SUE model. The SUE defines a stable situation only when no traveller in the network believes that his/her travel time (or in its general formulation: generalised cost) can be reduced by unilaterally changing his/her route (Daganzo and Sheffi, 1977). The SUE implies that travellers may perceive route costs differently than they actually are, and that every traveller takes the route which he/she perceives has the minimum cost. This concept of perception errors also allows the models to represent the non-perfect knowledge of the modeller, e.g. the failure to include a relevant component in the specification of the choice function.

The DUE is computationally attractive by implicitly 'specifying' the set of possibly used routes and preventing the use of many unattractive routes. The cut-off is, however, strict and induces the risk of leaving many possibly attractive paths unused. The SUE model allows some sort of 'smoothing' of this condition, by also assigning flow to paths being slightly costlier than the cheapest. It is, however, not only paths being slightly costlier that are required to be used: under the often-used assumptions of unbounded support of the underlying distribution(s), SUE models will assign some flow to all routes, no matter how costly they may be (see Watling et al. (2014) for more details on this). This requires full path enumeration, which is computationally infeasible for large-scale applications. In this way, SUE solutions are also affected by the addition of any paths (e.g. new network link), even if completely irrelevant for the trip being made. The issue is further complicated by the fact that only a sub-set of all possible routes will be identified in most practical SUE solution algorithms (section 2.2.4). Specifying this sub-set is a non-trivial task to be undertaken without any support from the underlying model framework (section 2.2.1). These characteristics of the DUE and SUE have led the PhD study to introduce alternative modelling frameworks (Watling et al., 2014; Rasmussen et al., 2014cd). These new generic frameworks combine the strengths of the DUE and SUE by permitting the *consistent* combination of (i) unused paths and (ii) the use of RUM models for the choice between used paths.

2.2.4 Solution approach

As the network size grows, it soon becomes very complex to find a solution that satisfies the equilibrium conditions of the DUE and, especially, the SUE. Solutions cannot be identified analytically for large-scale systems, but are rather found via some iterative procedure (Ben-

Akiva and Lerman, 1985). The procedure typically consists of three steps that iterate until the desired DUE or SUE is found: (i) use the present network performance to identify an auxiliary solution, (ii) shift flow towards the auxiliary solution, and (iii) load the new solution onto the network to obtain the resulting network performance. Some solution algorithms require step (i) to search for and include new relevant routes. Step (i) and (ii) reallocate the flow towards the currently best route(s) (DUE case) or towards the desired route choice distribution (SUE case, e.g., Dial, 1971; Sheffi, 1985; Cascetta et al., 1996). For stability reasons, this flow reallocation is typically realised in some gradual way that reduces the dynamics of the iterations (step (ii)). In large-scale applications the iteration process is not continued until full convergence is found. Rather, it is continued either until a pre-specified iteration number has been reached or until the solution found resembles the equilibrium solution within a certain threshold. Other applications iterate until link flows have stabilised, which, however, does not guarantee convergence (see Rasmussen et al. (2014c) for a further discussion of this). I.e. one might add a fourth step to the procedure above: (iv) terminate if stopping criterion is met.

Traditional SUE and DUE solution algorithms are formulated and solved either in the space of path flows or in the space of link flows. Link-based formulations are attractive by not having to store the paths, which is computationally demanding and requires a large computer memory. Additionally, higher congestion often induces more routes to be generated and thus increases the computational requirements and memory usage for path-based algorithms. Link-based algorithms do not require more memory as congestion increases, and the computation time per iteration is typically also the same (Rasmussen et al, 2014d). However, path-based formulations allow easier checking for and ensuring the consistency of the choice sets. Path-based formulations also allow for the inclusion of path-specific attributes in the choice function (e.g. congestion toll, ticket cost, or correction for path overlapping).

As mentioned earlier, the specification of the choice set is not-trivial (section 2.2.1 and section 2.2.3). This is especially evident for the SUE, due to the theoretical need to enumerate the universal choice set. Solution algorithms generate the choice set either implicitly in or explicitly prior to the flow allocation. Several algorithms exist for explicit choice set generation, such as e.g. probabilistic generation techniques (Cascetta and Papola, 2001; Frejinger, 2007), breadth first search with network reduction (Rieser-Schüssler et al. 2013), contrained enumeration or branch and bound algorithms (Friedrich et al., 2001; Hoogendoorn-Lanser et al., 2007; Prato and Bekhor, 2006; Van der Zijpp and Fiorenzo-Catalano, 2005) and various deterministic or stochastic shortest path algorithms (Dijkstra, 1959; Sheffi and Powell, 1982; Ben-Akiva et al., 1984; Akgün et al., 2000; Hunt and Kornhauser, 1997; Lombard and Church, 1993; Van der Zijpp and Fiorenzo-Catalano, 2005).

Implicit choice set generation is often done by, in each iteration, performing shortest path searches based on the travel costs obtained in the previous iteration. Two examples of widely applied link-based SUE solution algorithms with implicit choice set generation are the Method of Successive Averages All-or-Nothing assignment (MSA AoN, Sheffi and Powell, 1982) and the STOCH algorithm of Dial (1971) in combination with MSA (Sheffi, 1985). The algorithm combining STOCH and MSA can be used to solve the MNL SUE problem, without requiring simulation. The link-based MSA AoN can be applied to find any SUE solution where the distribution of the error term can be consistently aggregated from link level to path level. The MSA AoN however requires simulation. Fosgerau et al. (2013) proposed a link-based method to obtain a solution equivalent to the MNL with infinitely many alternatives. This is found without the need for path identification, and can be combined with e.g. MSA to find a corresponding SUE solution. Several path-based SUE solution algorithms are also available (e.g., Chen and Alfa, 1991; Damberg et al., 1996; Bekhor and Toledo, 2005; Zhou et al., 2012). These often solve for a solution among a pre-specified choice set.

Turning to the solution of DUE problems, several solution algorithms have been presented in literature (among many: Dafermos and Sparrow, 1969; Larsson and Patriksson, 1992; Han, 2007; Florian et al., 2009; Kumar and Peeta, 2010; Mounce and Carey, 2011). Some of these are very efficient and computationally attractive by e.g. not requiring simulation. Additionally, the problem of having to pre-specify a subset of routes to be used can be avoided (see Watling et al., 2014). Several link-based DUE solution algorithms exist. The link-based MSA AoN algorithm can be applied to solve the DUE problem (without simulating the link impedances), and the well-known algorithm by Frank and Wolfe (1956) is also a link-based DUE solution algorithm. This approximates an optimal step length in every iteration (and is thus more computationally demanding per iteration) and converges in fewer iterations than the fixed step-length MSA AoN.

As for the SUE, a lot of research goes into path-based assignment algorithms for DUE. They include the path-based Disaggegate Simplicial Decomposition (DSD, Larsson and Patriksson, 1992) algorithm and the Gradient Projection (GP, Bertsekas and Gafni, 1982) algorithm. These two are both similar in concept to the Frank-Wolfe algorithm, as they also approximate the optimal step length in every iteration. The DSD and GP have furthermore been extended to solve certain SUE problems, see Damberg et al. (1996), Bekhor and Toledo (2005), Bekhor et al. (2008b), and Zhou et al. (2012). Another branch of path-based DUE solution algorithms which are gaining more and more interest in literature is the algorithms based on path-swapping. This branch of solution algorithms has shown fast and stable convergence patterns when applied to small-scale networks (Han, 2007; Carey and Ge, 2012; Mounce and Carey, 2011; Nie, 2003). To the knowledge of the author, they have however not been tested in large-scale applications.

3 Conclusions and future research

The PhD study presents theoretical and empirical contributions to the state-of-the-art with the aim of facilitating the deployment of traffic assignment models in fully disaggregate activity-based model frameworks. Use of real-life data and large-scale application have been a priority of the study to allow the contributions to be adopted directly into operational real-life models (such as the ACTUM model). The study has successfully achieved this: the proposed methodologies have been shown to be operational and computationally attractive at a fully disaggregate level for large-scale applications and large-scale data sources. The contributions relate to (i) the generation of choice sets for individual trips as well as the use of disaggregate data in the estimation of utility functions, (ii) the difficulty of obtaining theoretical consistency between choice sets and route choice model in the solutions, and (iii) the generation of such consistent solutions via algorithms that are operational at a fully disaggregate level.

Moving to a fully integrated individual-based approach poses some challenges and present great potential. A main advantage is that the utility functions become individual-based. This removes the need to account for taste heterogeneity across travellers in the traffic assignment model. The RSUET model framework and solution algorithms thereby become especially attractive to apply; taste heterogeneity cannot be accounted for, but a rich specification of the utility function is allowed. Moreover, state-of-the-art choice models such as the PSL can be applied, and the utility function can account specifically for elements such as the different components of the travel time as well as non-linearities.

The RSUET leads not only to consistent, equilibrated and behaviourally realistic solutions, but also removes the need for simulation. This reduces the calculation time and, importantly, removes stochasticity in the outputs (which potentially has a major implication for project appraisals). Feasibility in terms of calculation time seems to be a major challenge for disaggregate models. The study has however shown that the new RSUET framework and solution algorithms provide extremely fast convergence, even for large-scale and highly congested networks.

Large data sources that contain information on the behaviour of individuals become more and more available. The study has demonstrated that such data have a huge potential to be exploited to improve the specification of the disaggregate models. The data often provide route-level information on behaviour, and thereby seem particularly relevant to use in the estimation of the path-based RSUET solution algorithms and the cost threshold of the RSUET. In addition to the validation of the new framework, the study used disaggregate data to (i) enrich the specification of the utility functions for car users by estimating models which include separately the different variables related to time, and (ii) validate a new public transport choice set generation method. However, the study not only contributed with ways in which data can be used in validation and estimation procedures. It also demonstrated its great potential to enrich the specification of the network and its attributes (i.e., GPS logs were used to calculate a reliability measure for each link, and network topology was used to identify turning movements).

3.1 DATA PROCESSING

From a data perspective, the technological development has made it possible to collect individual-based GPS data from large samples and over numerous days. The size of the resulting datasets has called for automated post-processing methods. The study contributes to the state-of-the-art by proposing a *new approach to post-process GPS data* that uses disaggregate digital information on the infrastructure (bus line alignment, time tables, information on link type, etc.). The approach was validated by use of data collected in the Greater Copenhagen area. This showed that utilising information from a highly complex disaggregate network leads to large improvements in the correct classification of trip components and mode of transport used. The findings highlight that it is possible and attractive to conduct travel surveys based on GPS data collection, even for complex multi-modal study areas.

One future extension of the method would be to identify the trip purpose directly from the raw data. This could be done by, for instance, a combination of using time-stamps of trip start/end points and overlay analysis with disaggregate land-use maps. Using time-stamps would ensure that activities are done at reasonable time periods (e.g., individual end at home in the evening and only shop within opening hours of stores) and the overlay analysis would help ensure the reasonability of the land-use at activity locations (e.g., classify purpose of trips ending at schools, shopping centres). Identification of the trip purpose would facilitate better overall model performance by for instance allowing the model estimation to be done by trip purpose. Another future extension could be to identify whether the GPS carriers perform joint travel, which may influence route choices and activity patterns. This could be identified by searching for correspondence of trip leg alignment (in time and space) across individuals, e.g. among family members or work colleagues.

3.2 TRAVEL TIME COMPONENTS

The study is the first to provide an estimation based on RP data, which considers *all the components related to travel time* concurrently, namely the free-flow, congestion, and reliability parts. The method was applied to the processed GPS data, and numerical evidence was given on the value of congestion and reliability for different traffic conditions and periods. Results suggested that the value of the different time components as well as the congestion multiplier is significantly higher in peak hours than outside peak hours. This seems reasonable because of possible higher penalties for drivers being late, and, consequently, possible higher time pressure. An interesting finding is that the rate of substitution between the travel time reliability and the total travel time did not change across time periods. Different user types and trip purposes could not be distinguished in the data, and a future study could seek to consider also this in the model estimation. Overall, the results showed the great potential of exploiting GPS data to increase the behavioural realism of models.

3.3 PUBLIC TRANSPORT CHOICE SET GENERATION

The study contributes to the state-of-the-art by the proposal of a *simulation-based method to generate choice sets* in a time-expanded public transport network. The method allows to account for advanced preference structures and taste heterogeneity across individuals and thereby supports the progress towards more disaggregate models. The output generated by different specifications of the choice function was compared to RP data. Very high coverage levels were generated, and it was found that the specification should account for taste heterogeneity in addition to the simulation of edge costs. The results also illustrated that the variance of the stochasticity should be high when the choice sets are used for estimation. However, it should be introduced with parsimony as too high a variance results in the generation of counter-intuitive paths.

Theoretical consistency issues exist across the choice set generation process and the model estimation process used to evaluate the method. Ideally, the assumed traveller preferences used for the choice set generation should match the preferences estimated subsequently. The approach used in the present study does not obtain this neither in the functional form (different distributions and different components, e.g. PSL correction in the estimation) nor in the parameter values. It would be an interesting future research direction to develop a scheme which obtains consistency across the components. One possible approach could be a scheme which iterates between the choice set generation method proposed (using normal and skew-normal distributions) and the mixed MNP estimator proposed by Bhat and Sidhartan (2012). At each iteration, the parameters estimated in the previous iteration would be used in the specification of the generation function of the choice set generation method. Iterations would continue until convergence is obtained in the estimated parameters between iterations.

3.4 RSUE/RSUET Model Framework

The difficulty of specifying the choice set generation mechanism in SUE solution algorithms for large-scale applications is linked to theoretical inconsistency originating from the theoretical – but practically impossible – need of the SUE to perform enumeration of the universal choice set. This is true in the typical setup of the SUE, adopting unbounded distributions. Based on this realisation, two *new model frameworks* and corresponding equilibrium formulations were introduced. Both models permit the consistent combination of (i) *equilibrated* non-universal choice sets and (ii) flow distribution according to the *RUM theory*. One model allows distinction between used and unused routes based on the distribution of the random error terms. The other model allows this distinction by posing restrictions on the costs of used/unused routes (RSUE model). These models answer the need for a theoretical framework which allows behavioural realistic flow distribution, while providing a means to distinguish attractive and non-attractive paths (rather than leaving this for the modeller).

The RSUE model seemed the most straightforward to apply in the short term given its connection to existing RUM-based models, and was further developed to include a threshold for the cost on used paths to fulfil (RSUET). The threshold ensures that no unattractive paths are used at equilibrium (while another condition ensures that no attractive paths are left unused). The threshold can be specified in many ways. The study focused on a formulation which defines it as proportional to the cost on the minimum cost route among used routes. This provides a nice behavioural interpretation of the mechanism which distinguishes attractive and non-attractive paths; there is a limit to how long detours travellers are willing to accept. Many existing models do not have such a plausible interpretation of this mechanism, if any. Among these are models based on random walk mechanisms that allow loops (e.g., Fosgerau et al., 2013) and simulation-based models where the random draws may cause completely unreasonable paths to be used.

Generic path-based solution algorithms and convergence measures were proposed for the RSUE and RSUET. The algorithms were validated by application to the highly complex Zealand network. Extremely fast and well-behaved convergence was seen to an equilibrated solution on consistent non-universal choice sets (across different congestion levels, scale parameters, and step-sizes). Comparisons to corresponding GPS traces and link counts verified very reasonable choice set composition and distribution of flow. The analyses are the first to be conducted in the new framework that, hopefully, will attract interest in future research.

The issue of consistent calibration and estimation of the model would be an interesting direction for future research. Though the papers have hypothesised about possible calibration procedures, no actual calibration has been done. In this regard, the increasingly available route-level data seem particularly relevant to utilise, due to the path-based formulation of the framework and the path-based solution algorithms.

Another future research direction would be to extend the framework and solution algorithms to explicitly consider the temporal dimension. Temporal disaggregation would be in line with the tendency of models to become more disaggregate. It also facilitates to fully exploit the potential of correct integration between disaggregate individual-based assignment models and activity-based demand models. It seems straightforward to extend the underlying conditions and the equilibrium formulations to include the temporal dimension. However, some issues would probably arise in relation to the proof of existence of one or several equilibrated solutions (as hypothesised in section 5.1 of Watling et al. (2014)).

The solution algorithms and software implementation would also need to be updated to account for the temporal dimension. There exist efficient algorithms that perform time-dependent shortest path searches. Especially the identification of only the single shortest path (as needed in the RSUE(min)/RSUET(min, Ω) application) seems to be efficient in time-space networks (e.g., Dean, 2004). The largest challenge would probably be to incorporate the temporal dimension in the network loading procedure in a way that makes the calculation time remain feasible for large-scale applications. Actually, we are currently working on incorporating the temporal dimension into the theoretical framework, the solution algorithms and the software implementation. Among this is the software implementation of an operational link-node network loading model similar to the one proposed in Bliemer (2007). The hope is to be able to present some initial results of this work soon.

Summarising, the study proposed methods to improve the specification of the utility function and the generation of the choice sets. The study also proposed a new consistent modelling framework and corresponding algorithms to very efficiently solve for equilibrated solutions. The study focused on large-scale applicability at a disaggregate level, and also proposed ways to use large-scale individual-level datasets to improve the specification of the models. The contributions allow integration of the traffic assignment model and the activity-based models at a fully disaggregate level, and thereby facilitates the capitalisation of the true potential of activity-based models.

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APPENDIX 1: RASMUSSEN ET AL. (2014A)

TIMETABLE-BASED SIMULATION METHOD FOR CHOICE SET GENERATION IN LARGE-SCALE PUBLIC TRANSPORT NETWORKS

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Abstract: Composition and size of the choice sets are a key for the correct estimation of and prediction by route choice models. This study investigates actual path choices of public transport users and assesses choice set quality in a multimodal transport network. A timetable-based simulation method was applied to generate choice sets for 5,131 real-life trips in the Greater Copenhagen Area. The method was found suitable for choice set generation in a large-scale multimodal public transport network, and the importance of the algorithm and the utility specification chosen was clearly highlighted. It was found that the level of stochasticity should be high to provide statistically efficient parameter estimates when the choice sets are used for estimation, but still introduced with parsimony, as significant increases translate into generating redundant and counterintuitive paths. Adding heterogeneity across travellers improved the results considerably, and as models are becoming more disaggregate, showed the importance of accounting for as much individual heterogeneity as possible.

Keywords: path choice; route choice; public transport networks; choice set generation; simulation-based approach

1 INTRODUCTION

In order to understand the determinants of choice of public transport modes and to optimise the yield of investments in public transport systems, it is essential the availability of traffic models able to capture the travellers' behaviour sensitivity to public transport system's attributes and to predict demand and path choices on public transport networks in a realistic manner.

Modelling path choice essentially consists of two parts, namely the generation of a choice set and the representation of the choice between the generated paths. The available paths can be generated either explicitly prior to the choice process or implicitly in the choice process, but explicit choice set generation allows full control over desired properties of the generated paths, size and composition of the choice sets, and flexibility of the model specification. Then, travellers are assumed to maximise their utility (i.e., minimise their cost) and hence to choose their preferred path in the set of available paths.

Recent studies have given increasing attention towards the importance of the size and the composition of choice sets for path choice (see, for overviews, Bovy (2009) and Prato (2009)), whether they are to be used for model estimation or for prediction purposes. When used for model estimation, choice sets should facilitate statistical consistency and efficiency, while when used for prediction, they should contain all scenario-relevant alternatives (Van Nes et al., 2008). As a result, it is crucial to generate a choice set including alternatives considered by travellers and excluding alternatives never considered (Prato and Bekhor, 2007; Bliemer and Bovy, 2008). However, there exists no objective definition of what constitutes a relevant path, and hence the assessment of the generated path choice sets relies upon the experience of the analyst rather than objective measures of choice set quality.

The literature in path choice shows that choice set generation has been extensively investigated for car users and small synthetic networks, and has drawn much less attention for public transport users and large-scale networks. Deterministic and stochastic techniques have been implemented to the generation of alternative paths for car users: variations of shortest path algorithms (e.g., Akgün et al., 2000; Hunt and Kornhauser, 1997; Lombard and Church, 1993; Van der Zijpp and Fiorenzo-Catalano, 2005); application of heuristic rules (e.g., Ben-Akiva et al., 1984; Azevedo et al., 1993; De la Barra et al., 1993); branch and bound algorithms (Hoogendoorn-Lanser et al., 2006; Prato and Bekhor, 2006); single and doubly stochastic simulation approaches (e.g., Nielsen, 2000; Bekhor et al., 2006; Bovy and Fiorenzo-Catalano, 2007); biased random walk algorithm (Frejinger et al., 2009); breadth first search with network reduction (Rieser-Schüssler et al., 2013). In metro networks, a heuristic approach pooling observations for the same origin-destination pair was applied in Santiago (Raveau et al., 2011). In multimodal networks, constrained enumeration was applied to a multimodal interregional hub-and-spoke transport corridor in the Netherlands (Hoogendoorn-Lanser et al., 2007), and a simulation-based doubly stochastic choice set generation method was tested on the same corridor (Bovy and Fiorenzo-Catalano, 2007). Moreover, aggregation of the network into 'route segments' with consequent approximation of travel time and waiting time calculations was applied to evaluate existing choice set generation methods from smart card data in Singapore (Tan et al., 2014), and a Google Map procedure was used to generate alternative routes in the public transport network of Montreal (Eluru et al., 2012).

The current study contributes to the literature on public transport path choice by proposing, implementing and validating a timetable-based simulation approach for the choice set generation of paths in large-scale multimodal networks. The importance of the current study lies not only in the solution of the challenges of generating paths in a multi-layered public transport system, but also in the implementation on a large-scale network with multiple public transport modes. Moreover, the current study assesses path choice sets by comparing the generated paths with real-life path choices in the public transport system of the Greater Copenhagen Area as well as evaluating the ability to produce consistent parameter estimates in model estimation. The data consisted of 5,131 observations of actual path choices collected as part of the Danish Travel Survey, which is a one-day travel diary with high level of detail for the collection of public transport paths. For each observation, corresponding choice sets were generated and assessed for various configurations of the generation (utility) function, thereby enabling recommendations of good configurations. Notably, different levels of comparison measures between generated and observed paths (i.e., line level, stop level) were considered.

The next section introduces the proposed timetable-based simulation method to generate choice sets for public transport path choice and describes how the generated choice sets can be evaluated. Then, the case study is presented including the configurations tested and how the generated choice sets were evaluated in the study. Next, the results of the assessment are presented, followed by a discussion and conclusions summarising the main findings of the study.

2 PROPOSED CHOICE SET GENERATION IN MULTIMODAL PUBLIC TRANSPORT NETWORKS

This section presents the timetable-based simulation method used to generate the path choice sets. Subsequently follows the introduction of methods for evaluating the choice set generation method and the choice sets generated.

2.1 TIMETABLE-BASED SIMULATION METHOD TO GENERATE PATH CHOICE SETS

The proposed method generates choice sets by repeated shortest path searches in a timetable-based public transport time-space network graph (i.e., a diachronic graph with spatial as well as temporal component of edges and nodes, see e.g. Cascetta (2001)). Attributes on the edges of the graph and individual preferences are simulated, and the simulation induces that different unique paths may be generated by repeated application of the search, and the union of these unique paths constitutes the choice set. The method uses generation (cost) functions, and the utility (cost) on path *i* for individual *m* is expressed as:

$$C_{im} = V_{im} + \varepsilon_{im} = \sum_j \beta_{jm} \cdot x_{ijm} + \varepsilon_{im} \tag{1}$$

where V_{im} is the systematic utility of path *i* for individual *m*, x_{ijm} is an attribute *j* of path *i* for individual *m*, and β_{jm} is a parameter that expresses the preference of individual *m* for the attribute *j* of path *i,* accounting for perception errors as well as elements not accounted for in the systematic part of the generation function. It should be noted that it is like if for each attribute *j* there exists a randomly distributed parameter β_j accounting for taste heterogeneity across individuals *m*. The parameters are drawn once from the distribution of β_j for each individual *m* in each iteration of the path search.

The generation of the paths is based on the assumption that the cost on path *i* is the sum of the costs on the edges belonging to path i in the time-space network graph:

$$C_{im} = \sum_{l \in \Gamma_{im}} \left(\sum_j \beta_{jm} \cdot x_{ljm} + \varepsilon_{lm} \right) \tag{2}$$

where Γ_{im} represent the set of edges belonging to path i for individual m, x_{ljm} is the attribute j on edge l for individual m, and ε_{lm} is a random variable on edge l for individual m accounting for perception errors as well as elements not accounted for in the systematic part of the cost function. The assumption of additivity allows the paths to be consistently generated via shortest path searches in the graph: the individual specific parameters β_{jm} are initially drawn before the search (in each iteration), and the impedances of the edges are then drawn as the shortest path tree is built (i.e., not all elements of the graph have to be simulated). In order to ensure consistency in the aggregation of the costs from edge- to path-level, it is necessary that (i) the error term follows a distribution which is additive in mean and variance, (ii) the error term has a variance proportional to the mean of the cost on the edge, and (iii) the cost function is specified as linear-in-parameters (Nielsen and Frederiksen, 2006). We use a gamma distribution for condition (i), we define a proportionality factor γ for condition (ii), and hence we have a distribution with mean $\sum_j \beta_{mj} \cdot x_{lmj}$ and variance $\gamma \cdot \sum_j \beta_{mj} \cdot x_{lmj}$. We note that this specification induces (since the mean of the cost is dependent on the random variables β_{jm}) the link error term to depend on individual m.

It should be noted that the simulation of the edge costs does not influence the network graph, but only the path search, namely the graph is the same from realisation to realisation. The network graph is based on the full (deterministic) timetable, and no vehicles or runs or passengers are thus simulated for the creation of the graph, only edge costs and preferences are simulated prior to the path search.

2.2 METHODS FOR EVALUATION OF CHOICE SETS

In lack of a direct objective measure of what constitutes a relevant path, choice set generation methods can be evaluated based on a combination of (i) the size of the choice sets generated, (ii) the ability to generate choice sets containing at least one path having high similarity to a corresponding observed path, and (iii) the ability to generate choice sets which facilitate consistent parameter estimates when used in the estimation of route choice models.

The first evaluation criterion relies on the analysis of the evolvement and size of the choice sets defined as the sets of unique paths generated by the repeated application of the simulation-based path-generation method. As timetable-based multimodal public transport networks are very detailed, the distinction between unique paths can be done on numerous levels of detail: (i) departure level, where a path is only considered unique if no other paths use the same departures of the same lines to and from the same stops; (ii) line level, where a path is unique only if no other paths use the same lines to/from the same stops; (iii) stop level, where a path is unique only if no other paths use lines with the same stopping pattern between the same to/from stops; (iv) trip leg mode sequence, where a path is unique only if no other path uses the same sequence of modes for the different trip legs used.

The second evaluation criterion relies on the analysis of whether the applied choice set generation method is able to generate paths similar to the observed path of an individual. This is performed through a measure of coverage, equal to the share of observations for which at least one path within the generated choice set has an overlap with the observed path equal to or above a certain threshold. The overlap can, as the identification of unique paths, be calculated according to different levels of detail. Another dimension to consider when specifying the overlap is the unit of measure, which could be e.g. overlap-in-time, overlap-in-utility and overlap-in-length. Using stop level as the level of detail and overlap-in-length as the unit of measure, the overlap $O_{i,stop,m}$ of the generated path i with the observed path of observation m can be computed as (Ramming, 2002):

$$O_{i,stop,m} = \frac{L_{mi}}{L_m} \tag{3}$$

where L_{mi} is the sum of length of overlapping elements between path i and the observed path for observation m, and L_m is the length of the observed path used by observation m.

This overlap measure can be computed for each generated path i for observation m, and let $O^{\max}_{stop,m}$ denote the best overlap (measured on stop level using overlap-in-length as unit of measure) among the paths generated for observation m. Then the coverage for an overlap-threshold equal to δ can be computed as (Ramming, 2002):

$$Cov_{stop}\left(\delta\right) = \frac{\sum_{n=1}^{M} I\left(O^{\max}_{stop,m} \geq \delta\right)}{M} \tag{4}$$

where M is the number of observations and $I(\cdot)$ is an indicator equal to 1 when the criteria is fulfilled and 0 otherwise.

A path choice set generation method should produce an array of relevant paths within a reasonable amount of iterations, and the observed path should be among these. Visual inspection combined with network knowledge could be used to evaluate whether counterintuitive paths are generated[1]. Such a procedure however could become tedious when having many observations and large-scale networks. Alternatively, whether redundant and/or possibly counterintuitive paths are generated could be evaluated at the aggregate level by comparing the increase in coverage to the increase in the average choice set size. Large increases in average choice set size combined with low improvements in coverage would indicate that redundant paths similar to already existing paths or counterintuitive paths were generated. An efficient algorithm is characterised by fast increases in coverage as well as average choice set sizes.

As the composition of the choice set influences the parameter estimates when used for model estimation purposes (e.g., Train, 2002; Van Nes et al., 2008), obtaining good coverage does not necessarily induce that parameters can be consistently estimated, for example alternatives might or not be similar to each other and choice sets might be different although reproducing at least once the chosen path. Consequently, the third evaluation criterion relies on the analysis of whether the proposed choice set generation method is able to generate path sets which include relevant alternatives with sufficiently varying attributes to facilitate statistically efficient estimates of model parameters. This analysis relies on the estimation of the same choice model on the choice sets generated at different stochasticity levels, and enables also the analysis of whether and to what extent the estimated parameters are stable across stochasticity levels.

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[1] In the study a counterintuitive path is defined as a path that is clearly less attractive than some other path connecting the same origin and destination because it has a considerably larger travel cost.

3 CASE STUDY: GREATER COPENHAGEN AREA

This section presents the case study. Section 3.1 introduces the data sources used in the study and the data preparation, while section 3.2 presents the different tested configurations of the path choice set generation method. Section 3.3 describes how the generated choice sets were evaluated.

3.1 DATA

3.1.1 OBSERVED PATHS

The current study used revealed preference data collected as part of the Danish Travel Survey, and the dataset consisted of 5,131 observed paths in the multimodal public transport network of the Greater Copenhagen Area. The survey is an ongoing questionnaire-based collection of one-day travel diaries and associated respondents' and households' socio-economic characteristics. The respondents are a representative sample of the Danish population between 10 and 84 years of age who provide detailed information on all their trips during the day, and since February 2009 answer specific questions investigating the path choice of trips using public transport. Respondents fill in information at a level of detail enabling the path to be reproduced by the analyst while still being fairly easy to fill in by the respondent. Addresses and purposes at start points, change points and end points of the trips, as well as detailed information about the modes used *en route*, are collected in the survey:

- *Walk, bike, car, etc.*
  - Length and travel time
- *Bus*
  - Waiting time, bus line, length and travel time
- *Suburban train (S-train)*
  - Waiting time, boarding station, S-train line, alighting station, length and travel time
- *Train, Metro*
  - Waiting time, boarding station, alighting station, length and travel time

In order to perform comparisons, the observed paths were map matched to the same digital network representation as the one used for the choice set generation. The paths were mapped at the line level identifying the line and the stops where boarding and alighting the different public transport modes occurred, but omitting identification of which actual departure was used. This map-matching level was chosen due to uncertainties regarding the stated travel times and departure times as well as possible delays on the day the paths were observed. The mapping has been documented in Rasmussen (2010) (see, for documentation in English, Anderson and Rasmussen, 2010).

Figure 1 presents the characteristics of the observed paths in the data set used, namely trip purpose, trip length, and number and mode of trip legs for multimodal paths. Most observations are either commuting or leisure trips, whereas only a few business trips have been observed in this representative sample of trips of the Danish population. Notably, trip characteristics appear similar between commuting and business trips, while education trips are similar with only a higher share of shorter trips below 10 km. In the Greater Copenhagen Area, the average commuting distance for public transport users is 21.0 km per direction, which is slightly higher than the average 17.1 km for the trips in the dataset. Leisure trips are generally shorter and consist of only one bus trip leg (53% of the observations), while other purposes observe less bus-only one-trip-leg trips (38%-42% of the observations).

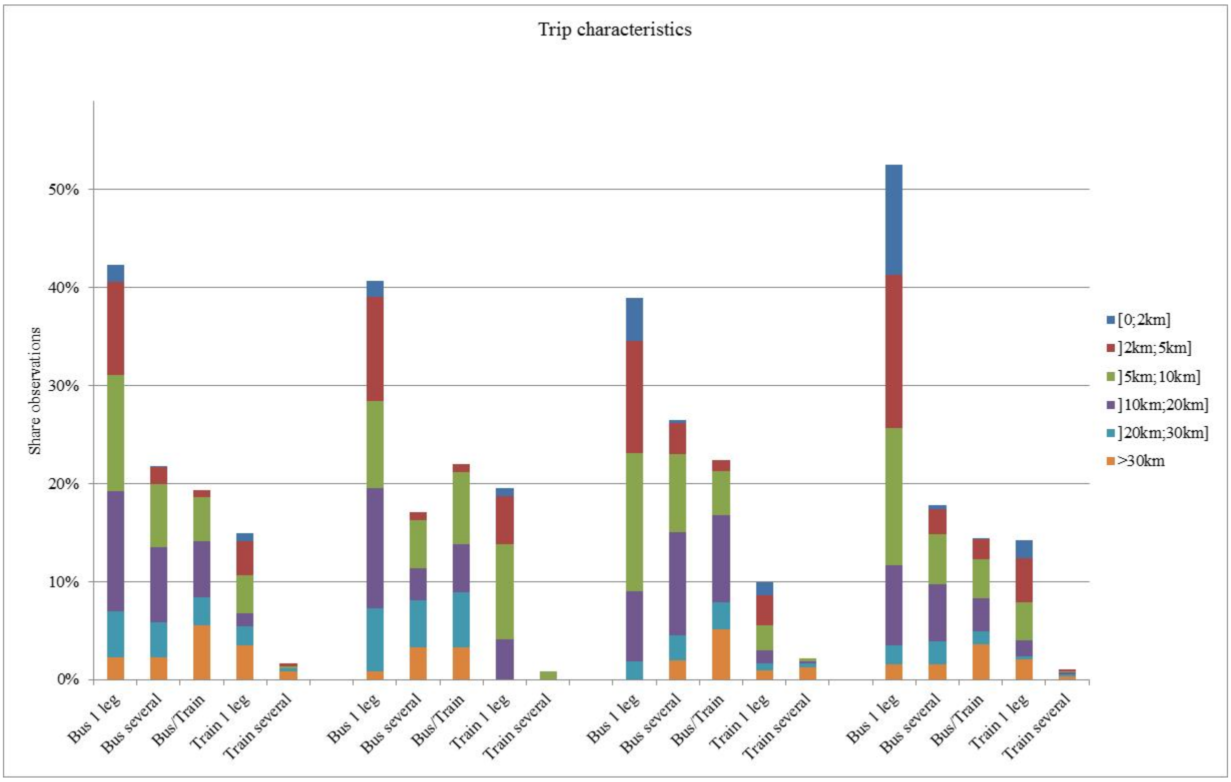

Figure 1 – Characteristics of the observed public transport trips

3.1.2 NETWORK DATA

The used digital network represents the Greater Copenhagen Area, in which approximately 2 million people live. The area is served by an array of different public transport modes, including numerous bus lines with different levels of service (i.e., regular, frequent, express, rapid), two metro lines, several Intercity and Regional train lines, seven S-train lines and various local train lines. The digital network representation is timetable-based and includes the departures of all the public transport in the area, namely 479 lines, 1,677 line variants, 5,652 stops and 635,027 daily stop departures. The data originates from The Danish National Transport Model (currently under development at DTU Transport), and the schedule is a digital representation of how the real-life public transport network was scheduled on November 10, 2010. Transfers are available between lines at every stop, but the most important transfers (e.g., between bus and train at the Copenhagen Central Station) are also represented in the graph through 560 transfer edges that are defined via a length-dependent impedance expressing walking time between the connected stops.

The analysis evaluates the proposed choice set generation method through the generation of choice sets corresponding to the observed paths. Therefore the start and end locations (addresses) of the observations were introduced into the network and linked to relevant public transport stops by connectors. The simplest approach to generate connectors would be to generate connectors to the nearest stop only. However, in order to facilitate the possibility of a wide array of alternative paths, a new approach to generating connectors between trip start and end locations and public transport stops was developed as a part of this study. The approach aims to generate connectors to all stops considered relevant by travellers, and thus connectors were generated according to the following criteria: (i) the 5 nearest bus stops served by bus lines with low service level within a distance of 2,500m; (ii) the 5 nearest bus stops served by bus lines with high service level within a distance of 5,000m; (iii) the 5 nearest train stations within a distance of 20km; (iv) the nearest bus stop on each of the A-, S or E- bus lines (high service level bus lines) within 20 km if not already generated by step (ii). The travel time on the connectors was calculated by using actual network distances. Summarising, the multimodal public transport network used for choice set generation consists of 5,652 stops, 560 transfer edges, 202,035 connector edges, 635,027 public transport run edges between stops.

3.2 CONFIGURATION OF THE TIMETABLE-BASED SIMULATION METHOD TO GENERATE PATH CHOICE SETS

In the current study, the detailed generation (cost) function used for generating the paths specified the cost of alternative path *i* for observation *m* at the path level as:

$$
\begin{aligned}
C_{im} &= V_{im} + \varepsilon_{im} = \beta_{walktime,m} \cdot TT_{walktime,im} + \beta_{waittime,m} \cdot TT_{waittime,im} + \beta_{changepen,m} \cdot N_{change,im} \\
&+ \beta_{conntime,m} \cdot TT_{conntime,im} + \beta_{waitzone,m} \cdot TT_{waitzone,im} + \beta_{IVT,train,m} \cdot TT_{IVT,train,im} \\
&+ \beta_{IVT,IC-train,m} \cdot TT_{IVT,IC-train,im} + \beta_{IVT,S-train,m} \cdot TT_{IVT,S-train,im} + \beta_{IVT,bus,m} \cdot TT_{IVT,bus,im} \\
&+ \beta_{IVT,metro,m} \cdot TT_{IVT,metro,im} + \varepsilon_{im}
\end{aligned} \tag{5}
$$

where for path *i* and observation *m* $TT_{walktime,im}$ and $TT_{waitime,im}$ are walking and waiting time when transferring, $N_{change,im}$ is the number of transfers, $TT_{conntime,im}$ is the time spent walking between the origin/destination and the first/last public transport stop, $TT_{waitzone,im}$ is the schedule delay, and $TT_{IVT,train,im}$, $TT_{IVT,ICtrain,im}$, $TT_{IVT,S\text{-}train,im}$ and $TT_{IVT,bus,im}$ are in-vehicle times spent respectively in regional trains, IC-trains, S-trains and buses. The corresponding parameters are distributed with mean β_j and variance $\alpha \cdot \beta_j$ where α is a scale parameter, while ε_{im} is the error term constituted as the sum of error terms drawn at the edge level.

In order to be able to recommend good formulations, three formulations of the generation (cost) function *(5)* were tested. To also be able to recommend levels of stochasticity introduced, each of these formulations was tested with nine levels of the variances of the distributions of error components and/or error term. This induced in total 27 configurations to test for each of the 5,131 observations. The choice of formulations to investigate was based on findings by Rasmussen (2010), who tested six different formulations on a limited number of observations. Consequently, in this present study, path choice sets were generated for three different formulations of the generation function *(5)*:

- *ErrTermOnly*: all β's not randomly distributed across the population, and ε_{im} Gamma distributed.
- *ErrCompAll*: all β's Log-Normal distributed, and no consideration of ε_{im} in the generation function.
- *ErrCompErrTerm*: all β's Log-Normal distributed, and ε_{im} Gamma distributed.

The distributions were chosen in order to avoid counterintuitive draws while still maintaining the theoretical assumptions: (i) negative values cannot be drawn from the Log-Normal distribution, securing to avoid counterintuitive cases where longer travel time generates lower cost; (ii) the Gamma distribution is additive in mean and variance and the variance is proportional to the mean; (iii) the consistency between the edge and the path level is maintained. Furthermore, the Gamma distribution has a finite support, whereby the risk of some alternative to have negative cost due to the error term can be avoided.

The nine levels of the scale parameters were defined based on starting values found in Rasmussen (2010) (see also Larsen et al., 2010) and are presented in Table 1. Consequently, ErrCompErrTerm_7 refers to a configuration where all parameters and the error term are distributed with a scale of $\alpha = \gamma = 1.5$. Rasmussen (2010) tested the levels denoted by _1, _2 and _3, and found that coverage increases with increasing size of the scale parameters. The present study additionally tested cases where the scale parameters are considerably higher in order to find the level from which the coverage does not continue to improve and possibly becomes worse by increasing the level of stochasticity. The parameters were drawn for each observation in each iteration, whereas the error terms were drawn at the edge level for each observation in each iteration. The respondents could report the departure time in 5 minute intervals. Accordingly, to account for this to allow different connections, a random departure time within a 10 minute interval around the recorded departure time was drawn before each path search.

Table 1 – Size of parameter that scale the variance of the distribution

Level of stochasticity	_1	_2	_3	_4	_5	_6	_7	_8	_9
γ	0.05	0.10	0.15	0.20	0.50	1.00	1.50	2.00	5.00
α	0.025	0.05	0.10	0.20	0.50	1.00	1.50	2.00	5.00

The means of the parameter values were based upon results estimated in Nielsen (2000), and shown in Table 2.

Table 2 – Parameter values in the generation function (source: Nielsen, 2000)

$\beta_{walktime}$	$\beta_{waittime}$	$\beta_{changepen,i}$	$\beta_{conntime}$	$\beta_{waitzone}$
38.0 DKK/h	38.0 DKK/h	7 DKK/change	45.0 DKK/h	16.0 DKK/h
$\beta_{IVT,train}$	$\beta_{IVT,IC\text{-}train}$	$\beta_{IVT,S\text{-}train}$	$\beta_{IVT,bus}$	$\beta_{IVT,metro}$
27.0 DKK/h	27.0 DKK/h	27.0 DKK/h	35.4 DKK/h	21.6 DKK/h

For each observation and configuration, 200 paths (i.e. 200 iterations of the path search) were generated between the corresponding origin and destination points of the observation. In total, 27,707,400 shortest path searches were conducted in the large-scale network (200 iterations, 27 configurations, 5,131 observations).

3.3 Evaluation of choice sets

The generated choice sets were evaluated by their ability to generate unique paths, reproduce the observed paths, and produce consistent parameter estimates when used in model estimation. As described, there are several levels of detail on which the paths could be distinguished and the overlap computed, and this section describes the choices made for the current case study.

The multimodal public transport network in the Greater Copenhagen Area is complex by often providing numerous different alternatives using the same sequence of modes for the trip legs used. These alternatives might however differ considerably from each other in terms of attributes such as e.g. travel times, and will need to be distinguished as different unique possibilities. Consequently, distinguishing between unique paths on the level of trip leg mode sequence was not considered attractive in this present study. By being timetable-based, the available digital representation of the network allowed distinguishing paths at the departure level. The generated paths were however to be compared to observed paths mapped at the line level, and so there was no need to distinguish between generated paths at the departure level. In this study, the distinction between paths has thus been done at the line level.

This study calculated the overlap between paths on the aggregate stop level for the public transport trip legs (excluding access/egress), as this is less sensitive to the correctness of the input data. As an example, several S-train lines share the same alignment and stopping pattern through the city of Copenhagen, and people might not remember which line they used for trips between stops in the segment, and may consequently report the wrong line. Additionally, using the stop level would lower the sensitivity towards delays experienced on the day of the reported trip, as such delay was not represented in the digital network representation used for the choice set generation. This study adopted length as the unit of measure for the overlap, and the coverage could thus be computed as in equation (4).

Using the observed paths and the corresponding choice sets generated, a choice model was estimated for each of the levels of stochasticity used. This was done in order to evaluate the ability to produce statistically efficient parameter estimates and to test whether these were stable across stochasticity levels. The study did not explore several different specifications of the model to be estimated, but rather used a Path Size Logit model formulation which in Anderson et al. (2014) was found to perform well. The utility function included in-vehicle travel time in different modes of transport, access/egress times, walking and waiting times when transferring, number and type of transfers, headway between departures dependent on time-of-day as well as correction for path overlapping using the PSC correction term presented in Bovy et al. (2009). Biogeme was used to conduct the maximum likelihood estimations (Bierlaire, 2009).

4 RESULTS

4.1 CHOICE SET SIZE

Ideally, the number of unique paths would stabilise after generating a variety of paths, indicating that no counterintuitive and redundant paths were added to the choice sets. The path generation would then be terminated when this 'stable' situation was reached. With a high level of stochasticity, the simulation however seemed to continue to generate new unique paths even after 200 iterations, whereas the choice set composition seemed to stabilise for the smallest levels of stochasticity. This was expected, as introducing more randomness in terms of larger variance around the mean might produce paths with minor deviations to the actually most attractive path as well as cause some obviously unattractive paths to become attractive.

The size of the generated choice sets is highly dependent on the formulation and the size of the stochasticity. This is indicated in Table 3, which lists various key figures describing the number of unique paths in the choice sets. As can be seen, the combined formulation generated the largest choice sets. Comparing formulations ErrTermOnly and ErrCompAll, the latter seemed to generate the largest choice sets for the lowest stochasticity levels. The opposite was observed in the cases with high level of stochasticity.

Table 3 – Choice set size characteristics at iteration 40 and 200

		ErrTermOnly								
	Iteration	_1	_2	_3	_4	_5	_6	_7	_8	_9
Min	40	1	1	1	1	1	1	1	1	2
	200	1	1	1	1	1	1	1	2	5
Mean	40	2.9	3.5	4.5	5.9	9.2	13.6	17.2	20.0	28.9
	200	3.8	5.0	6.9	10.2	19.7	36.0	51.6	65.7	117.8
Median	40	2	3	4	5	8	12	16	20	31
	200	3	4	5	8	15	29	45	60	126
Max	40	18	20	28	32	37	40	40	40	40
	200	31	39	61	84	132	181	192	198	200

	Iteration	ErrCompAll _1	_2	_3	_4	_5	_6	_7	_8	_9
Min	40	1	1	1	1	1	1	1	1	1
	200	1	1	1	1	1	1	1	2	4
Mean	40	3.7	4.7	5.5	6.1	8.4	10.2	11.2	11.8	13.6
	200	5.3	7.5	9.3	10.9	16.9	22.5	26.2	28.3	34.4
Median	40	3	4	5	5	8	10	11	11	13
	200	4	6	8	9	15	20	24	26	32
Max	40	20	22	25	27	30	32	34	33	36
	200	35	51	54	69	86	105	121	128	143

	Iteration	ErrCompErrTerm _1	_2	_3	_4	_5	_6	_7	_8	_9
Min	40	1	1	1	1	1	1	4	3	3
	200	1	1	1	1	2	4	7	8	9
Mean	40	4.6	6.3	8.0	9.9	15.4	20.4	23.1	25.0	29.2
	200	7.4	11.3	16.0	22.5	43.5	67.6	82.6	93.2	118.7
Median	40	4	5	7	9	15	20	23	25	29
	200	6	9	13	19	39	65	80	92	118
Max	40	23	31	32	34	39	40	40	40	40
	200	47	76	100	135	172	187	190	200	200

In general, higher levels of the stochasticity implied larger choice sets, especially for the two formulations including a distributed error term. This indicates that, when adding high stochasticity, the method becomes very efficient in terms of generating unique paths to the choice sets. Larger choice sets for higher stochasticity were also observed in Figure 2, which is an example of a commute trip between a suburb and the city centre of Copenhagen. As can be seen, the observed path is represented in the choice set for the different levels of stochasticity shown (formulation ErrCompErrTerm).

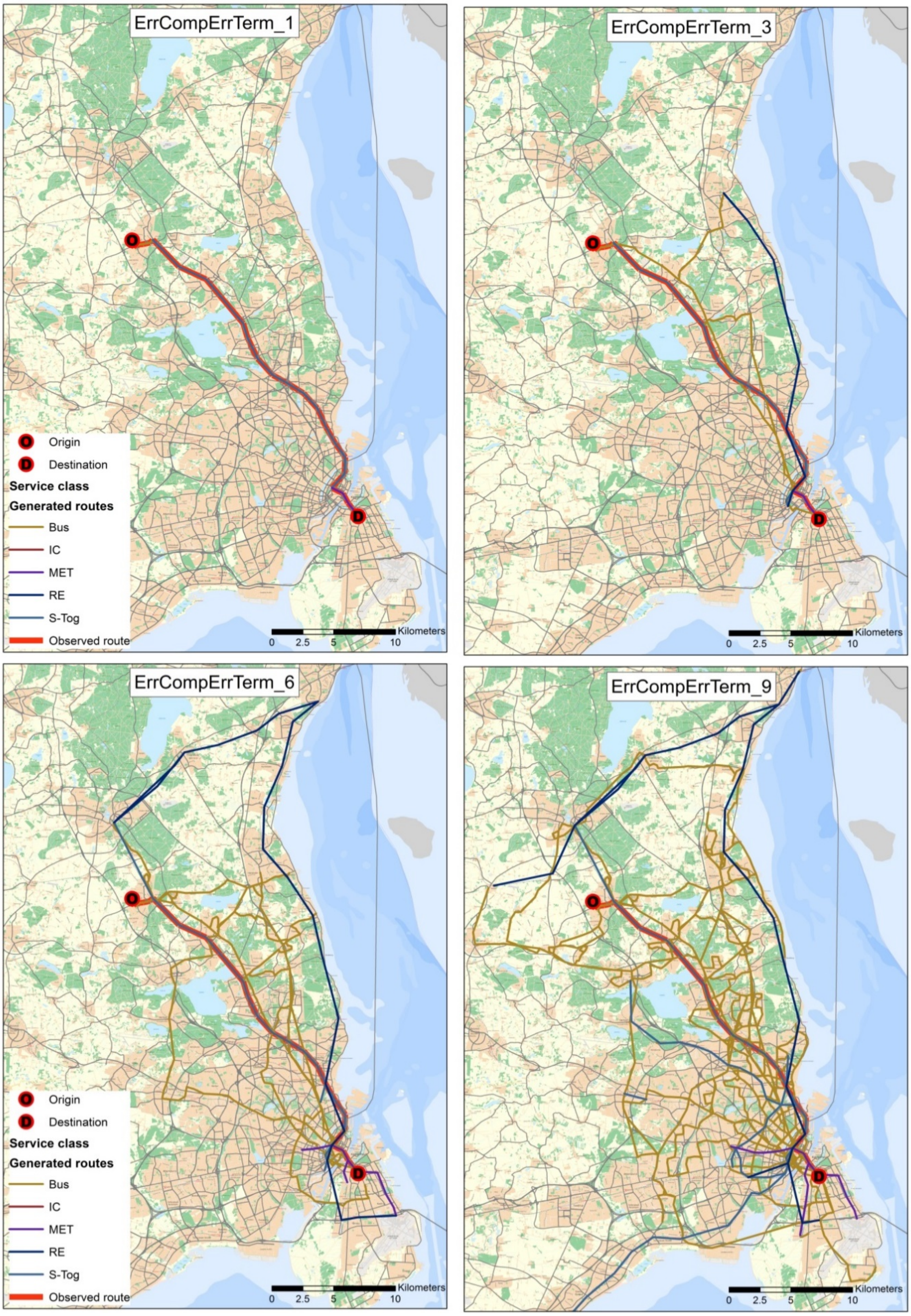

Figure 2 – Example of generated choice set with ErrCompErrTerm formulation and selected scale parameters

Figure 2 indicates a general tendency that has been verified by visual inspection in Geographic Information System of the choice sets generated for numerous observations: when using a high level of the stochasticity and after a number of iterations, new unique paths generated were redundant or counterintuitive. This suggests that it is undesirable to iterate until the number of unique paths stabilises.

4.2 COVERAGE

The observed path should be represented among the set of generated paths. Therefore, the improvement in coverage could supplement the choice set size as an additional indicator of performance. Applying an overlap threshold of 80%, a value often used in the literature (e.g., Ramming, 2002; Prato and Bekhor, 2007), induced the results illustrated in Figure 3.

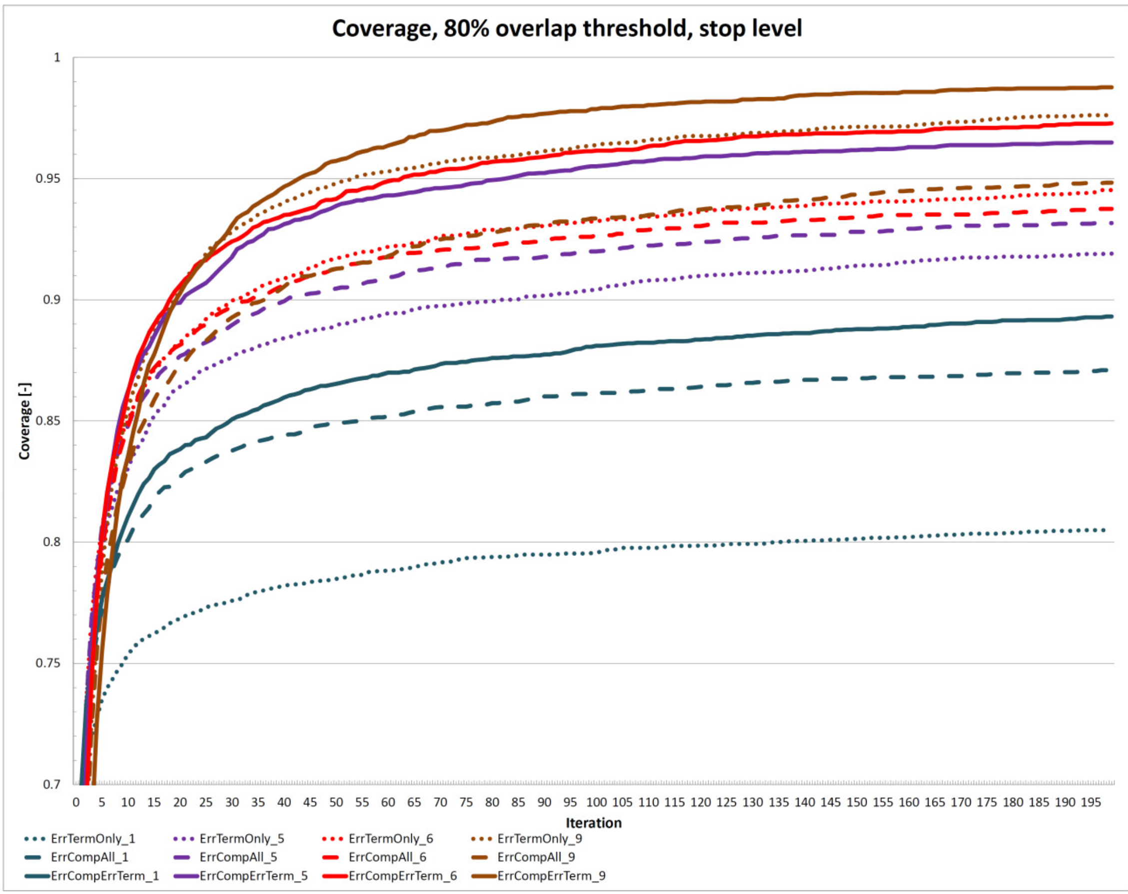

Figure 3 – Coverage with overlap threshold of 80% (stop level) for configuration _1, _5, _6 and _9 for all three formulations

The timetable-based simulation method produced in general very high coverage, especially when performing 40 iterations or more. Complete convergence was not seen within the 200 iterations, however the increment was rather small after 40 iterations. Figure 4 shows that setting a higher threshold of the overlap reduced the coverage, as expected, but the levels were still high, even for a threshold of 100%.

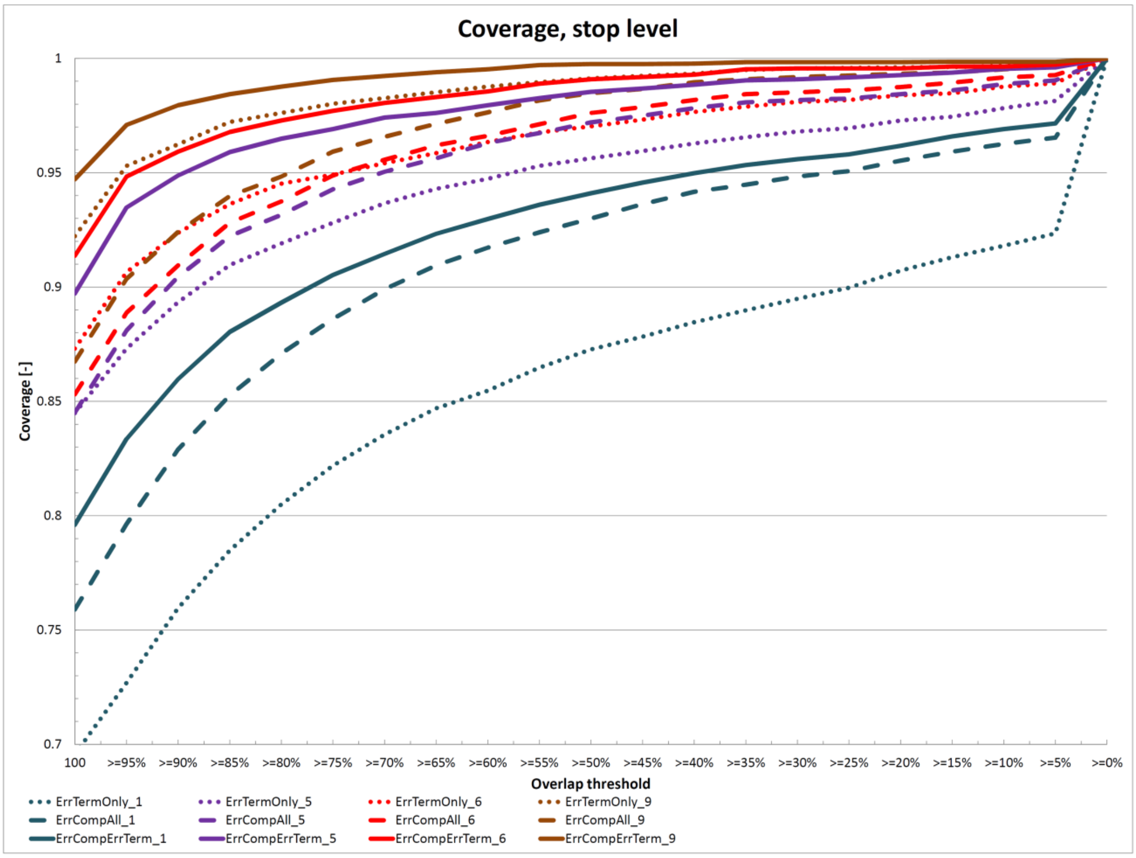

Figure 4 – Coverage of the timetable-based simulation for choice set generation (200 iterations) for configuration _1, _5, _6 and _9 for all three formulations

Rasmussen (2010) found that increasing the size of the scale parameter of the variance of the distributed terms does also increase the coverage. This was verified in the present analysis, and found valid even when applying very large scale parameters. However, the increase in coverage by increasing the scale parameters from 1 (configurations _6) to 5 (configurations _9) only induced approximately a 2 percentage-point increase in the coverage. This increase was at the cost of generating counterintuitive paths when using higher levels of stochasticity.

When comparing the coverage across the formulations, it can be seen that the formulation ErrCompErrTerm outperformed the two other formulations at low as well as high levels of stochasticity (see Table 4). This indicates that accounting for taste heterogeneity by adding distributed parameters to the single stochastic formulation (widely used in car choice set generation) improves the coverage. By comparing Tables 3 and 4, it can be seen that formulation ErrCompAll not only generated more unique paths, but also produced better coverage than formulation ErrTermOnly when applying low stochasticity (after 40 as well as 200 iterations). Accordingly, if only low variance is to be applied to either the taste parameters or a distributed error term, the best results in terms of coverage are generated by applying distributions to the parameters. The opposite was seen when applying high stochasticity.

Table 4 – Coverage levels obtained, comparison across formulations

		Iteration 40		Iteration 200	
		Low stochasticity	High stochasticity	Low stochasticity	High stochasticity
Formulation	ErrorTermOnly	78-83%	92-94%	81-86%	95-97%
	ErrCompAll	84-88%	90-91%	87-91%	93-95%
	ErrCompErrTerm	86-90%	94-95%	89-93%	97-99%

The coverage grew, dependent on formulation and size of variance, between 2.7 percentage points and 4.5 percentage points when doing 200 iterations rather than 40 iterations. This gain was however at the cost of a 5 fold increase in computation time and, in cases with very high variance, larger choice sets including counterintuitive paths.

4.3 MODEL ESTIMATION RESULTS

The Path Size Logit choice model with the utility function verbally described in section 3.3 was estimated for the choice sets generated by the different variants of formulation ErrCompErrTerm. The observed path was added to the choice set if not generated by the choice set generation method. The focus was on the ErrCompErrTerm formulation, as it was found to perform best in terms of coverage. In section 4.1 it was established that the configurations with high stochasticity produced counterintuitive paths. In order to further investigate the influence of the presence of counterintuitive paths on the model estimation results, an additional level of stochasticity, denoted as ErrCompErrTerm_10, with $\alpha = \gamma = 10$

were tested. Table 5 lists the parameter estimates scaled to the parameter for in-vehicle time for bus, thereby representing the rates of substitution to 1 minute of travel time spent in bus. Note that the PSC parameter estimate is not scaled, as such a measure does not have a direct behavioural interpretation. The table also lists the value of the t-test for the parameter estimates.

Table 5 – Comparison of estimates of Path Size Logit models

		_1		_3		_5		_7		_9		_10	
	Index	scaled	t-test	scaled	t-test	scaled	t-test	scaled	t-test	scaled	t-test	scaled	t-test
Headway													
Up to 6 min	1	-15.08	23.82	-4.97	23.12	-2.98	21.29	-3.74	22.84	-3.49	23.23	-3.03	20.94
Above 6 min	2	1.19	-6.76	0.45	-8.08	0.31	-9.34	0.26	-8.24	0.23	-7.69	0.35	-11.16
In-vehicle time													
IVT Bus	3	1.00	-8.68	1.00	-17.20	1.00	-25.96	1.00	-26.36	1.00	-27.39	1.00	-27.78
IVT Local train	4	1.13	-4.24	1.10	-11.38	0.94	-12.38	0.93	-9.97	0.94	-9.25	0.96	-10.05
IVT Metro	5	0.38	-1.48	0.57	-6.29	0.52	-8.86	0.50	-7.97	0.53	-9.03	0.49	-8.64
IVT Regional/IC-train ≤ 20 km	6	4.29	-12.51	2.36	-17.36	1.89	-19.54	1.66	-15.95	1.45	-14.75	1.48	-15.06
IVT Regional/IC-train > 20 km	7	0.26	-1.10	0.60	-7.75	0.69	-11.73	0.74	-10.92	0.75	-11.19	0.74	-11.22
IVT S-train	8	0.83	-5.90	0.84	-14.01	0.83	-21.15	0.79	-19.51	0.78	-20.01	0.79	-21.01
TT Access/Egress	9	-0.62	4.37	1.11	-8.04	1.63	-19.71	1.55	-20.44	1.65	-23.60	1.89	-27.00
Change													
Walking time	10	-3.84	9.42	-0.44	3.33	0.30	-3.85	0.44	-5.53	0.67	-8.50	0.75	-8.97
Waiting time	11	-4.88	22.86	-0.62	13.88	0.15	-4.97	0.44	-13.33	0.53	-17.05	0.48	-16.97
Bus to bus penalty	12	26.41	-11.67	16.33	-16.64	14.92	-26.39	10.84	-22.01	7.19	-17.06	9.69	-21.21
Bus to train penalty	13	35.89	-14.71	18.75	-18.38	15.50	-25.08	9.83	-16.39	5.00	-9.37	6.86	-12.62
Train to bus penalty	14	41.53	-16.42	20.83	-20.00	16.88	-27.09	11.57	-19.09	6.51	-12.35	7.94	-14.31
Train to train penalty	15	13.63	-6.91	10.75	-13.43	11.16	-24.98	7.47	-18.53	4.64	-13.81	6.96	-18.16
Path Size factor													
PSC [estimated parameter, non-scaled]	16	0.145	3.15	0.525	13.29	0.612	18.76	-0.065	-2.64	-0.643	-24.05	-0.353	-11.30
Number of estimated params:		16		16		16		16		16		16	
Number of observations:		5,131		5,131		5,131		5,131		5,131		5,131	
Null log-likelihood:		-9,785.6		-13,389.4		-18,557.5		-22,179.8		-24,278.9		-24,807.5	
Final log-likelihood:		-8,235.3		-11,174.9		-13,253.3		-15,791.2		-13,748.9		-10,783.0	
Likelihood ratio test:		3,100.5		4,428.8		10,608.3		12,777.1		21,060.1		28,049.1	
Adjusted rho-square:		0.157		0.164		0.285		0.287		0.433		0.565	

For configuration _1 and _3 (i.e. low variance) some parameters were non-significant and/or non-reasonable, e.g. with a decrease in the cost for increasing walking time. From configuration _5 onwards the parameters were all highly significant and with logical signs. Rates of substitution indicates as reasonable to associate less nuisance to 1 min. of travel in train (local train, S-train, Metro) than by bus, as these have a higher level of service and provide better possibility to work while travelling. However, the estimates indicate that, for short trips, one minute of travel on a regional or IC train is associated with higher nuisance than when travelling by bus. One possible explanation of this could be the somewhat more difficult boarding/alighting of the trains and the lower accessibility of trains' platforms. Another finding was that travellers prefer, reasonably, to travel with a high frequency line, which is seen through a negative parameter value for short headways.

Though highly statistically significant, it seems that the rate of substitution associated to travel time and headway were not stable across the cases with lower stochasticity, but only became stable after a higher level of stochasticity introduced. This is also reflected by overlapping confidence intervals in Figure 5, which shows (for high variance cases) 95% confidence intervals associated to the rates of substitution (the parameter indices are defined in Table 5).

From Table 5 it can be seen that the rates of substitution associated to the number of transfers do not appear stable across stochasticity levels. Figure 5 also shows highly non-overlapping confidence intervals of these rates of substitution for configurations _7, _9 and _10. However, in general these configurations found that changing to a bus (from train or bus) is associated with a higher nuisance (corresponds to 6.5-11.6 min. of in-vehicle bus travel time) than when changing to a train (from train or bus, corresponding to 4.6-9.8 min. of in-vehicle bus travel time). This seems reasonable, as train stations and train platforms typically provide better level-of-service than bus stops (e.g., better shelter for weather).

The PSC parameter is also highly significant in all cases and the negative sign of the parameter estimate (for all cases but _1, _3 and _5) was also expected. However, when observing Table 5 and Figure 5 it can be seen that the parameter estimate were highly non-stable across stochasticity levels, although given the dependence of the PSC on the choice set composition, it seems reasonable that this specific term will not be consistently estimated across choice set specifications.

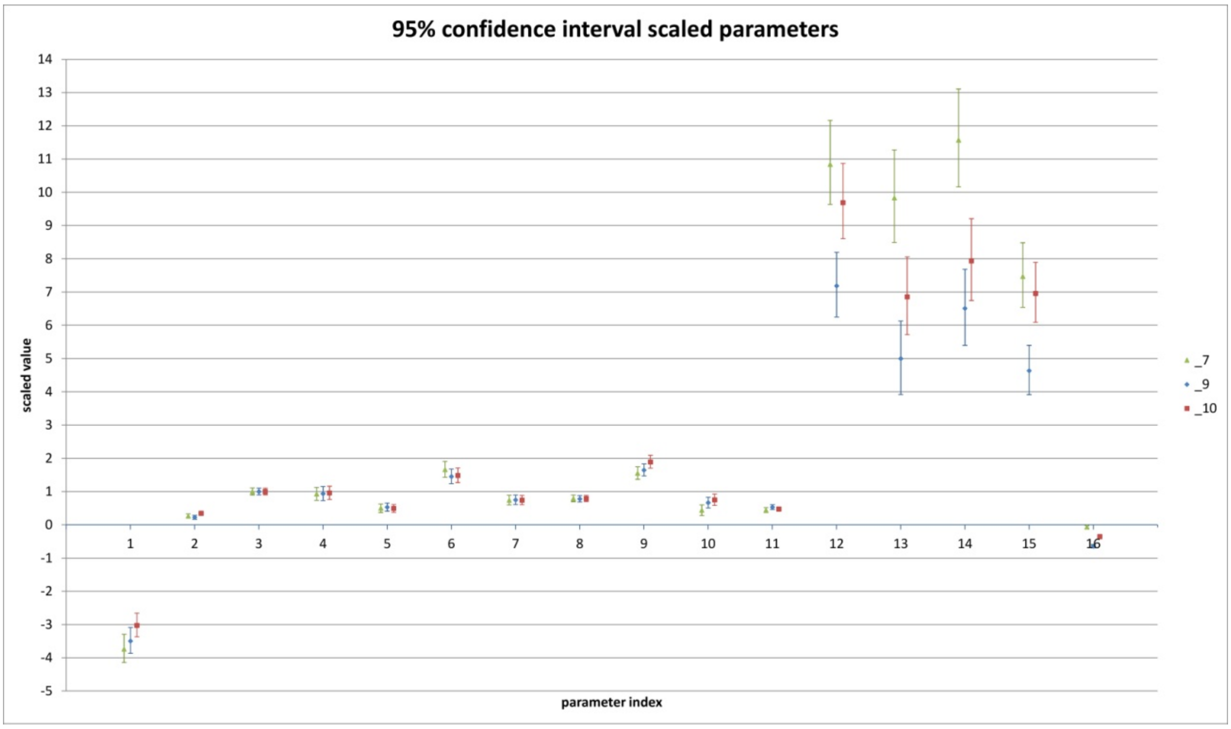

Figure 5 – Confidence intervals of the rates of substitution for the Path Size Logit model estimation on stochasticity levels _7, _9 and _10 for formulation ErrCompErrTerm

5 DISCUSSION

The current study investigated the generation of path choice sets in a complex real-life multimodal public transportation network. The analysis focused on 5,131 actual choices of public transport users in the Greater Copenhagen Area and the choice set quality was evaluated against these revealed preference data.

The study implemented a timetable-based simulation method for choice set generation of public transport paths. The model is very flexible regarding the configuration of the generation (cost) function, as it can capture similarities across alternatives and perception errors through a distributed error term as well as taste heterogeneity through distributed parameters. Various configurations were tested in order to be able to give recommendations. The cost function included in-vehicle time for all the public transport modes available in the Greater Copenhagen Area (i.e., bus, metro, train), waiting and walking (connecting) time at the stations, the number of transfers as well as network-distance-based access/egress time from/to the origin/destination.

Results show that the proposed timetable-based simulation method for choice set generation in general produced very high coverage, especially when using a high level of stochasticity. Adding parameters drawn from a log-normal distribution to account for taste heterogeneity improved the results considerably compared to the traditional single stochastic model with a gamma distributed error term. Distributing the parameters without having a distributed error term also generated good results, as it actually performed better than the traditional single stochastic formulations at low levels of stochasticity. The formulation was however outperformed by the doubly stochastic formulation, which generated the best results among the three formulations tested. When evaluating the coverage, it was important to bear in mind that while high coverage should be sought, it is usually not possible to obtain 100% coverage. This is due to possible (non-traceable) errors in the observed data as well as deviations between the real-life situation when collecting the observed data and the available network data. When comparing to coverage levels obtained elsewhere in the literature (e.g., Ramming, 2002; Prato and Bekhor, 2007), it was confirmed that very high coverage levels was found using the doubly stochastic formulation of the timetable-based simulation method. It should be noted that results are not directly comparable across different studies as different data sources (networks, observations) and methods were used. Additionally, results are highly dependent on the chosen overlap threshold as well as aggregation level (e.g., departure level, trip leg mode sequence). The choice of this should depend on, among others, the level of detail as well as accuracy of the available data. However, the current analysis indeed showed high coverage levels even at very high overlap thresholds and for both line level and stop level. Future research could seek to apply some of the numerous alternative methods for choice set generation proposed in literature (section 1) on the network and observations, thereby facilitating consistent comparison across methods.

For all formulations, the coverage seemed to increase when increasing the level of stochasticity. The improvements were small at high levels of stochasticity though, and our tests showed that adding too much stochasticity generated large choice sets with counterintuitive paths containing geospatial loops. Consequently, adding stochasticity to improve coverage should be done with parsimony. For the lowest levels of stochasticity, the size of the choice sets did not grow very fast, indicating that the same paths were generated over and over. For the highest levels of stochasticity, large choice sets were generated within a reasonable amount of iterations, corresponding to a very high efficiency in terms of generating alternatives. The observed path was also often among the initial alternatives generated to the choice sets, which was seen through a very high coverage level at 40 iterations. Actually, using 200 iterations rather than 40 iterations, at the cost of a 5-fold increase in calculation time, only improved the coverage marginally.

The study found that large choice sets containing counterintuitive paths were generated when increasing the scale parameters above 1. Such paths are not only behaviourally unrealistic, but may also influence the subsequent step where the choice sets are typically used for either estimation or prediction purposes. When used for prediction, the large choice sets could potentially pose a computational challenge if a path-based solution algorithm is used. When used for estimation, the study found statistically significant and reasonable parameter values when using choice sets generated at high stochasticity levels. Furthermore, apparently adding counterintuitive paths did not change the estimates considerably for the rates of substitution related to time. However, the rates of substitution associated to the number and type of transfers, though highly significant and at a logical level, did not stabilise above a certain stochasticity level. Accordingly, the parameter estimates for transfers are apparently highly dependent on the composition of the choice set.

The value of the rate of substitution of changing reported in other revealed preference studies also varies greatly between studies, ranging from 3.8 (relative to in-vehicle metro travel time) in Raveau et al. (2011) to 22.4 (relative to in-vehicle train travel time) in Vrtic and Axhausen (2002). None of the studies reporting evidence on the value of changes has investigated in detail the implication of varying choice set composition on the estimated values, often also because the choice set composition effect was simply not considered. An interesting future research direction would be to investigate whether the fluctuation found arises due to the discrete nature of the variable(s) with values typically in the lower end of the scale (0, 1 or 2). It would also be interesting in a future study to account for trip purpose in the specification of the generation function as well as in the choice model estimation, to see whether this would improve the coverage even further and possibly generate better model fit and more stable estimates for the parameters associated to changes.

Consistently with the existing route choice literature, the present study did not address the issue of consistency across the choice set generation component and the choice model component (estimation or prediction). Moreover, the specification of the generation function and the utility function of the estimation process were different: the functions contained different components (e.g. the utility function contained path-based attributes such as PSC correction), and the stochasticity and the size of this were specified differently. Some of these components and the defined stochasticity does not easily break down from path- to link-level, which are typically required by choice set generation methods as these adopt search-tree algorithms in the generation of paths. Ideally, theoretical consistency should be ensured across the choice set generation component and the choice model component by using the same specification of the utility and generation function. Only thereby does the 'hypothesis' about traveller preferences (used in the path generation) become consistent with the preferences actually estimated based on these. The development of methods which ensure this consistency across model components is an important future research direction.

6 CONCLUSIONS

This study investigates actual path choices of public transport users and assesses choice set quality against these. The study illustrates that the timetable-based simulation method for choice set generation of public transport paths is applicable to large-scale networks, produces good results in terms of coverage and the facilitation of consistent parameter estimates when the choice sets are used for estimation. It was found that the level of the introduced stochasticity should be defined with parsimony, as adding stochasticity translates into the generation of redundant and counterintuitive paths after a certain level. An interesting finding is however that the estimation results are not affected considerably by the presence of these (with the exception of the parameters associated to changes). Adding variability across people improved the results considerably, and the best results were seen with the doubly stochastic formulation when the level of the stochasticity introduced was high, although not too high.

ACKNOWLEDGEMENTS

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APPENDIX 2: RASMUSSEN ET AL. (2014B)

IMPROVED METHODS TO DEDUCT TRIP LEGS AND MODE FROM TRAVEL SURVEYS USING WEARABLE GPS DEVICES: A CASE STUDY FROM THE GREATER COPENHAGEN AREA

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Abstract: GPS data collection has become an important means of investigating travel behaviour. This is because such data ideally provide far more detailed information on route choice and travel patterns over a longer time period than possible from traditional travel survey methods. Wearing a GPS unit is furthermore less requiring for the respondents than filling out (large) questionnaires. It places however high requirements to the post-processing of the data. This study developed and tested a combined fuzzy logic and GIS-based algorithm to process raw GPS data. The algorithm is applied to GPS data collected in the highly complex large-scale multi-modal transport network of the Greater Copenhagen Area. It detects trips, trip legs and distinguishes between five modes of transport. The algorithm was validated by comparing with a control questionnaire. This showed that it (i) identified corresponding trip legs for 82% of the reported trip legs, (ii) avoided classifying non-trips such as scatter around activities as trip legs and (iii) identified the correct mode of transport for more than 90% of trip legs. The method thus makes it possible to use GPS for travel surveys in large-scale multi-modal networks.

Keywords: GPS data processing; revealed preference data; multimodal travel survey; handheld GPS; GIS

1 INTRODUCTION

Over the last 20 years Global Positioning Systems (GPS) have been applied in various investigations of transport-related issues. These applications include, among others, (i) evaluation of system performance, such as measuring historical and real-time congestion and flow levels (Quiroga and Bullock, 1998; Quiroga, 2000; Li et al., 2004; Herrera et al., 2010), (ii) analysis of travel behaviour, such as response to road pricing schemes (Nielsen, 2004; Liu et al., 2010), and (iii) estimation of route choice parameters in route choice models (Rich and Nielsen, 2007; Bierlaire et al., 2013; Prato et al., 2014).

In recent years much effort has been made to investigate the use of GPS devices as the data source for travel surveys (Wolf, 2000; Gong et al., 2011; Bolbol et al., 2012; Stopher et al., 2005, etc.). When compared to traditional travel diaries, collecting data via GPS devices ideally provides the investigator with far more detailed information on travel times, used routes and locations of activities. Another advantage of using GPS data is that it is not dependent on individuals' (possibly mis-) perception of travel time, travel distance and departure time (revealed preferences rather than stated preferences). In traditional travel diaries there is often a common problem of underreporting of trips (e.g., Stopher et al., 2007; Forrest and Pearson, 2005). This problem is likely to be reduced when using GPS as all movements of participants are logged (Stopher et al., 2008). Additionally, far less effort is required by the respondents as answering time-consuming questionnaires can be avoided. This enables larger sample sizes and data collection over a longer time period per respondent.

Today GPS units are sufficiently accurate, lightweight and have long enough battery-life to make multi-day individual-based data collection possible for all conducted trips (e.g. Stopher and Shen, 2011; Gong et al., 2011; Bolbol et al., 2012). Such a data collection facilitates complete analyses and better understanding of individuals' travel patterns. This includes choice of mode of transport, combination of modes, route choices in multi-modal transport networks and day-to-day variations.

Collection of GPS data generates very large data sets, containing millions of GPS logs and a lot of non-relevant data in the form of e.g. scatter. Manual processing of such data is highly unfeasible. The possibility to utilize such data thus relies heavily on the availability of computer-based analysis tools which automatically process the raw dataset and convert the processed data into a format which is usable in the subsequent analyses of e.g. mode and route choice and travel patterns. This paper describes the results obtained by applying the existing POSDAP (2012) algorithm to a multi-day individual-based GPS data set collected among families in the Greater Copenhagen Area. The POSDAP (2012) was developed by Schüssler and Axhausen (2009) and identifies trips, trip legs and mode used. In the present paper we extend this method by proposing a method that utilises a Geographical Information System (GIS) to identify the mode used. The extension additionally includes algorithms to detect and correct illogical mode chains and transfers. To enable comparisons this extended method was tested on the Greater Copenhagen Area case study. Corresponding traditional interview-based travel survey data were collected for each of the respondents for one of the days in the survey period. This made it possible to validate the results of the trip and mode identification algorithms.

Section 2 of the paper presents a review of the existing literature focused on using GPS as a travel survey method. The case study is introduced in section 3, while section 4 presents the method used to post-process the GPS data. Section 5 reports the results obtained by applying the methods to the case study and discuss further research directions. Section 6 relates the results to findings in similar studies and concludes the work. A preliminary version of the work was presented in Rasmussen et al. (2013).

2 LITERATURE REVIEW

The literature review is divided into two parts. Section 2.1 focuses on how GPS devices have been used in travel surveys, whereas section 2.2 focuses on approaches for post-processing raw GPS data.

2.1 GPS IN TRAVEL SURVEYS

Technology limited the first travel surveys using GPS to only being vehicle-based, as the devices were large and the power consumption was high (Wagner et al., 1996; Yalamanchili et al., 1999). These early studies sought mainly to supplement telephone-based travel surveys by collecting additional data to e.g. identify detailed route choices, verify exact time of day as well as detect unreported trips (Wolf, 2000). Additional trip information such as trip purpose was specified by the respondents when starting a trip (Yalamanchili et al., 1999; Du and Aultman-Hall, 2007). This was often done on a connected personal digital assistant (PDA).

Draijer et al. (2000) was the first study to expand a GPS-based data collection to support several modes of transport. Respondents were asked to wear a GPS and a PDA device on all trips. There was however a consistent underreporting of trips due to the size and weight of the devices (approx. 2kg). These trips were walking, cycling and public transport trips as well as trips with the purpose of shopping and visiting friends. The survey design demanded a constant effort from the participants, as the respondents were asked to turn the device on/off when starting/ending a trip and answer questions on the PDA. Several studies have combined GPS traces with additional information gathered by a travel survey questionnaire. Among these are the studies by de Jong and Mensonides (2003), Bohte and Maat (2009) and Tsui and Shalaby (2006). These used internet-based questionnaires where respondents needed to confirm the trips identified by the trip identification algorithm. As GPS devices have become smaller and lighter, multi-modal GPS based travel surveys have become extensively applied as travel survey method.

Much has been done to reduce the effort needed by the respondents, and many studies today therefore do not ask participants to provide trip information *en-route* (Schüssler and Axhausen, 2009; Stopher and Shen, 2011). This however sets higher requirements to the post-processing algorithms as these need to identify trip legs and mode of travel from raw GPS data consisting solely of time and space information. Later studies have proposed and analysed fully automatic GPS data processing methods (e.g., Schüssler and Axhausen, 2009; Bolbol et al., 2012). These do not require any questionnaire data in the post-processing. Schüssler and Axhausen (2009) processed GPS data collected in Switzerland with no additional information provided by the respondents. The data set included 4,882 participants wearing the GPS devices for 6.65 days on average. The results were compared to the existing (national) travel survey. This showed that in aggregate figures the trip and mode identification only deviate slightly from that of the census data. However, the study did not perform any disaggregate evaluation of individual data. This was done in Bolbol et al. (2012), where 81 respondents wore a GPS device for 2 weeks but also answered a travel diary questionnaire. Based on speed and acceleration only, the study designated each trip to one of six different modes. When comparing to the travel diary it was found that most modes could be inferred correctly. Some modes however had very similar speed and acceleration profiles, making it harder to distinguish between bus and metro, and between bus and bicycle.

Alternative approaches utilising information on local spatial information in the identification have also been proposed (e.g., Chen et al., 2010; Chung and Shalaby, 2005; Tsui and Shalaby, 2006; Bohte and Maat, 2009; Stopher et al., 2005; Gong et al., 2011; Schüssler, 2010). These methods and applications have shortcomings in either (i) not including modes which are important in an application to the Greater Copenhagen Area (rail is not included in Chung and Shalaby (2005), and bicycles are not included in Chen et al. (2010) and Gong et al. (2011)), (ii) relying on prompted recall surveys where participants need to verify their trips (Bohte and Maat, 2009; Stopher et al., 2005), and/or (iii) including a very small sample of participants (only 9 participants in Tsui and Shalaby (2006)). These shortcomings were addressed in the present study. Moreover, the study included the five most dominant modes in the Greater Copenhagen Area and 183 participants totalling 644 person days of travel. The five modes cover in total 97.5% of all trips undertaken in the Greater Copenhagen Area (according to the Danish National Travel Survey (Christiansen, 2012)) and the sample size is sufficiently large to validate the algorithms.

2.2 Post-processing of GPS data

Post-processing of raw GPS data typically involves four steps, namely (i) GPS data cleaning, (ii) trip and activity identification, (iii) trip segmentation into single-mode trip legs, and (iv) mode identification. The approach varies slightly between studies, e.g. steps ii) and iii) are performed jointly in Stopher et al. (2005). Some analyses subsequently apply additional steps. Chen et al. (2010), Stopher et al. (2005) and others infer the purpose of the trips identified, while e.g. Schüssler and Axhausen (2009) match the identified trip legs onto the corresponding modal networks.

Most analyses set off with a cleaning and filtering step, where systematic and random errors are removed from the data. This is often conducted by use of the number of satellites visible and the Horizontal Dilution Of Position (HDOP) (Nielsen and Jørgensen, 2004; Stopher et al., 2005). Random errors can be dealt with by including a data smoothing algorithm (Schüssler and Axhausen, 2009).

Trip end points (activity points) are often identified at a location where the device has been stationary for a period of time and/or if the spatial density of observations has been high for a period of time (Schüssler and Axhausen, 2009; Stopher et al., 2005). The result is a number of trips which are defined as being from one activity point to the next. This approach was evaluated in Schüssler (2010). The study finds that the algorithm correctly detected 97% of stated activities without detecting any false activities. Most studies further split trips into trip segments (or trip legs), defined by a change of mode. Correct trip segmentation is crucial for the subsequent identification of the mode of travel of the trip legs. de Jong and Mensonides (2003) divide trips into trip legs whenever the speed drops to 0 km/t, with the option to combine segments again if no mode change occurred. Schüssler (2010) and Tsui and Shalaby (2006) initially identify walking segments if speed and acceleration are low. This is done under the assumption that trip legs of all other modes are preceded or followed by such short walking segments (or by time gaps).

Several studies find that most modes can be identified by only using the speed and acceleration profiles gathered by the GPS device. Moreover, Bolbol et al. (2012) found that using the acceleration profile rather than the speed profile induces better results when distinguishing between modes. The best results were however found when combining the two profiles. This is an easy and efficient approach for the correct identification of some modes. Certain modes can however not be clearly distinguished by such an approach. For example Bolbol et al. (2012) found that bus and bicycle trips in the Greater London Area have similar speed and acceleration profiles. Tsui and Shalaby (2006) found that bus characteristics overlap with characteristics of several other modes.

Other techniques have been proposed to improve the mode detection. Among these are the application of map matching to mode-specific networks by use of GIS-software (e.g., Chen et al., 2010; Chung and Shalaby, 2005; Tsui and Shalaby, 2006; Bohte and Maat, 2009; Stopher et al., 2005; Gong et al., 2011; Schüssler, 2010). In Gong et al. (2011) rail and bus trip legs are identified based on the proximity of start and end locations to rail stations and bus stops. A similar approach for bus trip legs is proposed in Schüssler (2010). Using the proximity to bus stops of start and end locations to identify bus trips seems insufficient in urban areas where the bus network is extensive; trip legs starting and ending near bus stops might have been done by e.g. bicycle rather than by bus. Another approach is to utilise available information about the respondents implicitly in the identification of modes. Stopher et al. (2008) allow only car or bicycle as mode for a trip leg if the household has a car or bicycle at its disposal, respectively. However, these approaches have limitations if applied to a typical Scandinavian city where the bus network is dense and the ownership and use of bicycles is relatively high[1].

3 CASE STUDY: GREATER COPENHAGEN AREA

The study area covers the Greater Copenhagen Area in which approximately 2 million people live. The study utilised data which were collected as part of the ongoing research project 'Analyses of activity-based travel chains and sustainable mobility' (ACTUM). The dataset included 53 households, corresponding to 183 persons from 6 to 58 years of age. The households were sampled from the Danish National Travel Survey (Christiansen, 2012). All participants were asked to bring a GPS device on all trips undertaken within a period of 3-5 days. Additionally, each respondent was asked to fill in an internet-based travel diary corresponding to one of the days for which GPS data were also collected. This enabled a validation of the new fully automatic trip- and mode detection algorithm.

The travel diaries were linked to the recorded GPS observations resulting in travel diaries with corresponding GPS data for 101 person days. Consequently, there were 82 persons for which data could not be linked. An analysis identified that this was due to one of the following three reasons; (i) the respondent failed to answer the survey for a day where he/she also carried a GPS device, (ii) no or only few GPS data were collected for the day where the survey was filled in, or (iii) there was a large difference between the number of trips reported and what could be seen in the GPS traces.

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[1] 4 out of 5 inhabitants in Copenhagen have access to a bicycle (2012), and bicycling constitutes 36% (2012) of the trips (Municipality of Copenhagen, 2014). In Odense bicycling constitutes 24% (2012) of the trips (Municipality of Odense, 2014).

The GPS device was the wearable KVM BTT08M (KVM, 2013). It logged data every second, thereby facilitating a high level of accuracy for the identification of *en route* travel choices and the location of trip ends. The dataset contained in total 6,419,441 collected GPS points (observations) corresponding to 1,783 hours of travel (including stationary and error data), and was collected on 644 person days of travelling.

The GIS-based analyses utilised a detailed digital representation of the road and public transport networks of the Greater Copenhagen Area. The road network was based on the road network of NAVTEQ (2010) and was in a format that allowed for a complex map matching algorithm to be run (Nielsen and Jørgensen, 2004). The public transport network used for mode identification of rail trip legs was a digital representation of the rail line alignment. The analysis distinguishing between bus and car utilised a disaggregate public transport network representation containing information on bus route alignment and stop locations for all bus lines and bus line variants.

4 METHODOLOGY

The study developed and tested a fully automatic method to post-process GPS data without requiring any information about or from the GPS carrier. The method performed, and iterated between, a series of steps. These steps identified activities (trip ends), trip legs and the most probable mode chosen. The method was based on the automatic trip and mode detection algorithm developed in Schüssler and Axhausen (2009). This was modified in order to improve the results in three way; i) GIS analyses were used to better distinguish between modes with similar speed and acceleration characteristics, ii) advanced feedback loops between steps were used, allowing inconsistent mode-sequences to alter the trip leg detection algorithm, and iii) map matching was used to exclude non-trips and hinder wrongly splitting trips on motorways. The method consisted of a 6-step process as shown in Figure 1. The following subsections present a detailed description of the steps of the algorithm.

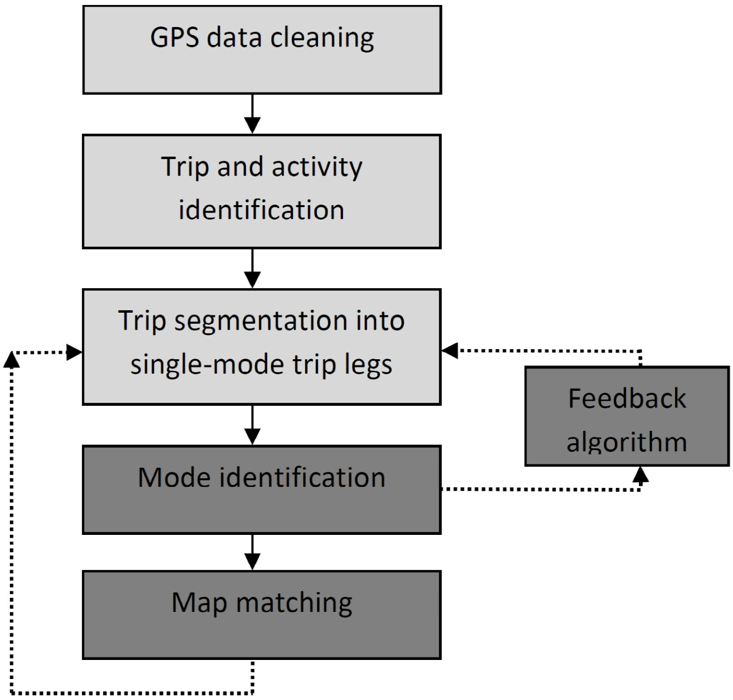


Figure 1 – Approach used in this study. Boxes highlighted in light grey denote steps that are similar to corresponding steps in Schüssler and Axhausen (2009). Boxes highlighted in dark grey are steps where this paper contributes with new, alternative methods

4.1 GPS DATA CLEANING

The parameter values used for data cleaning vary across studies (Stopher et al., 2005; Schüssler and Axhausen, 2009; Tsui and Shalaby, 2006; Stopher et al., 2005). Most studies however require four satellites to be visible (Stopher et al., 2005; Schüssler and Axhausen, 2009). This ensures obtaining coordinates in three dimensions. To get accurate positions sufficient dispersion of the satellites is often required. Most studies ensures this by requiring a HDOP value of less than 4-5 (Tsui and Shalaby, 2006; Schüssler and Axhausen, 2009). This study adopted the values from Schüssler and Axhausen (2009), which required a minimum of 4 satellites to be visible and a HDOP value lower than 4. In addition, only observations with altitude levels between -37 meters and +201 meters were regarded as acceptable. This corresponds to the altitude range in Denmark +/- 30 meters, where the +/- 30 meters equals three times the standard deviation of measurement for the GPS devices. Lastly, a Gauss kernel smoothing approach was used to remove systematic errors and perform data smoothing as suggested by Schüssler and Axhausen (2009).

4.2 TRIP AND ACTIVITY IDENTIFICATION

The trip and activity identification algorithm developed in Schüssler and Axhausen (2009) was applied to identify trips. The activities (trip ends) were identified as locations where the bearer of the GPS was stationary for a period of time. The specification of the length of this period of time was based on the best compromise between identifying short stops (e.g., picking up persons) and falsely detecting activities (e.g., when driving in congested traffic or waiting at traffic signals) (Chen et al., 2010; Tsui and Shalaby, 2006; Stopher et al., 2005; Wolf, 2000). Accordingly, using the rules put forth in Schüssler and Axhausen (2009), an activity point was defined if at least one of three criteria was met; (i) if there was a time gap between consecutive observations of 120 seconds or more, (ii) if the speed had been lower than 0.01 m/s for at least 60 seconds, or (iii) if the location of the GPS device was within a limited area for at least 60 seconds. The first situation occurred when the device had been stationary for a while, which caused the unit to turn off to save battery, or if the GPS signal was lost during a trip leg. The last observation before the time gap was flagged as 'beginning of (time) gap' if the signal was lost, and the first observation when the signal was re-established was flagged as 'end of (time) gap'. The last criterion was analysed by use of bundles of GPS points (Schüssler and Axhausen, 2009).The point density was calculated for every observation, corresponding to the number of observations within a 15 meter radius of the respective observation. An activity was flagged if this number exceeded 30 for at least two-thirds of the points in a sequence and the sequence lasted for more than 60 seconds (60 observations as the GPS device logged every second).

4.3 TRIP SEGMENTATION INTO TRIP LEGS

A trip between two activities might involve several trip legs with different modes of transport or changing between vehicles of the same mode (e.g. changing between train lines). The split of trips into trip legs was done by applying the approach of Schüssler and Axhausen (2009). Trip legs were identified by assuming that a *short* walking segment is needed between modes or when changing vehicles, e.g. from bike to train or from bus to another bus. The first and last points of such segments were denoted as 'start of walk' and 'end of walk', respectively. The walking segments were identified by means of the unique characteristics of walking (low acceleration and low speed). If an identified walking segment was *long* (90 seconds or longer), it was defined as a separate trip leg.

A new trip leg was specified if 3 criteria were fulfilled. The first criterion required the speed in the mode change (or walking segment) to never exceed 2.00 m/s and the acceleration to never exceed 0.1 m/s^2. The second criterion required the length of the derived trip legs to be at least 90 seconds for walking trip legs and at least 120 seconds for trip legs of all other modes. The last criterion required the short walking segment between two trip legs to include observations that were flagged as a 'start of walk' (or 'end of gap') and an 'end of walk' (or 'beginning of gap') (see section 4.2). These criteria are similar to the ones applied in Schüssler and Axhausen (2009), but a test of different values of the thresholds led to a slight modification of some of the thresholds. Schüssler and Axhausen (2009) used a maximum speed of 2.78m/s and a minimum duration of walking trip legs of 60 seconds.

4.4 MODE IDENTIFICATION

Each trip leg was associated with a most probable mode of transport. The identification process was partly based on speed and acceleration profiles, similar to the approach used in Schüssler and Axhausen (2009) and in Bolbol et al. (2012). The driving conditions in the Greater Copenhagen Area however range from being slow moving traffic through congested urban areas to fast moving traffic on motorways. Additionally, in urban areas it is hard to distinguish whether the respondent is driving in a bus or following close behind it in a car, or even biking next to it. These factors cause problems distinguishing between modes solely based on acceleration and speed profiles.

The present study therefore developed the three-step mode identification process illustrated in Figure 2. This process was based on analyses using the speed and acceleration profiles as well as more advanced analyses conducted in GIS software. The steps are explained further in the following subsections.

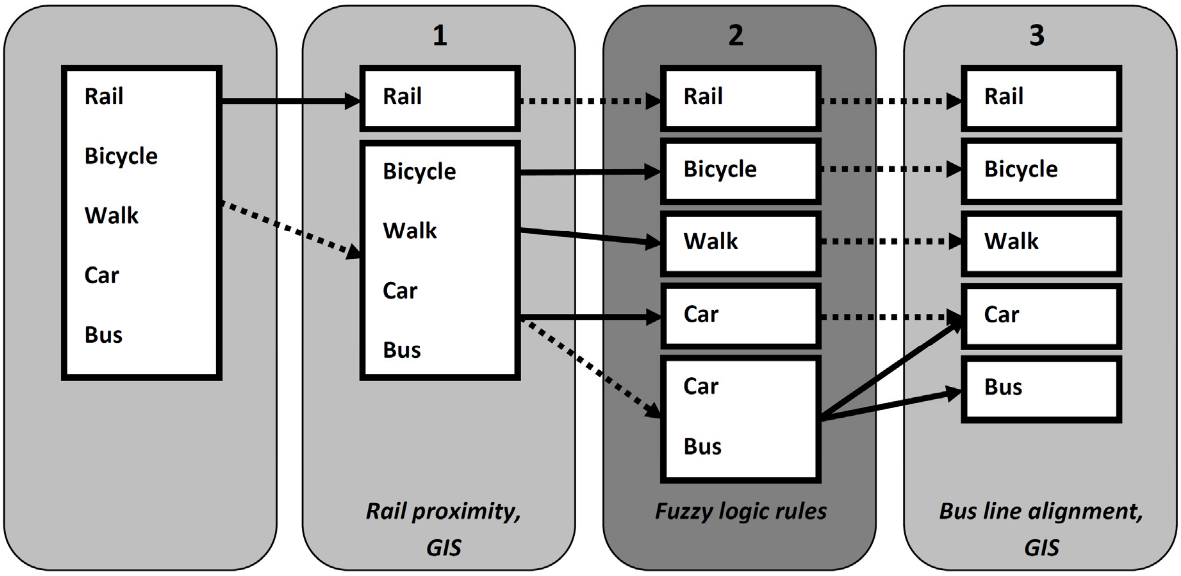
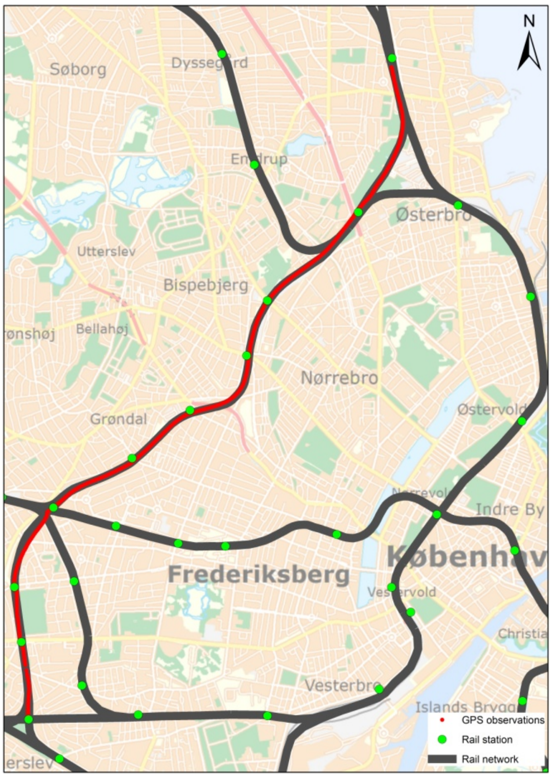


Figure 2 – The stepwise mode classification algorithm. Continuous arrows denote mode classification whereas dotted arrows denote no change from previous step. Step 2 is directly adopted from Schüssler and Axhausen (2009), but with adapted fuzzy logic rules

4.4.1 STEP 1: RAIL PROXIMITY

Rail networks are typically characterised by not having the same spatial location as the street and path network (with the exception of on-street light rail and tram lines). This is also the case in the Greater Copenhagen Area, and trip legs using rail could easily be distinguished from others by their close proximity to the alignment of the rail network. Consequently, the first step identified rail trip legs based on the proximity of observations to the rail network. If more than 75% of the observations in one trip leg were located less than 25 meters from the rail network, the trip segment was classified as being a rail trip. The 75% was chosen instead of 100% to account for potential small errors in the digital representation of the network as possible measurement errors of the GPS units. Additionally, the length of a rail trip leg was required to be at least 250 meters. This was done to avoid classifying walking trips on railway station platforms as rail trips. 250 meters was chosen as this is less than the shortest distance between railway stops in the Greater Copenhagen Area, but longer than most within-platform walking trips. An example of a successfully identified rail trip leg is shown in Figure 3. In this example almost 98% of the observations for the trip leg were located within 25 meters of the rail network.



Trip leg information

Date:	09-11-2011
Starting time:	16:07:01
Ending time:	16:22:53
Trip length:	15 min 52 sec
Number of obs.:	947
Number of obs. within 25m:	924
Percentage rail:	97.6%

Figure 3 – Example of a rail trip leg (shown with red) on the Danish S-train ring line identified by the rail proximity algorithm. The railway network is highlighted by bold black lines, and stations with green dots

4.4.2 Step 2: Fuzzy logic rules

The next step was to determine the mode of travel of the remaining trip legs by applying the fuzzy logic method developed by Schüssler and Axhausen (2009). The distinction between walk, bicycle, car and bus was done by applying certain logic rules to the speed and acceleration profiles of the trip legs. The modes were found to be best distinguished if using the median speed together with peak values of speed and acceleration. Most studies represent the peak values using 75-95 percentiles rather than extremes to take into account outliers (Stopher et al., 2005; Gong et al., 2011; Tsui and Shalaby, 2006; Schüssler and Axhausen, 2009). As in Schüssler and Axhausen (2009) the study used the 95th percentiles of speed and acceleration in addition to the median speed.

Each profile was divided into three or four (possibly overlapping) intervals as proposed by Schüssler and Axhausen (2009). The division was based on an empirical analysis of the sample of trip legs in the data for which the mode was known, see Figure 4. Combining these defined intervals across the profiles by applying certain fuzzy logic rules facilitated the mode identification. The fuzzy logic rules applied are reported in Table 1.

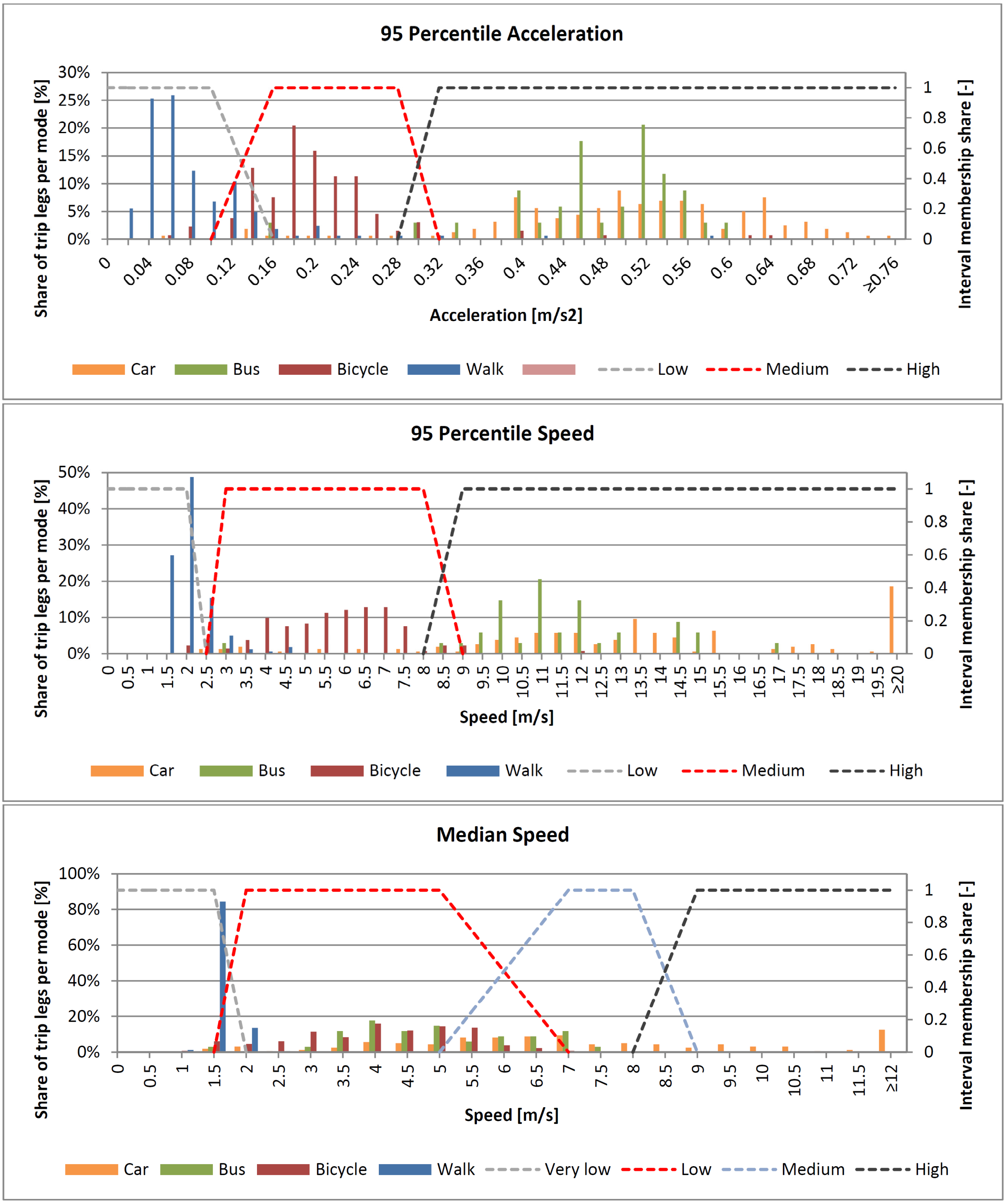


Figure 4 – The distributions of the 95th percentiles of speed and acceleration and the median speed for the subset of trip legs for which the mode is known (rail excluded) from the control questionnaire

Table 1 – Fuzzy logic rules applied

95^{th} percentile acc.	95^{th} percentile speed	Median speed	Mode classification
Low	Low	Very low	Walk
Low	Medium	Very low	Walk
Low	High	Very low	Bike
Medium	Low	Very low	Walk
Medium	Medium	Very low	Walk
Medium	High	Very low	Bike
High		Very low	Car
Low	Low	Low	Walk
Low	Medium	Low	Bike
Low	High	Low	Bike
Medium	Low	Low	Bike
Medium	Medium	Low	Bike
Medium	High	Low	Car
High	Low	Low	Bus
High	Medium	Low	Car
High	High	Low	Bus
Low	Low	Medium	Bike
Low	Medium	Medium	Bike
Low	High	Medium	Car
Medium	Low	Medium	Bike
Medium	Medium	Medium	Bike
Medium	High	Medium	Car
High	Low	Medium	Car
High	Medium	Medium	Car
High	High	Medium	Bus
Low		High	Car
Low		High	Car
Low		High	Car

Even though most trip legs were distinguishable based on a combination of speed and acceleration, this method did not uniquely separate modes (Figure 4). Walk and bicycle were the two modes which could be identified with a high chance of success based on the profiles. This is because these had the least overlaps with the other modes due to the consistently low maximum speed. These two modes could also be distinguished from each other when combining the profiles.

Trip legs undertaken by car and bus were difficult to distinguish from each other based on the profiles. More advanced analyses were needed, and a two-fold approach was used to identify bus trips. Initially a separation was done based on the profiles, clearly distinguishing between trips which were assumed to *definitely* be car trips and trips which were *either* car or bus trips. Hence, all trip legs which conferred with the speed and acceleration intervals for bus trips were classified as *potential* bus trips. Through this, the sample of initially classified *potential* bus trips included all actual bus trips and a large subset of car trips. Subsequently, the identification of *actual* bus trips among this set of initially classified *potential* bus trips were done in step 3 (section 4.4.3). The trip legs not classified as bus trips in step 3 were assumed to be car trips and added to the set of already identified car trip legs.

4.4.3 Step 3: Bus line alignment

A new approach for separating car and bus trips was developed in the study. This was based on a thorough analysis of coherence between GPS-recorded stop locations and bus line bus stops. Moreover, the subset of *potential* bus trip legs was analysed to identify whether they follow the stopping pattern of any bus line.

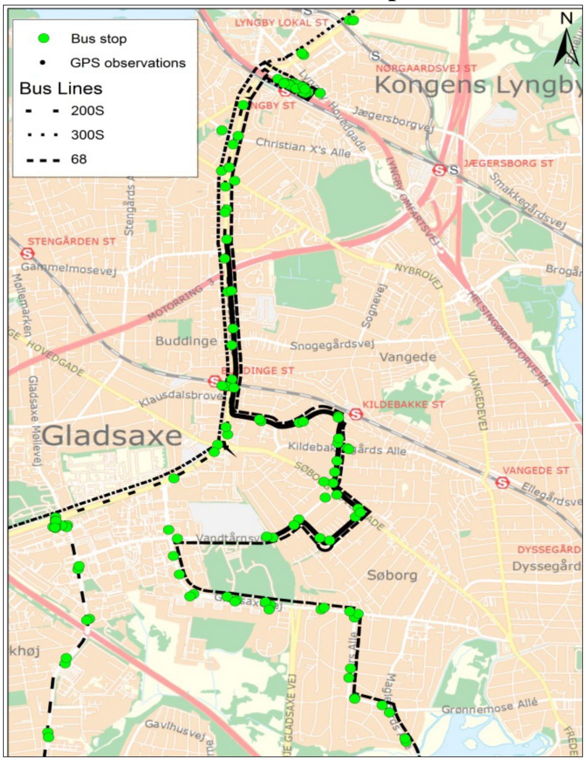
The initial step of the identification specified that IF at least 15 GPS observations were located less than 25 meters from a bus stop, THEN the algorithm flagged the trip leg as stopping at the bus stop (note that one GPS record was stored per second). Next, IF the GPS recorded stops at at least 60% of potential bus stops between boarding and alighting stops on any bus line, THEN the trip leg was flagged as a *probable* bus trip (on bus lines fulfilling this criterion). The rather low percentage of 60% was applied to take into account bus routes with few passengers where the bus often does not stop at all bus stops. The threshold was set at 80% for high demand bus lines.

Subsequently, the trip legs in the sample of *probable* bus trip legs were analysed with regards to the location of their start and end. The start point *and* end point of each trip leg were analysed to see if they were both located less than 100 meters from a bus stop on any of the bus lines identified previously. IF this was the case, THEN the trip leg was classified as a bus trip. Otherwise the trip leg was classified as a car trip. An additional benefit of applying this method was the identification of the most probable actual bus line used.

In some cases the trip leg identification algorithm split one actual bus trip leg into several trip legs due to long stopping times. To take this into account, the algorithm analysed the mode classified for the trip leg prior to and the trip leg subsequent to a flagged trip leg (within a timeframe of +/-300 seconds). If any of these were classified as car trip legs, they were reclassified as a *probable* bus trip leg. This was done based on the assumption that within a short timeframe it is more probable that two consecutive trip legs with similar speed and acceleration characteristics are of the same mode (rather than a change from car to bus or vice versa).

Figure 5 shows two examples of the application of the method. The example to the left is an actual bus trip, whereas the example to the right is an actual car trip. The first part of the analyses showed that there (for both examples) were clusters of observations at a large percentage of stops associated to several bus lines. The second part of the analysis determined whether the trip leg started and ended in the close proximity of any of the bus stops on the bus lines identified. In the example to the left the GPS carrier stopped at 14 out of 19 bus stops of bus line 68. The trip leg also began and ended close to bus stops on this line. This caused the trip leg to be correctly classified as a bus trip leg. In the example to the right, the GPS carrier stopped at several stops along bus line 161. The trip leg was however correctly classified as a car trip leg. This was because the trip leg started and ended more than 100 meters away from a bus stop served by bus line 161.

Actual bus trip



Trip information

Date	09-11-2011
Starting time:	16:41:24
Ending time:	16:53:18
Trip length:	11 min 54 sec

Bus line stop overlap	# stops	Hit%
200S	2/2	100%
300S	2/2	100%
68	14/19	74%

Origin stop:	68
Destination stop:	200S/300S/68
Classified as:	Bus trip
Actual bus line:	68

Actual car trip

Trip information

Date:	09-11-2011
Starting time:	06:42:26
Ending time:	06:49:07
Trip length:	6 min 41 sec

Bus line stop overlap	# stops	Hit%
161	3/5	60%

Origin stop:	None
Destination stop:	None
Classified as:	Car trip
Actual bus line:	-

Figure 5 – Example of results from the bus stop algorithm. The lines represent relevant bus lines, while black and green dots represent GPS points and stops, respectively

4.5 ALGORITHMIC FEEDBACK

The trip leg and mode identification was improved by identifying and correcting illogical mode shift patterns in a subsequent feedback step. This was done to avoid wrong trip leg splitting and modal classification due to irregular changes in speed and/or acceleration for a trip leg. Such irregularities could e.g. arise in congested stop-and-go traffic. The feedback algorithm used simple rules to identify irregular mode shifts and was based on a set of probable mode transfers. For example, it is likely that a bicycle trip leg follows or precedes a bus trip leg as some passengers might bicycle to and from the bus stop. It is however not very likely that a bicycle trip leg is followed by a car trip leg with only a short time gap between the trip legs.

Specifically, the algorithm searched for sets of two consecutive trip legs ending and beginning within 120 seconds and 25 meters of each other for which the mode-sequence was *non-likely*. The trip legs concerned were merged into one single leg in such cases. The mode for this merged trip leg was classified by re-running the mode classification on the merged trip leg. Table 2 lists the rules used to identify *non-likely* mode changes.

The feedback algorithm additionally searched for sets of three consecutive trip legs fulfilling certain criteria. Sequences were found where the first and third trip leg were identified as a car trip, but with the second trip leg identified as another mode. If certain criteria on the distance in space and time were fulfilled (see Table 2), the three trip legs were joined and assumed to be a car trip. A similar approach was used for other sequences of three trip legs, see Table 2. Such an approach ensured successful connection of trip legs that were mistakenly split due to congestion.

Table 2 – Feedback algorithms that join trip legs. ΔTime and ΔSpace refer to temporal and spatial distances between end of trip A and start of trip B (C for 3 leg cases), respectively. Order is the classification of the 2 (3) trip legs and Classification is the mode assigned to the merged trip leg

	ΔTime	ΔSpace	Order	Classification
2 consecutive trip legs A→B	≤ 120sec.	≤ 25m	Car-Bicycle Bicycle-Car	Rerun mode identification
			Bus-Car Car-Bus	*Probable* Bus trip, c.f. 4.4.3
3 consecutive trip legs A→B→C	≤ 300sec.	≥ 25m	Car – Bus – Car	Car trip
			Car – Bicycle – Car	Car trip
			Car – Train – Car	Car trip
			Car – Walk – Car	Car trip
			Bicycle – Bus – Car	Car trip
			Bicycle – Bicycle – Car	Car trip
			Bicycle – Bus – Car	Car trip
			Bicycle – Train – Car	Car trip
			Bicycle – Walk – Car	Car trip
			Car – Bus – Bicycle	Car trip
			Car – Bicycle – Bicycle	Car trip
			Car – Train – Bicycle	Car trip
			Car – Walk – Bicycle	Car trip

4.6 MAP MATCHING

In the last component of the algorithm, all trip legs (except trip legs identified as rail trips) were map matched to the NAVTEQ road network (NAVTEQ, 2010). This was done using a map matching algorithm developed at DTU Transport (Nielsen and Jørgensen, 2004). The map matching served two purposes; (i) to detect and correct trip legs which were wrongly split due to congestion on motorways, and (ii) to remove non-trips. Non-trips are e.g. short trip legs generated as a consequence of the GPS device being turned on when no trip was actually undertaken. The first instance was defined as the case when the matched last link and first link of two consecutive trip legs were either a motorway or ramp. These two trip legs were then merged into one. The mode of the merged trip leg was determined by re-running the mode classification on the merged trip leg. Non-trips were identified as instances where either no GPS observations could be map matched or where less than half of the mapped route was found by mapping of actual GPS observations[2]. Such non-trips were then discarded. This ensured that short non-trips were successfully removed, but also caused the removal of some actual walking or bicycle trips (e.g. short walking trips through parks, etc.).

5 RESULTS

Two configurations of the method were tested on the available dataset:

(i) *Algorithm 1* including trip leg and mode identification as well as feedback algorithm (sections 4.1-4.5), but excluding the map matching algorithm,

(ii) *Algorithm 2* including *Algorithm 1* and the map matching algorithm (sections 4.1-4.6).

The effect of including the map matching could be evaluated by comparing the results generated by the two algorithms. The study also evaluated an algorithm similar to *Algorithm 1* but without the feedback algorithm. The results of this evaluation are however not reported, as only 11 trip legs were connected by the feedback algorithm (i.e. almost identical results).

A 'traditional' *Baseline algorithm* was also tested for comparison. This included trip leg and mode identification as proposed by Schüssler and Axhausen (2009), i.e. with the mode identification step based solely on fuzzy logic rules. Different configurations of intervals as well as rules were tested for each of the three algorithms, and the best configuration was chosen for each algorithm.

The above algorithms were run on the full dataset consisting of approximately 664 person days of travel. The following analyses however only used the data subset where the travel mode was known from the additional questionnaire. This included trips that were directly connected to the travel diary data supplied by the respondents as well as trips where in-depth investigation made it possible to deduct the travel information manually. The results of the mode identification were evaluated using two assessment measures. The first measure is the *success rate* which denotes the number of correctly classified trip legs by the algorithm as percentage of the number of *observed* trip legs of that mode. The second measure is the

[2] In cases where only a part of the observed route can be map matched, the map matching algorithm generates the shortest path between links to which observations can be mapped.

confidence rate which denotes the number of correctly classified trip legs by the algorithm as percentage of the number of trip legs of that mode identified by the algorithm. Thus, the latter refers to the percentage of trip legs in the output of the algorithm where the mode was correctly identified. Hence, the first measure relates to the *observed* travel survey trip legs whereas the second measure relates to the trip legs in the output of the *algorithm* which may also include non-trips (see section 4.6).

5.1 TRIP LEG IDENTIFICATION

The total number of trip legs identified was 754, 744 and 464 if using the *Baseline algorithm*, *Algorithm 1* and *Algorithm 2,* respectively. This compares to the total number of reported trip legs in the subset of the travel survey of 521. Three sources of error influenced these numbers:

(i) There were trip legs in the travel survey where no corresponding GPS trip legs could be identified. This could be due to either the respondent not wearing the GPS device, the GPS device not being able to get an acceptable signal or the device not functioning properly.

(ii) Some trip legs were identified by the algorithm even though no corresponding trip information was reported by the respondents in the diary. This error was partly due to underreporting by the respondents. Underreporting has also been observed in other studies including Stopher et al. (2007) and Wolf et al. (2003). Another reason was the identification of non-trips (see section 4.6).

(iii) The algorithm sometimes wrongly separated a trip leg into several trip legs due to long dwell times while travelling. This could for example occur in stop-and-go congested traffic. The opposite was also observed, namely that several actual trip legs were identified as one trip leg if the dwell time(s) between trip legs was very low. We found examples of this happening when a fast travelling cyclist transferred to a local train or bus without any waiting time at the station.

Figure 6 illustrates the results of a comparison between the trip legs identified by the algorithms and the trip legs reported by the respondents.

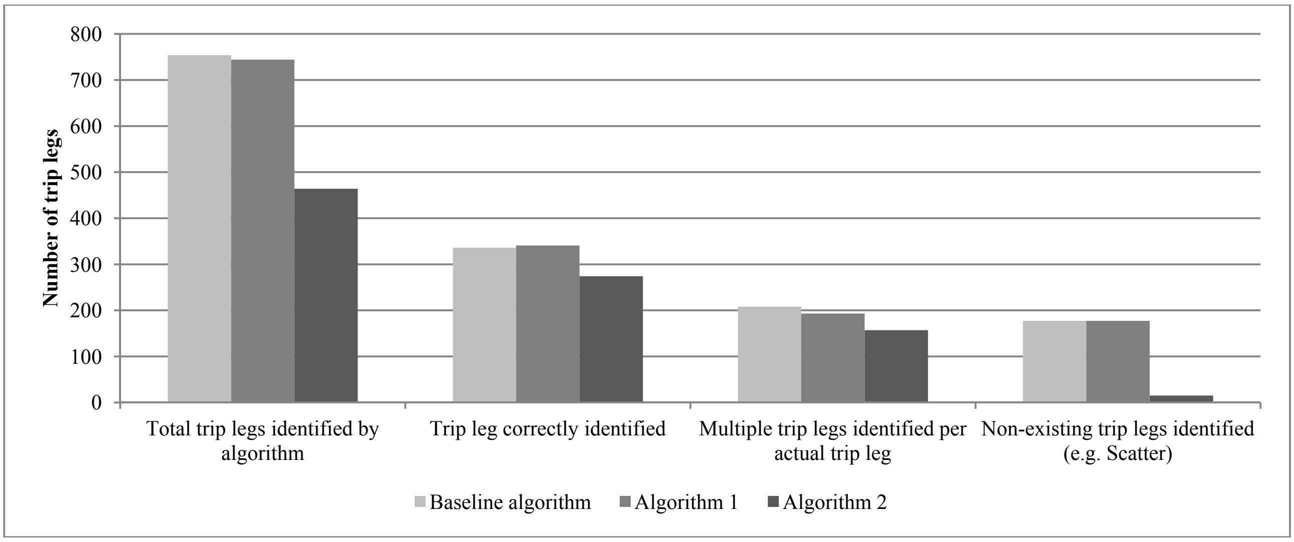


Figure 6 – Classification of trip legs identified by the algorithms

The *Baseline algorithm* generated trip legs with the correct origin and destination for 45% of the identified trip legs. 28% of the identified trip legs were partial trip legs, i.e. one reported trip leg was identified as two or more trip legs by the algorithm. A further 24% of the identified trip legs were non-trips which should not have been detected as a trip leg. Non-trips include random scatter and short trip legs which are not actual trips, e.g. walking around the workplace etc.. The remaining 4% represents trip legs that either included several actual trip legs (not split correctly) or trips where observations had a too low quality for general usage.

The stepwise mode classification algorithm and the feedback algorithm improved the results (*Algorithm 1*). Fewer trip legs were identified and more trip legs were correctly identified. Additionally, the feedback algorithm caused fewer actual trip legs to be wrongly split into several trip legs, as eleven partial trip legs were successfully connected into actual complete trip legs. The map matching algorithm of *Algorithm 2* connected further nine trip legs successfully into four actual trip legs and 143 trip legs were correctly removed from the sample, cf. Figure 6. However, further analysis showed that some trip legs which should be merged remained unconnected. Overall, the best results were achieved when using *Algorithm 2* as this identifies the entire actual trip leg as one trip leg in 59% of the cases, and the entire actual trip leg as one or several trip legs in 93% of the cases. Additionally, the percentage of wrongly identified trip legs (non-trips) dropped to a very low level (3%).

Figure 7 illustrates the distribution of lengths of the trip legs reported and identified by the algorithms. The distribution for *Algorithm 2* fits best with the distribution of the reported trip legs. This suggests that many of the non-trips removed by the map matching algorithm of *Algorithm 2* are short trip legs. We note that the map matching also removed trip legs which were reported in the diary. Figure 7 indicates that these wrongly removed trip legs are short trips. The results presented in section 5.2 support this by determining that most of the wrongly removed trip legs were bicycle or walking trips.

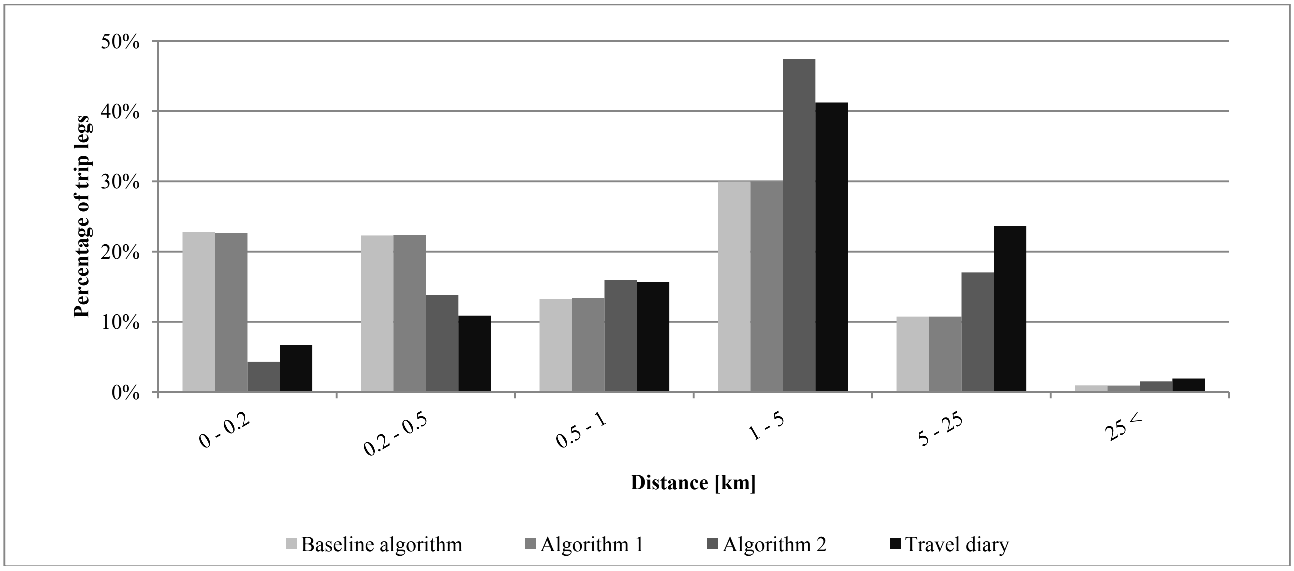


Figure 7 – Trip length for identified GPS trip legs compared to stated travel survey trip lengths

5.2 MODE IDENTIFICATION

This section presents the methods' capability to identify the correct mode of transport of the trip legs. Table 3 reports the results of a disaggregate comparison between the mode identified by the algorithm and the actual chosen mode (*success rate*) for the *Baseline algorithm*. Approximately 82% of the trip legs were assigned the correct mode of transport when only considering trip legs which were actual trip legs. Table 4 reports the corresponding results for *Algorithm 1*. The success rate obtained is 90%. Consequently, results were improved considerably by including the stepwise mode classification algorithm and feedback algorithm. Especially the method to identify rail trips was very efficient – using the fuzzy logic rules caused 24% of the rail trip legs to be correctly identified, whereas the corresponding number for *Algorithm 1* was 97%. Applying the method to identify car and bus trips also improved the results considerably, as the success rate for bus rose from 38% to 73% while the success rate for car rose from 82% to 93%. The success rates for walking and bicycling reduced slightly for *Algorithm 1* when compared to the *Baseline algorithm*.

Table 3 – The results of the mode identification when using Baseline algorithm (compared to reported mode use)

Observed / Algorithm	Walk	Bicycle	Bus	Car	Rail	Non-trips	Confidence rate
Walk	184	12	2	6	-	111	58.4%
Bicycle	9	121	-	13	-	52	62.1%
Bus	-	1	14	9	-	2	53.8%
Car	-	4	21	143	25	12	69.8%
Rail	-	-	-	3	8	2	61.5%
Other	-	-	-	-	-	1	-
Total	193	138	37	174	33	180	**62.3%**
Success rate	95.3%	87.7%	37.8%	82.2%	24.2%	-	**81.7%**

Table 3 and Table 4 however also highlight a weakness of the two approaches. Both approaches identified many trip legs which were not reported in the diary (non-trips generated due to e.g. scatter). This induced the confidence rates to be 62% and 69% for the *Baseline algorithm* and *Algorithm 1*, respectively. The stepwise mode classification algorithm classified generated trip legs considerably better, especially for bicycle, bus (no generated bus trip legs were wrongly classified) and rail. Summarising, the comparison between the *Baseline algorithm* and *Algorithm 1* showed that applying the stepwise mode classification algorithm and the feedback algorithm improved the mode classification (especially the success rate) considerably. The two algorithms however – as also found in section 5.1 – identified too many non-trips.

Table 4 – The results of the mode identification when using Algorithm 1 (compared to reported mode use)

Algorithm \ Observed	Walk	Bicycle	Bus	Car	Rail	Non-trips	Confidence rate
Walk	180	11	2	2	-	111	58.6%
Bicycle	2	114	-	6	-	15	83.2%
Bus	-	-	27	-	-	-	100.0%
Car	4	8	8	156	1	48	69.3%
Rail	3	-	-	-	33	4	82.5%
Other	3	1	-	3	-	2	-
Total	192	134	37	167	34	180	**68.5%**
Success rate	93.8%	85.1%	73.0%	93.4%	97.1%	-	**90.4%**

Many of such non-trips were removed when adding the map matching algorithm of *Algorithm 2* (see section 5.1). This improved the overall confidence rate from 69% to 85% (Table 5). The improvement in confidence rate was however at the cost of also removing a large number of generated trip legs for which a corresponding observed trip leg exists. Specifically, many actual trip legs undertaken by foot or bicycle were discarded by the map matching algorithm. This was probably a consequence of the map matching being conducted on a road network not including bicycle and footpaths. The row denoted by 'Success rate (all)' highlights this issue. The measure represents the share of the *total* number of observed trip legs for which a generated trip leg with the correct mode was identified.

All trip legs identified as bus by the proposed algorithms were correctly classified (confidence rate). However, the success rates of 73-77% generated for bus were the lowest success rates obtained across modes. A disaggregate analysis identified two primary reasons for these lower percentages. The first reason was problems associated with the trip leg identification algorithm and the feedback algorithm. The trip leg identification algorithm caused some actual bus trips to wrongly be split into several trip legs due to congestion, longer dwell times, etc. The feedback algorithm subsequently failed to identify and reconnect these. The other reason was the actual stopping pattern of the buses. At times some buses may have skipped a large percentage of stops, e.g. during evening hours where fewer passengers board the bus.

Table 5 – The results of the mode identification when using Algorithm 2 (compared to reported mode use)

Algorithm \ Observed	Walk	Bicycle	Bus	Car	Rail	Non-trips	Confidence rate
Walk	75	6	1	1	-	13	78.1%
Bicycle	1	104	-	5	-	3	92.0%
Bus	-	-	27	-	-	-	100.0%
Car	1	7	7	152	1	19	81.3%
Rail	-	-	-	-	33	2	94.3%
Other	1	1	-	1	-	1	-
Total	78	118	35	159	34	38	**84.6%**
Total (all)	192	134	37	167	34	-	
Success rate	96.2%	88.1%	77.1%	95.6%	97.1%	-	**92.2%**
Success rate (all)	39.1%	77.6%	73.0%	91.0%	97.1%	-	**(69.3%)**

5.3 Further work

The method to identify trips and trip legs were adopted directly from POSDAP (2012). We found that at times the approach wrongly split trips into several trip legs, and that this also influenced the results of the mode classification. Though the algorithmic feedback captured some of these wrongly split trip legs, more research is needed in the correct detection of trip legs. New methods could be developed which use the available disaggregate digital representation of the infrastructure, possibly in combination with congestion measures. Such an approach could e.g. hinder that trips are wrongly split into several trip legs when queuing at intersections.

The present study analysed all generated trip legs and found that many of these do not have a corresponding observed trip leg reported in the travel diary. This can partly be because of underreporting, but the analysis found that many non-trips were identified around activity locations. While the method developed and tested in this present study aimed at removing such non-trips (through the map matching step), the other studies reviewed do not seem to explicitly deal with these (important) non-trips. The map matching algorithm succeeded in removing most of the non-trips, however at the cost of also removing trip legs which were actually performed. These wrongly removed trip legs were primarily walking and bicycle trips. The incorrect removal of these could be partly explained by the use of the street network for the map matching. Additional research could test whether expanding the network to also include paths would further improve the results.

6 CONCLUSIONS

Automated post-processing procedures are essential to have available in order to facilitate the use of GPS data for transport surveys. The paper has presented a fully automated and disaggregate method to process raw GPS data and classify trips, trip legs and the most probable mode of transport used. The method is applicable to all cases where data is collected as individual-based GPS traces and where detailed digital information on the local infrastructure is available. This study applied the method to GPS data collected in the Greater Copenhagen Area, for which a detailed digital representation of the infrastructure is available.

The deployment of the method does not require the respondents to provide any information beyond the GPS traces. It is however important to note that the parameters used in e.g. the segmentation of the speed and acceleration profiles may need to be adapted/calibrated to fit the characteristics of the specific case. To do this, it is necessary to have available a control sample of corresponding revealed or stated information of trips undertaken by the respondents (e.g. trip start and end time and location, mode chosen etc.).

The method performs, and iterates between, a series of steps. While being based on the automatic trip and mode detection algorithm developed in Schüssler and Axhausen (2009), the method contributes by utilizing (i) available disaggregate information on the local infrastructure to conduct GIS analyses to better distinguish between modes with similar speed and acceleration characteristics, (ii) advanced feedback loops between steps, allowing inconsistent mode-sequences to alter the trip leg detection, and (iii) map matching to exclude non-trips and hinder wrongly splitting of trips on motorways.

Two variants of the method proposed were tested, one algorithm with the map matching step and one without it. This showed that including map matching improves the success rates by removing many non-trips, however at the cost of also removing some actually performed (primarily walking) trips. Both variants produced success rates above 90% when comparing to the control sample. These results are promising in comparison to the overall success rates obtained in other studies. Gong et al. (2011), Chen et al. (2010) and Bolbol et al. (2012)

obtained success rates of 82.6%, 79.1% and 87.4%[3] respectively. Chung and Shalaby (2005) obtained a success rate of 91.6% in their study including 60 trips. Especially the success rates of 77% for bus and 97% for rail are high when compared to other studies; Gong et al. (2011) and Bolbol et al. (2012) obtained success rates of 35.7% and 84.1% for rail, and 62.5% and 58.29% for bus. The current study also applied the method proposed by Schüssler and Axhausen (2009) on the same dataset. This allowed evaluating whether the high success rates were generated due to special circumstances related to the case study rather than improvements in the methodology. Success rates of 24% and 38% were obtained for rail and bus, respectively, when using this existing algorithm. This verified that the high success rates for the two proposed algorithms were generated as a result of applying the suggested advanced feedback algorithm and utilising the available disaggregate network data.

The study has contributed to literature by demonstrating much improved fit rates in the detection of trips, trip legs and mode of transport used. Through this we believe that the abilities of automatic post-processing methods are causing travel surveys based on GPS data collection to be highly attractive, even for complex multi-modal study areas.

ACKNOWLEDGEMENTS

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[3] Success rate has been calculated as the rate is not presented in Bolbol et al. (2012).

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APPENDIX 3: PRATO ET AL. (2014)

ESTIMATING VALUE OF CONGESTION AND OF RELIABILITY FROM OBSERVATION OF ROUTE CHOICE BEHAVIOR OF CAR DRIVERS

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Abstract: In recent years, a consensus has been reached about the relevance of calculating the value of congestion and the value of reliability for better understanding and therefore better prediction of travel behavior. The current study proposed a RP approach that used a large amount of GPS data from probe vehicles to provide insight into actual behavior in choosing a route. Mixed path size correction logit models were estimated from samples of 5,759 observations in the peak period and 7,964 observations in the off-peak period, while a mean-variance model was specified to consider both congestion and reliability terms. Results illustrated that the value of time and the value of congestion were significantly higher in the peak period because of possible higher penalties for drivers being late and consequently possible higher time pressure. Moreover, results showed that the marginal rate of substitution between travel time reliability and total travel time did not vary across periods and traffic conditions, with the obvious caveat that the absolute values were significantly higher for the peak period. Last, results showed the immense potential of exploiting the growing availability of large amounts of data from cheap and enhanced technology to obtain estimates of the monetary value of different travel time components from the observation of actual behavior, with arguably potential significant impact on the realism of large-scale models.

1 INTRODUCTION

Travel demand studies place a significant emphasis on the estimation of the value of time (VOT), namely the monetary value assigned by travelers to their travel time savings. Theoretical foundations of the VOT concept have been thoroughly reviewed in the recent past (Jara-Diaz, 2007; Small and Verhoef, 2007), and empirical estimates of the VOT have been extensively provided by researchers and practitioners and meticulously reviewed (Wardman, 2001; Zamparini and Reggiani, 2007; Shires and de Jong, 2009; Abrantes and Wardman, 2011).

Recently, consensus has been reached about the importance of calculating two values that are related to the VOT: (*a*) the value of congestion, as the VOT varies with traffic conditions and relates to the complexity of driving conditions and the emergence of feelings of frustration and danger when more vehicles are present on the road (see, e.g., Fosgerau et al., 2007; Wardman and Ibañes, 2012*]*, and (*b*) the value of reliability, as the VOT relates to the predictability of travel time and the repeatability of the travel experience without additional costs that are attributable to uncertainty (see, e.g., Li et al., 2010; Carrion and Levinson, 2012).

Evidence of the value of congestion dates back about 40 years according to a report that 'the estimated coefficient of auto congestion time was about thirty percent larger (in magnitude) than that of auto non-congestion time' (Train, 1976). About 10 years later, a stated preference (SP) study in the United Kingdom provided the first estimates of the value of congestion, which ranged between 1.28 and 1.46 for different trip purposes (Wardman, 1986). Then, a large variety of studies estimated the value of congestion, with values ranging from about 1.00 to more than 2.50, with higher valuation of congested time for commuters and methods inclining heavily towards SP studies of mode and route choice, and rarely towards revealed preference (RP) studies, with higher valuation of congested time for unlabeled SP studies (Wardman and Ibañes, 2012). Values of congestion have been mentioned to contain an element of reliability, and some practitioners have supported the inclusion of a reliability variable alongside free-flow and congested time variables (Wardman and Ibañes, 2012).

Evidence of the value of reliability dates back about 30 years, according to the classification of travel time variability, into three categories: (a) interday variability, (b) interperiod variability, and (c) intervehicle variability (Bates et al., 1987). About 5 to 10 years later, SP studies in the United States presented the first estimates of the value of reliability ratio, with values between 1.31 and 3.29 for car commuters (Noland and Small, 1995; Noland et al., 1998). Then, a large number of studies offered estimates of the value of reliability ratio, with values ranging from about 0.50 to more than 3.00 and differences across periods and methods leaning mostly towards SP studies and less frequently towards RP, with slightly higher values obtained for the latter over the former (Li et al., 2010; Carrion and Levinson, 2012). Clear differences between estimates of the value of reliability did not emerge when one considers the two major approaches of the mean-variance model and the scheduling model.

The current study contributes to both lines of research by (a) providing additional evidence about the value of congestion and the value of reliability from actual observed behavior and (b) answering the need to include measures of reliability when values of congestion are calculated as, so far, only SP approaches consider congestion and reliability variables concurrently (see Wardman and Ibañes, 2012). More importantly, the current study illustrates how to exploit the growing availability of large amounts of data from cheap and enhanced technology to obtain estimates of the monetary value of travel time components from the observation of actual behavior. Being able to make use of the availability of large amounts of data is arguably extremely important for the realism of large-scale models being developed worldwide.

The current study presents the estimation of the value of congestion and the value of reliability from the observation of drivers' behavior in route choice. Car drivers were using a vehicle equipped with GPS in the Greater Copenhagen, Denmark, area in 2011, and recorded GPS points were matched to the network of the Danish National Model [Landstrafikmodel (LTM)] for construction of a data set of observed route choices. Route choice behavior was modeled with a two-stage approach consisting of choice set generation and model estimation: (a) a doubly stochastic generation method was applied to the origin-destination pair of each observed route for producing route choice sets, and (b) mixed path size correction logit (MPSCL) models were estimated for modeling route choice behavior. As travel times were available from the LTM assignment, travel time variables could be calculated for each link as free-flow time and congested time to estimate two time parameters and calculate the value of congestion. As distributions of travel times were available from probe vehicles driving through the network, and as the literature suggests the use of differences of percentiles of travel time distributions as variance measures in RP studies (Carrion and Levinson, 2012), travel time reliability could be calculated for each link as the difference between the 90th and the 50th percentiles to compute the value of reliability ratios. Accordingly, the MPSCL were formulated as mean-variance models that accounted for variability of travel times on the network, heterogeneity in the preference for the time components, and similarity across alternative routes.

The remainder of the paper is organized as follows. The next section describes the data, the technique of choice set generation, and the MPSCL model that were applied in the current study. Then, the results of choice set generation and model estimation are presented, and the values of congestion and reliability are illustrated. Last, conclusions are drawn from the current study.

2 METHODS

2.1 DATA

Route choice behavior was observed in 2011 for 169 drivers who lived in the Greater Copenhagen area and used vehicles equipped with GPS devices that allowed collection of traces of their routes. The traces were matched to the LTM network, and the obtained routes were filtered by removing (*a*) routes that were shorter than 1 km for the likely impossibility to identify alternative routes and (*b*) routes that were filled with shortest-path linkage between two matched parts for the presence of gaps in the GPS traces. The map-matching and filtering process produced an initial data set of 17,115 observed routes.

The LTM network consists of 34,251 links covering the entire country and accounts for a variety of road types: motorways, highways, national roads, regional roads, major rural roads, minor rural roads, major urban roads, and minor urban roads. The LTM network was preloaded with traffic volumes from the LTM traffic assignment, and hence the travel time was expressed as the sum of free-flow time and congested time for each link of the LTM network during each of 10 periods corresponding to the most disaggregate temporal resolution of the LTM.

For the LTM network, speed observations were collected from probe vehicles. These observations allowed the calculation of travel time reliability for each link and each of the 10 periods as the difference between the 90th and the 50th percentiles of the observed travel time distribution in accordance with the following procedure: (*a*) if the link had at least 10 observations, then the difference was calculated directly from its time distribution; (*b*) if the link had fewer than 10 observations, then the difference was calculated from the time distribution on that link joined with the immediately preceding and following links belonging to the same road; and (*c*) if the joined links had fewer than 10 observations, then the difference was calculated from the time distribution on those links joined with the adjacent links belonging to the same road type within a radius of 2 km.

For the LTM network, the cost of driving through each link was calculated as the product of its length and the marginal cost of driving that the Danish Ministry of Transport defined in 2011 as being equal to 0.89 Danish Krone (DKr) per kilometer (about US$0.16/km or €0.12/km) and corresponding to the consumption of fuel, oil, tires, and battery.

2.2 Choice Set Generation

The initial 17,115 observed routes were considered for the estimation of route choice models and hence were required to have alternative routes generated, given the two-stage approach to modeling route choice behavior (Prato, 2009). Reasons of behavioral plausibility and computational efficiency advised to apply a doubly stochastic method of route generation that produces alternative routes (Prato, 2009, 2012) under the assumption that drivers might perceive costs with error and that drivers' perceptions might differ from one another (Nielsen, 2000; Bovy and Fiorenzo-Catalano, 2007).

The generating function of the applied doubly stochastic method accounted for heterogeneity in the two components of the travel time (i.e., free-flow time and congested time) and error in length. The parameters of the travel time components were assumed to be lognormally distributed to avoid unreasonable positive preferences for time, and the error term was assumed to be gamma distributed to allow additivity across links in the routes. The distributions of parameters and the error term were calibrated in accordance with the utility functions used in the LTM traffic assignment, and random values were extracted 100 times from the respective distributions for each observation n and for the period when observation n occurred. Accordingly, 100 alternative routes were produced for each of the 17,115 observed routes and then postprocessed before model estimation.

The post-processing included (*a*) defining the choice set composition for each observation, (*b*) removing observations for which the choice set contained only one alternative route, (*c*) removing observations for which the choice set did not contain any alternative reasonably similar to the observed route, and (*d*) computing the variables for model estimation. The first step removed duplicate routes generated during the 100 iterations and composed each choice set of each observation with alternative routes that were unique in not having any exact duplicate in the choice set. The second step removed observations for which the model could not be estimated because no alternative to the chosen route existed. The third step calculated for each route in each choice set the coverage measure cov_g to verify its consistency with the observed route with a certain tolerance (Prato, 2009):

$$\text{cov}_g = \frac{\sum_{n=1}^{N} I\left(O_{ng} \geq \delta\right)}{N} = \frac{\sum_{n=1}^{N} I\left(\frac{L_{ng}}{L_n} \geq \delta\right)}{N} \tag{1}$$

where

$I(\bullet)$ = function equal to 1 when its argument is true and 0 otherwise

L_{ng} = overlapping length between the route generated by method g and observed route n,

O_{ng} = overlap percentage of the route generated by method g and observed route n,

L_n = length of observed route n,

N = number of observations at the third post-processing step, and

δ = overlap threshold between generated and observed routes.

Observations for which the choice set did not contain at least one route overlapping the observed one for more than 80% were not considered for model estimation because of being inconsistent with the observed behavior (Prato, 2009).

The fourth post-processing step calculated the following variables for each alternative route in the choice set of observation n for the period during which observation n occurred: (*a*) free-flow time, (*b*) congested time, (*c*) travel time reliability, (*d*) cost, (*e*) number of left turns, and (*f*) number of right turns. Specifically, for each route, free-flow time, congested time, and cost were summed over the links composing the route; travel time reliability was calculated as the difference between the 90th and the 50th percentile of the travel time distribution over the route; and numbers of turns were calculated from the topography of the network.

2.3 Route Choice Model

For each observation n, a linear-in-parameter utility function V_j was specified for each route j within the choice set to estimate a path size correction logit (PSCL) model (Bovy et al., 2008):

$$V_j = \beta_{fft} fft_j + \beta_{congt} congt_j + \beta_{trel} trel_j + \beta_{\cos t} \cos t_j + \beta_{left} left_j + \beta_{right} right_j \quad (2)$$

where

fft_j = free-flow time,
$congt_j$ = congested time,
$trel_j$ = travel time reliability,
$cost_j$ = cost,
$left_j$ = number of left turns,
$right_j$ = number of right turns, and
β_x = parameters to be estimated.

Socioeconomic characteristics of the drivers were not available because the objective of the data collection was originally to collect information about congestion in the study area.

The specification of the utility function corresponds to the specification of a mean-variance model. Although debate exists about the use of mean-variance versus scheduling models, for the current study, travel time distributions on the network links were available rather than preferred arrival time (Li et al., 2010; Carrion and Levinson, 2012). Accordingly, the model selection was data driven rather than theory driven; nevertheless, the mean-variance approach is advantageous for the straightforward calculation of the value of congestion and the value of reliability (Li et al., 2010).

The probability of selecting the observed route i in the choice set C_n is equal to (Bovy et al., 2008)

$$P_i = \frac{\exp\left(V_i + \beta_{psc} psc_i\right)}{\sum_{j \in C_n} \exp\left(V_j + \beta_{psc} psc_j\right)} \tag{3}$$

where psc_j is the path size correction and β_{psc} is a parameter to be estimated. Path size correction psc_j captures the similarity across alternative routes within choice set C_n and is defined as (Bovy et al., 2008)

$$psc_j = -\sum_{a \in \Gamma_j} \left(\frac{L_a}{L_j} \ln \sum_{j \in C_n} \delta_{aj} \right) \tag{4}$$

where

L_j = length of route j,

L_a = length of link a,

Γ_j = set of links belonging to route j, and

δ_{aj} = link-path incidence dummy (equal to 1 if link a belongs to route j and 0 otherwise).

The current study considered preference heterogeneity across drivers, and hence an MPSCL model was estimated in which the βs were random parameters distributed with probability density function $f(\beta \mid \theta)$ characterized by parameters θ. Accordingly, the probability of driver n to select route i needed to be integrated over the distribution of the βs:

$$P_i = \int \frac{\exp\left(V_i + \beta_{psc} psc_i\right)}{\sum_{j \in C_n} \exp\left(V_j + \beta_{psc} psc_j\right)} f\left(\beta \middle| \theta\right) d\beta \tag{5}$$

Possible distributions for time and cost components included lognormal, the constrained triangular, and the Johnson SB because of the avoidance of positive preferences for travel time, while possible distribution for the turn components included the normal as well because not all drivers might prefer the most direct route. Given the distribution of the parameters, the probability did not have a closed-form expression, and hence the maximization of the likelihood function consisted of simulating the multidimensional integral:

$$SLL = \sum_{n=1}^{N}\sum_{j=1}^{J} d_{ni} \ln\left\{\frac{1}{R}\sum_{r=1}^{R}\left[\frac{\exp\left(V_i^r + \beta_{psc} psc_i\right)}{\sum_{j\in C_n}\exp\left(V_j^r + \beta_{psc} psc_j\right)}\right]\right\} \quad (6)$$

where

SLL = simulated log likelihood,

N = number of observations,

J = number of alternative routes,

d_{ni} = 1 if driver n has selected route i and 0 otherwise, and

r = one of the R random draws required for integral simulation, and, in superscript, an instance of a draw from the distribution of the random parameters βs that realizes the utility function V_j^r.

The parameters β, θ and β_{psc} were restricted so as not to vary across different observations of the same driver and were estimated in the current study by using 1,000 random draws from a modified Latin hypercube sampling method (Hess et al., 2006) in the freeware Biogeme (Bierlaire, 2008).

Given the estimated values of the βs, the value of congestion was calculated as the ratio between the estimates of the parameters β_{congt} and β_{fft}, while, for comparison, the value of reliability ratio was calculated as the ratio between the estimates of the parameters β_{trel} and β_{fft} and the ratio between the estimate of β_{trel} and the combination of β_{fft} and β_{congt} for the average level of congestion *conglev* [i.e., (1- *conglev*)$\cdot\beta_{fft}$ + *conglev* $\cdot\beta_{congt}$], where *conglev* is the percentage of congested travel time with respect to the total travel time. Because the values were distributed, the mean, standard deviations, and confidence intervals were calculated analytically, as illustrated by Daly et al. (2012).

3 RESULTS

3.1 GENERATED CHOICE SETS

The post-processing of the initial 17,115 observations initially removed 1,838 observations because they each contained only one alternative, and then an additional 841 observations were removed because they were behaviorally inconsistent with the observed routes in their overlap, being below an acceptable 80% threshold (Prato, 2009).

The distribution of the coverage over the cumulative distribution of the observations is presented in Figure 1, which shows that the doubly stochastic method of route generation replicated link by link almost 85% of the observed routes and reproduced, with 80% overlap, about 95% of the routes. These results suggest that the doubly stochastic method was behaviorally consistent in its ability to replicate the observed routes within the generated sets. In addition, the doubly stochastic method was computationally efficient in its capacity to generate 100 alternative routes for 17,115 observations in a couple of hours in the large-scale LTM network thanks to the programming in the C# language of a module for route set generation in ArcGIS.

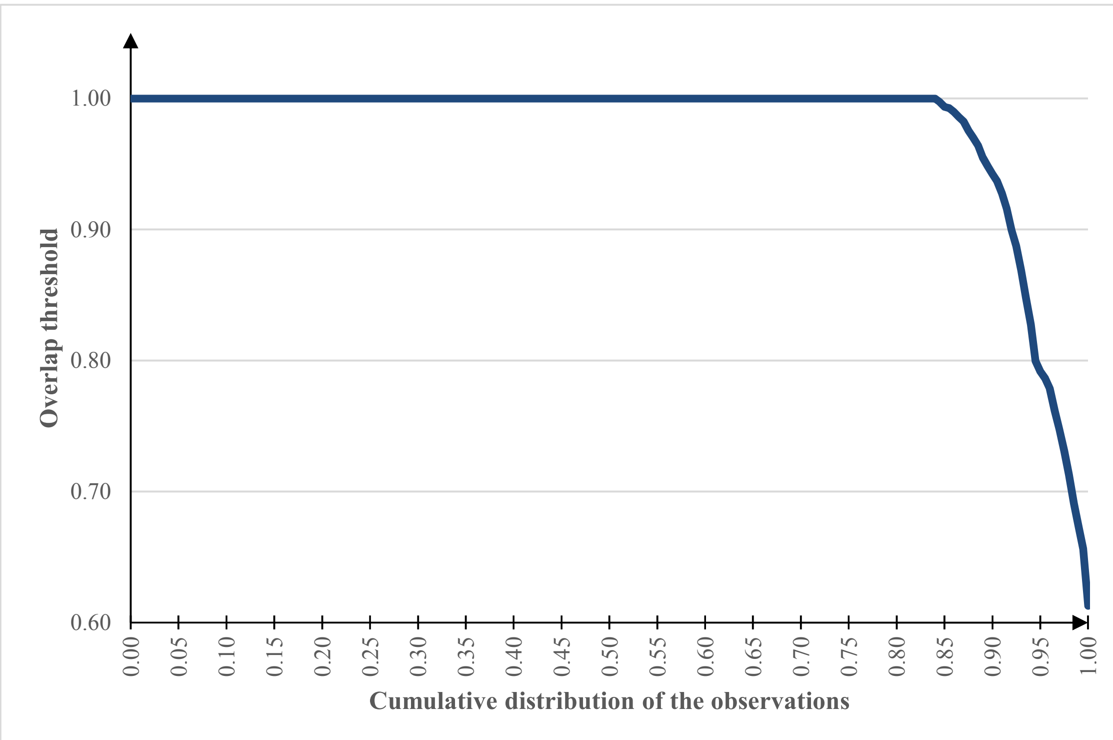

Figure 1 – Coverage distribution

For the remaining 14,436 observations, the distribution of the choice set size is presented in Figure 2, which shows that (*a*) the number of unique routes within the generated choice sets covers the entire range from 2 to 100, (*b*) the median is equal to 17 routes, (*c*) two-thirds of the observations have 35 alternatives or less, and (*d*) only one-fourth of the observations have 50 alternatives or more.

The average travel time on the routes is approximately 12.5 min, with a 90% confidence interval between 4.8 and 29.3 min. The average level of congestion on the routes (i.e., the ratio between congested and free-flow time) is approximately 17.8% over the entire day, with expectedly higher value in the morning and afternoon peaks (34.3%) and lower value in the midday and evening off peaks (8.8%). These averages compare reasonably well with recently published levels of congestion in Copenhagen (TomTom, 2012). The average level of reliability (i.e., the difference between the 90th and the 50th percentiles of the travel time distribution on the routes) is approximately 30.9% of the travel time over the entire day, with intuitively larger value in the morning and the afternoon peaks (40.4%) and smaller value in the morning and evening off-peaks (22.8%).

The analysis was conducted for the 10 periods in the LTM, and similarities emerged in the observed levels of congestion and reliability between the two periods in the morning peak hours (i.e., 7 to 8 a.m. and 8 to 9 a.m.) and the three periods in the afternoon peak (3 to 4 p.m., 4 to 5 p.m., 5 to 6 p.m.), as well as between the midday off-peak period (9 a.m. to 3 p.m.) and the evening off-peak period (6 to 9 p.m.). Only a limited number of observations were in the remaining periods of late evening and early morning, when free-flow conditions are usually observed.

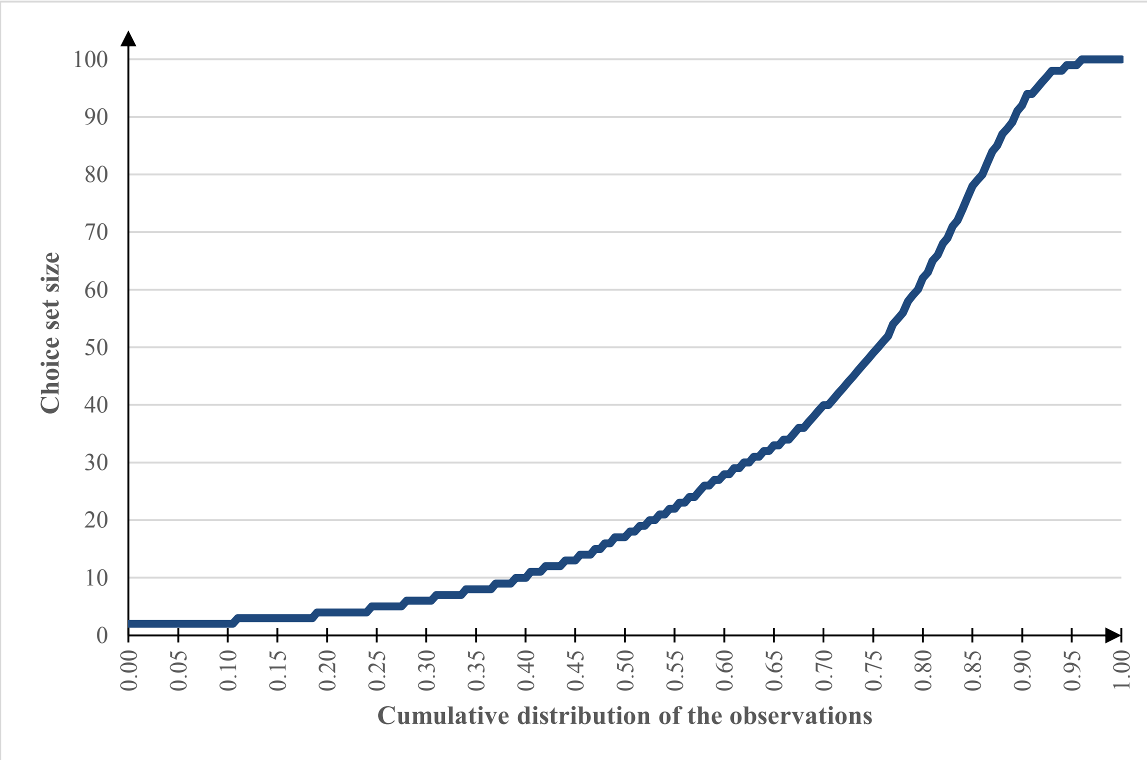

Figure 2 – Choice set size distribution

3.2 Estimates of Route Choice Model

Initially, PSCL models of route choice behavior were estimated for the morning peak hours (7 to 9 a.m.), midday off-peak hours, afternoon peak hours (3 to 6 p.m.) and evening off-peak hours. The statistical comparability of the estimates across models suggested estimation of two models for 5,759 observations in the morning and afternoon peak periods and 7,964 observations in the midday and evening off-peak periods. The comparability of the estimates could be related to similar purposes, as peak hours have a prevalence of mandatory trips with distribution indeed spread over 2 h in the morning and 3 h in the afternoon, and off-peak hours have a prevalence of nonmandatory trips with almost uniform distribution over the 6 h at midday and the 3 h in the evening. However, information on trip purposes was not available in the current study.

Estimates of the two PSCL models for peak and off-peak periods are presented in Table 1. As expected, the parameters of the travel time components express that drivers seek to minimize free-flow time, congested time, and time reliability when driving in both peak and off-peak periods. In addition, with respect to free-flow time, drivers appear more sensitive to congested time and even more sensitive to time reliability. Also predictably, drivers look for minimizing travel costs and searching for the most direct routes, with left turns being perceived as more time consuming than right turns. Finally, the parameters of the path size correction appear in line with the theory that hypothesizes penalties to the utility of routes highly similar to alternative ones in the choice set.

Estimation of the MPSCL models for peak and off-peak periods considered several possibilities for the distributions of the parameters. Normal and triangular distributions were considered for left and right turns, but the parameters expressing heterogeneity across drivers were not significantly different from 0, and hence the final models considered fixed coefficients for these two variables. The lognormal, the constrained triangular, and the Johnson SB distributions were considered for travel time variables and cost to control for their sign to be negative. The parameters expressing heterogeneity of taste for cost were not significantly different from 0, and hence the final models also considered a fixed coefficient for cost. The parameters accounting for heterogeneity in the preferences of drivers for time components were significant, and lognormal distributions provided the models with the best fit. However, correlation of the three parameters was estimated and was nonsignificant.

Table 1 – PSCL Estimates

	Peak period		Off-Peak Period	
Variable	Est.	*t*-Test	Est.	*t*-Test
Free-flow time (min)	-0.473	-12.10	-0.338	-14.51
Congested time (min)	-0.689	-17.50	-0.424	-18.27
Travel time reliability (min)	-0.843	-48.47	-0.524	-45.57
Cost (DKr)	-0.221	-3.08	-0.272	-6.10
Left turns (unit)	-0.618	-31.21	-0.636	-43.86
Right turns (unit)	-0.431	-26.94	-0.446	-36.86
Ln (path size)	0.935	17.61	0.904	25.54
Number of observations	5759		7964	
Null log-likelihood	-15607.99		-23987.90	
Final log-likelihood	-8870.99		-15959.49	
Adjusted rho-square	0.431		0.334	

Estimates of the two MPSCL models for peak and off-peak periods are presented in Table 2. The lognormal distributions of the parameters for the travel time components are presented with the two parameters μ and σ of the distribution. Both models improve in goodness of fit with respect to the respective PSCL models, as likelihood ratio tests show the rejection of the hypothesis that the correct model is without distributed parameters for travel time components for both the peak model [likelihood ratio test (*LRT) = 193.05, degrees of freedom (df) = 3, p = .0000*] and the off-peak model (*LRT = 90.23, df = 3, p = .0000*). Estimates of the parameters of the cost and the turns maintained comparable values and expressed the same negative preference for long and tortuous routes. Estimates of the parameters of the travel time components maintained the negative preference for free-flow time, congested time, and time reliability, as well as the relatively stronger sensitivity. Their means in the lognormal distributions are, respectively, -0.494, -0.740 and -0.891 for the peak period and -0.351, -0.443 and -0.536 for the off-peak period. In addition, their standard deviations in the lognormal distributions are, respectively, 0.130, 0.234 and 0.352 for the peak period and 0.057, 0.083 and 0.152 for the off-peak period.

Table 2 – MPSCL Estimates

	Peak Period		Off-Peak Period	
Variable	Est.	*t*-Test	Est.	*t*-Test
Free-flow time (μ, min)	-0.739	-13.26	-1.060	-15.12
Free-flow time (σ, min)	0.259	2.10	0.161	2.53
Congested time (μ, min)	-0.348	-5.53	-0.831	-11.94
Congested time (σ, min)	0.309	2.36	0.186	2.13
Travel time reliability (μ, min)	-0.189	-6.90	-0.653	-17.84
Travel time reliability (σ, min)	0.381	3.07	0.243	3.57
Cost (DKr)	-0.256	-3.25	-0.299	-5.64
Left turns (unit)	-0.665	-32.76	-0.644	-44.11
Right turns (unit)	-0.467	-27.63	-0.455	-38.24
Ln (path size)	0.929	16.92	0.897	24.64
Number of observations	5759		7964	
Null log-likelihood	-15607.99		-23987.90	
Final log-likelihood	-8774.47		-15914.37	
Adjusted rho-square	0.437		0.336	

3.3 VALUE OF CONGESTION AND VALUE OF RELIABILITY RATIO

Estimates of the PSCL and MPSCL models allowed calculation of the VOT for free-flow time and congested time, the value of reliability, the value of congestion, and the value of reliability ratio that are presented in Table 3.

The values of the PSCL and the mean values of the MPSCL are comparable and highlight the same trends. Relative to the off-peak period, in the peak period the VOT is higher for all three time components, the value of congestion is greater, and the value of reliability ratio is larger. Intuitively, drivers in the peak period are likely to commute and hence are required to avoid being late; those conditions make them more sensitive to time generally, to congestion particularly, and to uncertainty especially. Conversely, drivers in the off-peak period experience less congestion and have less restrictive time constraints; those conditions make them less sensitive to the eventuality of being late. In addition, consideration of taste heterogeneity suggests much larger variation in the preferences of drivers during the peak period.

The VOT combining free-flow and congested time produced reasonable values for Denmark. When an average congestion of about 35% in the peak period is considered, the VOT is approximately DKr149.0/h (about US$26.5/h or €20.0/h) for the PSCL model and has a mean equal to DKr135.8/h (about US$24.2/h or €18.2/h) and standard deviation equal to DKr40.7/h (about US$7.3/h or €5.5/h) for the MPSCL model. When an average congestion of about 9% in the off-peak period is considered, the VOT is roughly DKr76.3/h (about US$13.6/h or €10.2/h) for the PSCL model and has a mean equal to DKr72.0/h (about US$12.8/h or €9.7/h) and standard deviation equal to DKr12.0/h (about US$2.1/h or €1.6/h) for the MPSCL model.

When the PSCL model is considered, the value of congestion is equal to 1.46 in the peak period and 1.25 in the off-peak period. The value of reliability ratio with respect to free-flow time is equal to 1.78 for the peak period and 1.55 for the off-peak period. When factoring the 35% and 9% average congestion level in the two periods, respectively, the value of reliability ratio amounts to 1.54 and 1.51, respectively. For the MPSCL model, the mean of the value of congestion is equal to 1.50 in the peak period and 1.26 in the off-peak period. The mean of the value of reliability ratio with respect to the free-flow time is equal to 1.80 for the peak period and 1.53 for the off-peak period, but again, considering the average congestion levels makes the values amount to more comparable values of 1.53 and 1.49, respectively. Apparently, the value of reliability ratio with respect to the total travel time is similar between peak and off-peak periods, regardless of the level of congestion in the two periods.

Table 3 – Value of Congestion and Value of reliability Ratio

	Peak Period		Off-Peak Period	
Measure	PSCL	MPSCL	PSCL	MPSCL
VOT free-flow time (mean, DKr/h)	128.42	115.64	74.56	70.38
VOT free-flow time (st.dev., DKr/h)	-	30.43	-	11.37
VOT congested time (mean, DKr/h)	187.17	173.27	93.51	88.91
VOT congested time (st.dev., DKr/h)	-	54.75	-	16.68
Value of reliability (mean, DKr/h)	228.95	208.45	115.59	107.51
Value of reliability (st.dev., DKr/h)	-	82.45	-	26.54
Value of congestion (mean)	1.46	1.50	1.25	1.26
Value of reliability ratio (mean)	1.54	1.53	1.51	1.49

As the distribution of the random parameters is lognormal and hence asymmetric, Table 4 presents the 90% and 95% confidence intervals of the VOT of free-flow time, the VOT of congested time, the value of reliability, the value of congestion, and the value of reliability ratio. Confidence intervals are calculated analytically, and results show significant dispersion in the VOT of the three time components for both peak and off-peak periods (Daly et al., 2012). When the value of reliability ratio with respect to travel time is considered and the level of congestion in the peak and off-peak periods are factored, the confidence intervals are fairly similar: for peak and off-peak periods, the intervals are, respectively, 1.26 to 1.75 and 1.29 to 1.67 at the 90% level, and 1.22 to 1.81 and 1.26 to 1.72 at the 95% level.

Table 4 – Confidence Intervals of Value of Congestion and Value of Reliability Ratio from MPSCL Estimates

Measure	Peak Period	Off-Peak Period
Confidence interval 90%		
VOT free-flow time (DKr/h)	73.07 - 171.16	53.36 - 90.47
VOT congested time (DKr/h)	99.47 - 274.43	64.35 - 118.66
Value of reliability (DKr/h)	103.54 - 362.90	69.96 - 155.72
Value of congestion (mean)	1.36 - 1.60	1.21 - 1.31
Value of reliability ratio (mean)[a]	1.26 - 1.75	1.29 - 1.67
Confidence interval 95%		
VOT free-flow time (DKr/h)	67.35 - 185.70	50.72 - 95.16
VOT congested time (DKr/h)	90.25 - 302.45	60.69 - 125.82
Value of reliability (DKr/h)	91.82 - 409.22	64.80 - 168.12
Value of congestion (mean)	1.34 - 1.63	1.20 - 1.32
Value of reliability ratio (mean)[a]	1.22 - 1.81	1.26 - 1.72

[a] *calculated with respect to the VOT of the total travel time by considering the average congestion level in the peak period at 35% and in the off-peak period at 9%.*

4 CONCLUSIONS

As consensus has been reached in recent years about the importance of evaluating the value of congestion and the value of reliability to improve understanding and hence to better predict travel behavior, the current study contributes to both lines of research by providing additional evidence about the value of congestion and the value of reliability from observed behavior and proposes an RP approach that answers the need to include measures of reliability when value of congestion is calculated. A sample of 5,759 observations in the morning and afternoon peak periods and a sample of 7,964 observations in the midday and evening off-peak periods were used for the estimation of MPSCL models with lognormally distributed travel time variables and fixed parameters for cost and turn variables.

When estimates of the MPSCL models are considered, the value of congestion is on average equal to 1.50 in the peak periods with an interval of 1.36 to 1.60 at the 90% confidence level, and 1.26 in the off-peak periods with an interval of 1.21 to 1.31 at the 90% confidence level. The estimated values of congestion from the current RP study for the peak periods are in line with previous findings from Danish SP studies that estimated value of congestions between 1.31 and 1.65 in congested conditions (Nielsen, 2004; Nielsen et al., 2002; Fosgerau, 2006) and are lower than previous results from another Danish RP study that calculated values between 1.65 and 2.00 in congested conditions (Rich and Nielsen, 2007). Notably, no previous Danish studies examined less congested conditions or considered reliability. The estimated values of congestion for the peak periods are also in agreement with previous findings from the United Kingdom (Abrantes and Wardman, 2011) and Australia (Rose et al., 2008). The estimated values are generally lower than the ones from other RP studies and more in agreement with the ones from a variety of SP studies in various regions (Wardman and Ibañes, 2012).

When the estimates of the MPSCL models are considered and the average levels of congestion are factored, the value of reliability ratio is on average equal to 1.53 in the peak periods with an interval of 1.26 to 1.75 at the 90% confidence level and 1.49 in the off-peak periods with an interval of 1.29 to 1.67 at the same confidence level. These ratios suggest that the marginal rate of substitution between travel time reliability and total travel time does not vary across periods and traffic conditions, with the obvious caveat that the absolute values are higher for commuters experiencing possible penalties for being late and consequently possible time pressure. Notably, these are the first value of reliability ratios estimated in the Danish context. The estimated ratios are lower than the ones from some RP studies (Lam and Small, 2001; Small et al., 2005; Bhat and Sardesai, 2006) and more in accordance with the ones from a variety of SP and RP studies (Small et al., 1995; Small et al., 1999; Liu et al., 2007). In contrast with the value of congestion, no clear disagreement exists here with other RP studies or agreement with previous SP studies.

Some limitations of the current study should be acknowledged. In relation to data, the procedure for the calculation of travel time reliability would benefit from a larger collection of speeds. However, the current study is the first to exploit large amounts of data from cheap and enhanced technology to estimate jointly the value of congestion and the value of reliability from observed behavior, and the limitation may be considered marginal when the results are examined. From the perspective of the models, the selection of a mean-variance model is driven by data availability of observed travel time distributions rather than theoretical considerations. However, Bates et al. (2001) have discussed and Fosgerau and Karlstrom (2010) have shown theoretically that the scheduling model can be approximated by a mean-variance model under reasonable conditions. From a general perspective, the representation of heterogeneity would benefit from both collection of socioeconomic characteristics of the drivers and estimation of a latent class model to provide insight into the differences in value of time, congestion and reliability across the population. However, given that the data in the current study were collected for various purposes, this endeavor is left for further research.

In summary, the current study provides valuable information on the value of congestion and the value of reliability in different traffic conditions and periods. Furthermore, it suggests a roadmap for exploiting the growing amount of information becoming available through the high penetration of cheap and efficient technology that collects data about travel times and speeds on the world's networks. Moreover, the continuous need in large-scale models for better and more insightful information seems to have a potential application by having linear-in-parameter utility functions that may be used within traffic assignment models that would improve their behavioral realism through consideration of not only travel time but their components in the free-flow, congested, and reliability parts. Possible research directions include the use of these utility functions within traffic assignment, the estimation of observed heterogeneity by collecting information about socioeconomic characteristics of the drivers and their trip purposes, and the consideration of alternative decision paradigms such as lexicographic and regret minimization behavior.

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APPENDIX 4: WATLING ET AL. (2014)

STOCHASTIC USER EQUILIBRIUM WITH EQUILIBRATED CHOICE SETS: PART I – MODEL FORMULATIONS UNDER ALTERNATIVE DISTRIBUTIONS AND RESTRICTIONS

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Abstract: The aim of this paper is to remove the known limitations of Deterministic and Stochastic User Equilibrium (DUE and SUE), namely that only routes with the minimum cost are used in DUE, and that all possible routes are used in SUE regardless of their costs. We achieve this by combining the advantages of the two principles, namely the definition of unused routes in DUE and mis-perception in SUE, such that the resulting choice sets of used routes are equilibrated. Two model families are formulated to address this issue: the first is a generalised version of SUE permitting bounded and discrete error distributions; the second is a Restricted SUE model with an additional constraint that must be satisfied for unused paths. The overall advantage of these model families consists in their ability to combine the unused routes with the use of random utility models for used routes, without the need to pre-specify the choice set. We present model specifications within these families, show illustrative examples, evaluate their relative merits, and identify key directions for further research.

Keywords: Choice Set; Random Utility; Traffic Assignment; Stochastic User Equilibrium.

1 INTRODUCTION

The Stochastic User Equilibrium (SUE) traffic assignment model was first proposed as an approach for investigating congested road networks by Daganzo and Sheffi (1977). Though not a necessary requirement, Daganzo and Sheffi gave the model theoretical appeal by basing it on the Random Utility Model (RUM), a well-known approach for modelling the discrete choice behaviour of agents. RUM permits the inclusion of random error structures which may be used to capture the uncertainty of travellers in terms of perception errors, and the uncertainty of the modeller in terms of unobserved attributes or unobserved heterogeneity. In addition, when the variance in the error terms approaches zero, SUE is able to approximate Deterministic User Equilibrium (DUE, Wardrop, 1952) to an arbitrary accuracy, and so it has a claim to be a generalisation of DUE (Baillon and Cominetti, 2008; Cominetti et al., 2012).

Though originally proposed with only logit (Fisk, 1980; Miyagi, 1985), probit (Daganzo, 1979, 1982) and potentially nested logit (Williams, 1977) models in mind, the SUE approach has since been extended to accommodate a range of alternative choice models within the RUM family. However, a common feature of both the original logit-/probit-based modelds and their later developments/generalisations is (as remarked by Damberg et al. (1996), in relation to the logit model) 'that of every route receiving a positive flow in the equilibrium state, regardless of its travel cost'. While only considering actual minimum cost routes, as in DUE, seems difficult to justify, moving to a case where all routes are used seems equally questionable.

To investigate the seriousness of this point, consider the well-known and frequently studied Sioux Falls network (LeBlanc et al., 1975). Using the DUE link travel costs[1], we randomly generated a sample of paths which are *unused* in the DUE solution by replicating 10,000 times the following procedure for each Origin-Destination (OD) movement: i) perturbing the DUE link travel costs with a Normally distributed error (with a variance = DUE link cost); ii) performing a shortest path search on the perturbed costs; iii) storing this path if not previously generated, and if its actual (unperturbed) cost is greater than the DUE route cost.

[1] Link flows and travel costs were downloaded on December 10 2012 from the web-page of Professor Hillel Bar-Gera (http://www.bgu.ac.il/~bargera/tntp/): The solution is an approximate DUE solution, with an Average Excess Cost (normalised gap) of $3.9 \cdot 10^{-15}$.

For the resulting set of distinct unused paths, we calculated their *relative* travel cost with respect to the DUE travel cost on the corresponding OD movement. The frequency distribution of the relative travel costs for unused paths, across all OD movements, is illustrated in Figure 1.

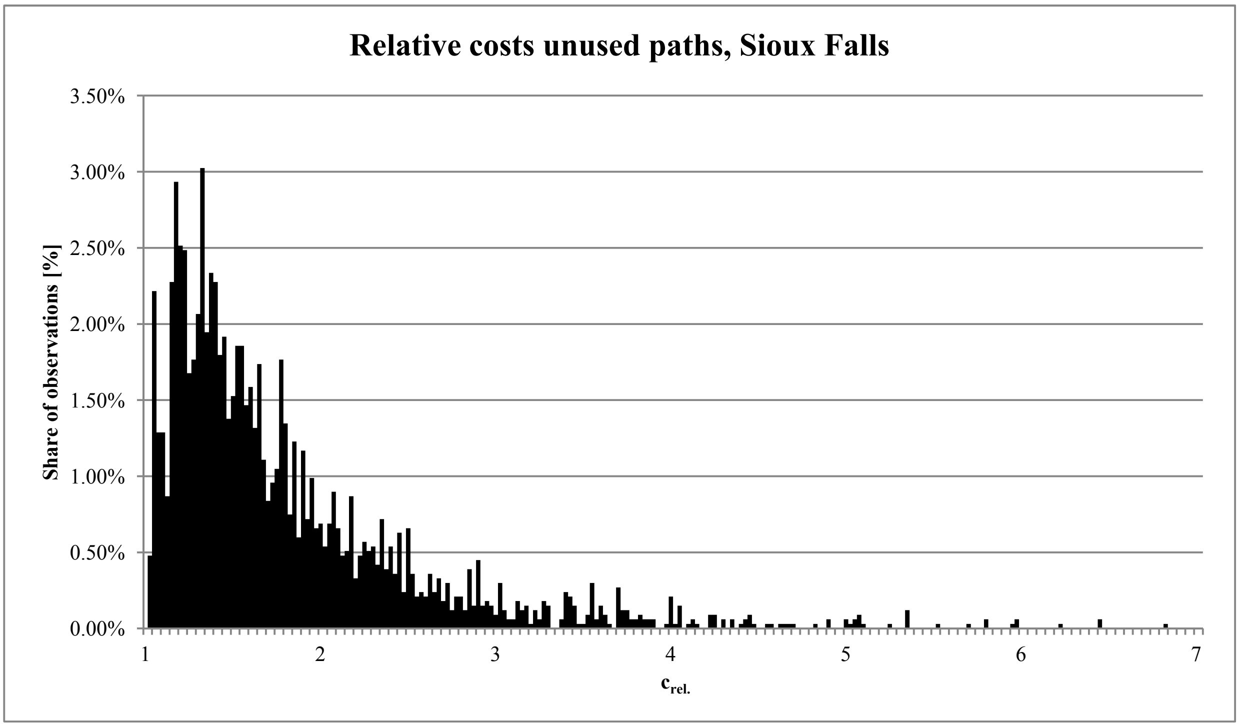

Figure 1 – Distribution of relative travel costs of random sample of unused DUE paths. Share of sampled paths as function of path cost relative to cost on corresponding minimum cost path

It can be seen that there are a significant number of unused paths with travel cost only a little greater than the travel cost of the used routes from the DUE solution. In reality, travellers may not *perceive* these paths as more costly (due to their mis-information or the modeller not being able to capture their personal preference structure in the specification of the cost function), and consequently we may expect travellers to use them, which they will not in the DUE model. On the other hand, there is a significant right-hand tail of unused routes which are between two and seven times as costly as the used routes from the DUE solution, yet all such routes should be used in the case of a perfect SUE solution, however circuitous and seemingly implausible is the route. While SUE will only assign small amounts of demand to the most costly routes, their existence means that at least in principle any SUE algorithm has the *aim* to consider all such routes, resulting in high computational requirements.

These issues are particularly evident in large-scale network models, which are becoming increasingly common, spanning the regional (e.g., Bar-Gera et al., 2012; Ben-Akiva et al., 2012; Kumar et al., 2012), national (see, e.g., Lundqvist and Mattson, 2001), and transnational scales (e.g., Burgess et al., 2008; Hansen, 2009; Petersen et al., 2009), which lead to extremely large feasible route choice sets. It is useful to imagine a network construction process for such a problem, in which we begin with a city network and then 'grow' it to a regional scale. At each step of the growth, new links are added, and every OD movement may have potential new routes which should theoretically attract flow at the SUE solution, however unattractive the route may seem. In this way, SUE is not as 'scaleable' as DUE; we can add unappealing routes to a DUE solution, and as long as the travel costs are higher than any used route, the solution will be unaffected.

In the present paper, and a companion paper (Rasmussen et al., 2014), we are particularly interested in developing a theoretical foundation for SUE-style approaches which does not suffer from such scaleability problems. Of course we are aware that practical solution algorithms, after a finite number of iterations, will typically produce an *estimated* SUE solution which includes only a subset of the available routes. However, such algorithms are an undesirable way of addressing the problem for several reasons. Firstly, the number of used paths is typically directly connected to the number of iterations, and so we confound the problem of convergence to the desired (SUE) conditions with the issue of which permitted paths it might be reasonable to use. Secondly, it means that the paths that are used are sensitive to the initial conditions and to the algorithm adopted. Thirdly, as we solve the problem only at the algorithmic stage, we do not have a criterion for judging whether one subset of used paths is more reasonable than another (as we do in DUE), since all that we know from SUE is that all permitted paths should be used.

An alternative way of addressing this problem would be to consider it at the model specification stage, in terms of the paths that are permitted. At one extreme, in a sense progressing in the opposite direction to the present paper, Bell (1995), Akamatsu (1996) and Maher and Hughes (1997) argue for a model in which path restrictions are *lifted*, so that even paths with multiple cycles are (implicitly) assigned some flow; they this has the advantage of avoiding any path enumeration. From a quite different viewpoint, Bovy (2009) argues for the behavioural realism of models in which only a subset of all paths is permitted, stating that people do not choose their path from the full *Universal Choice Set* of alternatives, but rather from a *Master Choice Set* of paths considered relevant. To this end, several studies have developed methods to pre-generate a fixed Master Choice Set, such as distance-bounded enumeration (Leurent, 1997), constrained enumeration (Friedrich et al., 2001; Prato and Bekhor, 2006), probabilistic generation techniques (Cascetta and Papola, 2001; Frejinger et al., 2009) and various deterministic or stochastic shortest path algorithms (e.g., Dijkstra, 1959; Sheffi and Powell, 1982; Ben-Akiva et al., 1984).

While the objective of using more limited path sets accords with our present study, we believe there are several disadvantages to SUE models based on a pre-generated Master Choice Set:

- Path generation methods typically require as input an estimate of level-of-service variables in order to generate plausible paths. These might be estimated by observing on-street travel times, using values generated by another model (e.g., DUE travel costs), or using fixed measures as a proxy (e.g., distance, free-flow speed). Regardless of the adopted method, an *inconsistency* problem arises, as the travel costs assumed for choice set generation will not be the same as those arisen from solving for SUE based on that choice set.
- The pre-defined choice set may exclude path options that, having solved for SUE on that choice set, would seem to be attractive based on the link travel costs of that solution. While it is undoubtedly true that travellers in real-life are not aware of *all* available paths, we have no independent evidence for excluding such apparently viable paths from consideration.
- Several of the methods are based on Monte Carlo simulation, which introduces a lack of repeatability in the final estimated solution due to this additional source of randomness.
- The notion of a fixed Master Choice Set implies that the set of considered paths is independent of any policy measure, whereas plausibly infrastructure improvements or tolling may make attractive some previously unattractive alternative paths or *vice versa*.

With these comments in mind, the present paper considers how we might *consistently integrate* the problem of distinguishing used and unused paths within the theoretical framework of SUE. The aim is to define not only an equilibrated flow solution but also an *Equilibrated Choice Set* in which the *equilibrium conditions* specify that some available routes should be unused, even at a perfect equilibrium (i.e., *it has nothing to do with the limitations of finite iterations of a solution algorithm*). In doing so, we aim to retain the basic simplicity and therefore applicability for large-scale problems that has made DUE so attractive in the past, in terms of its distinction between potentially used and definitely unused paths. We do this in a way that allows a connection with RUM and SUE. On the other hand, we aim to avoid an unappealing feature of SUE in assigning some traffic to *all* feasible paths, however unattractive, which is both behaviourally unrealistic and creates problems in devising convergent algorithms for large-scale networks.

The specific purpose of this paper is to set out two distinct methodological approaches for handling equilibrated choice sets in the framework of SUE, each approach leading to a family of techniques. Section 2 introduces the notation. Section 3 sets out the first approach, formulating a generalised version of SUE to admit a range of probability distributions. Section 4 formulates an alternative approach, namely a Restricted SUE model in which the choice set equilibration is handled through additional constraints that must be satisfied by unused alternatives. In both sections 3 and 4, simple illustrative examples are used to communicate the key concepts. In section 5 we evaluate the relative merits of these approaches, including possible extensions to the basic models, and set out the key areas for development in each. Finally, in section 6, we draw the main conclusions and wider future research directions.

2 NOTATION AND REFERENCE DEFINITIONS

We first introduce the basic common notation adopted in the paper. Most of this is the standard familiar notation for SUE problems, although two subtle and important variants are introduced and exploited later. We consider a network as a directed graph consisting of links a (a=1, 2,…, A) and origin-destination (OD) pairs m (m=1, 2,…, M). We define the demand d_m for OD-pair m composing a non-negative M-dimensional vector $\mathbf{d}$, the index set R_m of all[2] simple paths (without cycles) for each OD-pair m, the number N_m of paths in R_m and the union R of the sets R_m. The route index sets are constructed so that $R = \{1, 2,\ldots,N\}$, where $N = \sum_{m=1}^{M} N_m$.

Denote the flow on path $r \in R_m$ between OD-pair m as x_{mr} and let $\mathbf{x} = \left(x_{11}, x_{12}, \ldots, x_{1N_1}, x_{21}, x_{22}, \ldots, x_{2N_2}, \ldots, x_{M1}, x_{M2}, \ldots, x_{MN_M}\right)$ be the N-dimensional flow-vector on the universal choice set across all M OD-pairs, so that the notation x_{mr} refers to element number $r + \sum_{k=1}^{m-1} N_m$ in the N-dimensional vector $\mathbf{x}$. Denote the flow on link a (a=1, 2,..., A) as f_a and let $\mathbf{f} = \left(f_1, f_2, \ldots, f_a, \ldots, f_A\right)$ be the A-dimensional link flow-vector where f_a refers to element number a in $\mathbf{f}$.

The convex set of demand-feasible non-negative path flow solutions G is given by:

$$G = \left\{ \mathbf{x} \in \mathbb{R}_+^N : \sum_{r=1}^{N_m} x_{mr} = d_m, m = 1, 2, \ldots, M \right\} \tag{1}$$

where $\mathbb{R}_+^N$ denotes the N-dimensional, non-negative Euclidean space.

[2] We shall suppose that there are no pre-defined restrictions on the set of available routes, other than that they are acyclic, but our methods apply equally if R_m is pre-defined such that other routes are excluded, leading to some smaller Master Choice Set. We have avoided referring to this, so as not to confuse the reader between such pre-defined exclusions from the choice set, and those paths that emerge as unused from the equilibration process

Next, define δ_{amr} equal to 1 if link a is part of path r for OD-pair m and zero otherwise. Then the convex set of demand-feasible link flows is:

$$F = \left\{ \mathbf{f} \in \mathbb{R}_{+}^{A} : \mathrm{f}_a = \sum_{m=1}^{M} \sum_{r=1}^{N_m} \delta_{amr} \cdot x_{mr}, a = 1,2,...,A, \mathbf{x} \in G \right\} \tag{2}$$

In vector/matrix notation, let $\mathbf{x}$ and $\mathbf{f}$ be column vectors, and define $\mathbf{\Delta}$ as the $A \times N$-dimensional link-path incidence matrix. Then the relationship between link and path flows may be written as $\mathbf{f} = \mathbf{\Delta x}$. We suppose that the travel cost on path r for OD-pair m is additive in the link travel costs of the utilised links:

$$c_{mr}(\mathbf{x}) = \sum_{a=1}^{A} \delta_{amr} \cdot t_a(\Delta \mathbf{x}) \qquad (r \in R_m;\ m = 1,2,\ldots,M;\ \mathbf{x} \in G) \tag{3}$$

Define $\mathbf{t}(\mathbf{f})$ ($\mathbf{t} : \mathbf{R}_{+}^{A} \rightarrow \mathbf{R}_{+}^{A}$) as the vector of generalised link travel cost functions, and $\mathbf{c}(\mathbf{x})$ ($\mathbf{t} : \mathbf{R}_{+}^{N} \rightarrow \mathbf{R}_{+}^{N}$) as the vector of generalised route travel cost functions. Then the relationships between path and link flows, and between link and path costs, may be succinctly written as:

$$\mathbf{f} = \mathbf{\Delta x} \quad \text{and} \quad \mathbf{c(x)} = \mathbf{\Delta}^{\mathrm{T}} \mathbf{t(\Delta x)} \tag{4}$$

Our particular interest is in SUE models, which capture traveller heterogeneity and mis-perceptions through first positing random utilities U_{mr} for each route:

$$U_{mr} = -\theta \cdot c_{mr}(\mathbf{x}) + \xi_{mr} \qquad \left(r \in R_m; m = 1,2,...,M \right) \tag{5}$$

where $\boldsymbol{\xi} = \left\{ \xi_{mr} : r \in R_m, m = 1,2,...,M \right\}$ are random variables following some given joint probability distribution, and $\theta > 0$ is a given parameter. We then define the following functions as the probability relations:

$$P_{mr}(\mathbf{c}(\mathbf{x})) = \Pr(-\theta \cdot c_{mr}(\mathbf{x}) + \xi_{mr} \geq -\theta \cdot c_{ms}(\mathbf{x}) + \xi_{ms}, \forall s \in R_m) \qquad \left(r \in R_m; m = 1,...,M \right) \tag{6}$$

These relations express the probability that path r between OD-pair m will have a perceived utility greater than or equal to the utilities of all alternative paths in the universal set of routes for that OD-pair, when the random utilities are $-\theta \cdot \mathbf{c}(\mathbf{x})+\boldsymbol{\xi}$ and the generalised path travel costs are $\mathbf{c}(\mathbf{x})$.

The two slight variants to the standard SUE formulation are introduced for their relevance in subsequent sections. Firstly, we define:

$$Q_{mr}(\mathbf{c}(\mathbf{x})) = \Pr(-\theta \cdot c_{mr}(\mathbf{x}) + \xi_{mr} > -\theta \cdot c_{ms}(\mathbf{x}) + \xi_{ms}, \forall s \in R_m) \qquad \left(r \in R_m; m = 1,\ldots,M\right) \quad (7)$$

with the only distinction with $P_{mr}(\mathbf{c}(\mathbf{x}))$ being the strict inequality inside the probability statement.

Secondly, we define:

$$P_{mr}(\mathbf{c}(\mathbf{x}) | \tilde{R}_m) = \Pr\left(-\theta \cdot c_{mr}(\mathbf{x}) + \xi_{mr} \geq -\theta \cdot c_{ms}(\mathbf{x}) + \xi_{ms}, \forall s \in \tilde{R}_m\right) \quad \left(r \in \tilde{R}_m \subseteq R_m; m = 1,2,\ldots,M\right) \quad (8)$$

for any non-empty subset $\tilde{R}_m$ of R_m (m = 1,2,…,M). That is to say, whenever such a subset is not specified, we suppose P_{mr} refers to the universal set.

Moreover, we define for completeness the reference concepts in network equilibrium analysis (see, e.g., Sheffi, 1985; Patriksson, 1994):

Definition 1: Wardrop conditions

For any OD movement, the generalised travel costs on all paths actually used are equal, and less than or equal to the cost that would be experienced by a traveller on any unused path for that OD movement.

Definition 2: Deterministic User Equilibrium (DUE)

The route flow vector x ∈ *G is a DUE solution if and only if, for some* π_m (m = 1,2,…,M):

$$x_{mr} > 0 \Rightarrow c_{mr}(\mathbf{x}) = \pi_m \qquad \forall r \in R_m \quad m = 1,2,\ldots,M \quad (9)$$

$$x_{mr} = 0 \Rightarrow c_{mr}(\mathbf{x}) \geq \pi_m \qquad \forall r \in R_m \quad m = 1,2,\ldots,M \quad (10)$$

Definition 3: Stochastic User Equilibrium (SUE)

Given probability relations $P_{mr}(.)$ ($r \in R_m$; $m = 1,2,\ldots,M$) of the form given above, where ξ is a vector of <u>*continuous*</u> *random variables, the route flow vector* x $\in$ *G is a SUE solution if and only if:*

$$x_{mr} = d_m \cdot P_{mr}(\mathbf{c}(\mathbf{x})) \qquad (r \in R_m; m = 1,2,\ldots,M)\,. \tag{11}$$

3 Stochastic User Equilibrium with General Error Distributions

3.1 Specification and definition

The generalised cost may be expressed as a function of several factors, and if travellers were asked about their *perceptions* of travel time or trip length, then two things are certain: they would not include ∞ as a possibility, and they would give a round (likely whole-number) answer in whatever units they use. For such perceived factors, the reality is thus discrete and bounded. Many real-life phenomena are also discrete (e.g., due to the limitations of measurement equipment) and bounded, yet we choose to represent their likely values - for mathematical convenience - by continuous and unbounded variables. For example, we cannot measure vehicle speeds to a strictly continuous precision and we know that no infinitely fast vehicles exist, yet we might represent their probability distribution as a Lognormal distribution. In a similar way, when using RUM to represent travellers' heterogeneity/mis-perception inside an SUE model, it has become standard practice to use continuous and unbounded distributions for the random error terms. However, this mathematical convenience comes at a price, namely the SUE model must assign some flow to all alternative routes in the Master Choice Set, however unattractive they are.

In order to address this issue, it is useful to first reflect on the behavioural assumptions underlying SUE, which are typically not stated in the way that the Wardrop conditions are as the foundation for DUE.

Definition 4: Stochastic User Conditions (continuous distributions)

For any OD movement, the proportion of travellers on a path is equal to the probability that that path has a perceived utility greater than [or equal to] the perceived utility of all alternative paths.

The Wardrop conditions are written more as a plausibility test, they are not a definitive statement of the flows on alternative routes; even knowing some pattern of travel costs consistent with these conditions, we cannot even definitely state which routes are used - only those that are *potentially* used (those with equal minimum cost) and definitely unused (those with greater cost) - so we certainly have no chance to say with what proportions the routes are chosen. In contrast, the Stochastic User conditions make an explicit statement of the proportion of flow on alternative routes. This level of specificity emerges directly due to the assumption of a continuous error distribution in the RUM, since then the probability is zero of two routes being tied for maximum perceived utility. This also means that it is irrelevant whether we specify the condition with or without the *'[or equal to]'* part of the statement.

We then propose a more general set of conditions, which make no premise on the form of the probability distribution of random error terms:

Definition 5: General Stochastic User Conditions

For any OD movement, the proportion of travellers on a path is:

(i) than greater or equal to the probability that {that path has a perceived utility strictly greater than the perceived utility on all alternative paths}, and

(ii) less than or equal to the probability that {that path has a perceived utility greater than or equal to the perceived utility on all alternative paths}.

This definition captures the fact that, when there is a non-zero probability that the perceived utilities could be exactly equal, then there is not an exact equality definition for how proportion and probability may be related. In this way, (Def. 5) thus is more similar to the definition of Wardrop conditions (Def. 1).

Proposition 1

The General Stochastic User conditions contain Wardrop's conditions as a special case, when travellers make no perception errors (i.e. when discrete probability mass one is placed at zero perception error).

Proof

As defined earlier we suppose the deterministic part of utility being equal to $-\theta$ multiplied by the route travel cost, for scale parameter $\theta > 0$. When there are no stochastic elements in utility, comparing routes is invariant to θ and without loss of generality we may assume $\theta = 1$. Thus, we can replace maximising perceived utilities in the General Stochastic User conditions with minimising travel costs. The probability that a 'path has a perceived utility strictly greater the perceived utility on all alternative paths' is then either 1 (if the path has a strictly lower cost than all other paths) or 0 (if its cost is greater than or equal to all other costs). Similarly, the second General Stochastic User condition translates to a probability 1 for a path if it has cost lower than or equal to the cost on all other paths, and 0 if is its cost is greater than all other paths. We can then consider three cases: (i) a path that has cost strictly greater than all other paths for that movement has flow proportion bounded below and above by 0, and therefore is zero (i.e., is unused); (ii) a path that has cost less than or equal to that of all other paths has flow proportion bounded below by 0 and above by 1 (i.e., it is potentially used, but may be unused); (iii) a path that has cost strictly less than that of all other paths has flow proportion bounded above and below by 1 (i.e., it is the only used path). Used paths are definitely in case (iii) and potentially in case (ii): in case (iii) there is only one used route, so clearly then all used routes have equal cost; if we identify the subset of routes to which case (ii) applies (which may be both used and unused), then the only way any route in this subset is no worse than any other route in the subset is if all routes in the subset have equal cost, and so certainly all *used* routes in this subset will have equal cost to one another. Thus, we have verified the first component of Wardrop's conditions ('travel costs on all paths actually used are equal'). Note also that, by the same argument, any *unused* path in this subset satisfying case (ii) must also have equal cost to the used paths in that subset; this satisfies the second component of Wardrop's conditions ('travel costs on all paths actually used are less than or equal to the cost that would be experienced by a traveller on any unused path for that OD movement'). By construction, cases (i) and (iii) also satisfy this second component, thus completing the proof. □

This result is significant as it implies that, within the framework of the General Stochastic User conditions, we have at least one special case that has unused alternatives; in contrast, the standard Stochastic User conditions for continuous-only error distributions cannot include this special case, and can only at best approximate Wardrop, and then only through assigning some flow to all paths. This leads us to posit the following formulation of an equilibrium solution corresponding to the General Stochastic User conditions. In fact, these conditions were mentioned in passing by Daganzo and Sheffi (1977), and in the midst of a proof by Sheffi (1985), but it seems that they were never proposed or explored as a model in their own right, and it seems their possibility has since been forgotten (and certainly their relation to dealing with equilibrated choice sets has not been explored).

Definition 6: Stochastic User Equilibrium with General Error distribution (SUEGE)

Given probability relations $P_{mr}(.)$ and $Q_{mr}(.)$ $(r \in R_m;\ m = 1,2,\ldots,M)$ *of the form defined in section 2, the route flow* x $\in G$ *is a SUEGE if and only if:*

$$d_m \cdot Q_{mr}(\mathbf{c}(\mathbf{x})) \le x_{mr} \le d_m \cdot P_{mr}(\mathbf{c}(\mathbf{x})) \quad (r \in R_m; m = 1,2,...,M) \ . \tag{12}$$

Proposition 2

(i) Suppose that, in the definition of $P_{mr}(.)$ *and* $Q_{mr}(.)$, *the* $\{\xi_{mr} : r \in R_m, m = 1,2,...,M\}$ *are discrete random variables, each with probability mass 1 at the value of zero. Then* x *is a SUEGE if and only if* x *is a DUE.*

(ii) Alternatively, suppose that the $\{\xi_{mr} : r \in R_m, m = 1,2,...,M\}$ *are continuous random variables. Then* x *is a SUEGE if and only if* x *is a SUE.*

Proof

The fact that SUEGE contains DUE as a special case follows directly by applying (Prop. 1) at the route flows **x** and costs **c**(**x**). The fact that SUEGE contains SUE as a special case arises due to the fact that for continuous-only distributions $P_{mr}(\mathbf{c}(\mathbf{x})) = Q_{mr}(\mathbf{c}(\mathbf{x}))$, and so the only solution to the SUEGE inequality is $x_{mr} = d_m \cdot P_{mr}(\mathbf{c}(\mathbf{x}))$, i.e. SUE. □

3.2 INSTANCES OF SUEGE MODELS

SUEGE generalises both the standard DUE and SUE models. However, the existence of examples of SUEGE with unused paths has been shown only for the special case of DUE where the stochastic element disappears, not for SUEGE models with stochastic terms that incorporate unused alternatives. The existence of such examples would satisfy the aim of the present study to have an equilibrated (but non-universal) choice set. The two examples below establish the existence of such examples for the cases of discrete and continuous bounded error distributions.

Example 1: SUEGE with discrete bounded error distribution

Consider a network serving an OD demand of d_1=100 and consisting of three parallel links/paths with separable link travel cost functions $t_1(\mathbf{f}) = 8 + f_1/10 \quad t_2(\mathbf{f}) = 18 + f_2/15 \quad t_3(\mathbf{f}) = 25 + f_3/50$, and hence route travel cost functions $c_{11}(\mathbf{x}) = 8 + x_{11}/10 \quad c_{12}(\mathbf{x}) = 18 + x_{12}/15 \quad c_{13}(\mathbf{x}) = 25 + x_{13}/50$.

Suppose that θ = 1, and that the three discrete error distributions in the RUM are statistically independent between routes, and are given by:

$$\Pr(\xi_{11} = 0) = 0.7,\ \Pr(\xi_{11} = 5) = 0.3,$$
$$\Pr(\xi_{12} = 0) = 0.6,\ \Pr(\xi_{12} = 10) = 0.4,$$
$$\Pr(\xi_{13} = -5) = 0.2,\ \Pr(\xi_{13} = 0) = 0.8.$$

Consider the flow **x*** = (70, 30, 0). The corresponding travel costs are c(x*) = (15, 20, 25).

Then:

$$\begin{aligned} Q_{11}(\mathbf{c}(\mathbf{x}^*)) &= \Pr(-\theta \cdot c_{11}(\mathbf{x}^*) + \xi_{11} > \max(-\theta \cdot c_{12}(\mathbf{x}^*) + \xi_{12}, -\theta \cdot c_{13}(\mathbf{x}^*) + \xi_{13})) \\ &= \Pr(-c_{11}(\mathbf{x}^*) + \xi_{11} > \max(-c_{12}(\mathbf{x}^*) + \xi_{12}, -c_{13}(\mathbf{x}^*) + \xi_{13})) \end{aligned}$$

and we consider the 2^3 = 8 states for the three error terms across the routes, adding the appropriate joint probability whenever the condition in the Q_{11} probability is satisfied. The states 1-8 for $(-c_{11}(\mathbf{x}^*) + \xi_{11}, -c_{12}(\mathbf{x}^*) + \xi_{12}, -c_{13}(\mathbf{x}^*) + \xi_{13})$ are respectively (–15,–20,–30), (–15,–10,–30), (–10,–20,–30), (–10,–10,–30), (–15,–20,–25), (–15,–10,–25), (–10,–20,–25), (–10,–10,–25). State 1 occurs (due to the independence assumption) with probability

$0.7 \cdot 0.6 \cdot 0.2 = 0.084$, and the remaining seven state probabilities are 0.056, 0.036, 0.024, 0.336, 0.224, 0.144, 0.096. Only states 1, 3, 5, 7 satisfy the condition $-c_{11}(\mathbf{x}^*) + \xi_{11} > \max(-c_{12}(\mathbf{x}^*) + \xi_{12}, -c_{13}(\mathbf{x}^*) + \xi_{13})$, and so:

$$\begin{aligned} Q_{11}(\mathbf{c}(\mathbf{x}^*)) &= \Pr(-c_{11}(\mathbf{x}^*) + \xi_{11} > \max(-c_{12}(\mathbf{x}^*) + \xi_{12}, -c_{13}(\mathbf{x}^*) + \xi_{13})) \\ &= \Pr(-c_{11}(\mathbf{x}^*) + \xi_{11} > -c_{12}(\mathbf{x}^*) + \xi_{12}) \\ &= 0.084 + 0.036 + 0.336 + 0.144 = 0.6. \end{aligned}$$

For P_{11} we have additionally states 4 and 8 that satisfy the equality condition, hence:

$$\begin{aligned} P_{11}(\mathbf{c}(\mathbf{x}^*)) &= \Pr(-c_{11}(\mathbf{x}^*) + \xi_{11} \geq \max(-c_{12}(\mathbf{x}^*) + \xi_{12}, -c_{13}(\mathbf{x}^*) + \xi_{13})) \\ &= Q_{11}(\mathbf{c}(\mathbf{x}^*)) + 0.024 + 0.096 = 0.72. \end{aligned}$$

By repeating this process:

$$\begin{aligned} Q_{12}(\mathbf{c}(\mathbf{x}^*)) &= \Pr(-c_{12}(\mathbf{x}^*) + \xi_{12} > \max(-c_{11}(\mathbf{x}^*) + \xi_{11}, -c_{13}(\mathbf{x}^*) + \xi_{13})) \\ &= 0.056 + 0.224 = 0.28. \end{aligned}$$

$$P_{12}(\mathbf{c}(\mathbf{x}^*)) = Q_{12}(\mathbf{c}(\mathbf{x}^*)) + 0.024 + 0.096 = 0.4$$

and finally:

$$Q_{13}(\mathbf{c}(\mathbf{x}^*)) = P_{13}(\mathbf{c}(\mathbf{x}^*)) = 0$$

Hence, the SUEGE conditions would require the flow $\mathbf{x}^*$ to satisfy (in addition to demand feasibility):

$$100 \cdot 0.6 \leq x_{11}^* \leq 100 \cdot 0.72;\ 100 \cdot 0.28 \leq x_{12}^* \leq 100 \cdot 0.4;\ 100 \cdot 0 \leq x_{13}^* \leq 100 \cdot 0$$

i.e.:

$$60 \leq x_{11}^* \leq 72;\ 28 \leq x_{12}^* \leq 40;\ x_{13}^* = 0\,.$$

Since the given $\mathbf{x}^*$ = (70, 30, 0) satisfies this condition, it is indeed a SUEGE solution, and it notably consists of an equilibrated but non-universal choice set. We searched for alternative SUEGE solutions on the network but found that only the $\mathbf{x}^*$ above exists. It is important to note however that while only one SUEGE solution exists for the example, several SUEGE solutions may exist for other networks.

Example 2: SUEGE with continuous bounded error distribution

Again, consider a network serving an OD demand of d_1=100 and consisting of three parallel links/paths, again with θ= 1, and now with link cost functions:

$$t_1(\mathbf{f}) = 7.5 + f_1/10 \quad t_2(\mathbf{f}) = 15 + f_2/5 \quad t_3(\mathbf{f}) = 25 + f_3/50 \ ,$$

Now suppose three *continuous* bounded error distributions in the RUM, again statistically independent between routes:

$$\xi_{11} \sim \text{Uniform}(0,5),\ \xi_{12} \sim \text{Uniform}(0,10),\ \xi_{13} \sim \text{Uniform}(-5,0)\ .$$

Consider the flow allocation **x*** = (75, 25, 0), with **c**(**x***) = (15, 20, 25). In this case, the SUEGE conditions reduce to an equality-based (SUE) fixed point problem, and we need consider only the P functions:

$$P_{11}(\mathbf{c}(\mathbf{x}^*)) = \Pr(-c_{11}(\mathbf{x}^*) + \xi_{11} \geq \max(-c_{12}(\mathbf{x}^*) + \xi_{12}, -c_{13}(\mathbf{x}^*) + \xi_{13}))\ .$$

The three random utilities, $-c_{1r}(\mathbf{x}^*) + \xi_{1r}$ (r = 1,2,3), are thus distributed uniformly on the intervals (–15, –10), (–20, –10), and (–30,–25), and so for these random variables it is the case that:

$$\max(-c_{12}(\mathbf{x}^*) + \xi_{12}, -c_{13}(\mathbf{x}^*) + \xi_{13}) = -c_{12}(\mathbf{x}^*) + \xi_{12}$$

since the interval on which $-c_{13}(\mathbf{x}^*) + \xi_{13}$ is defined is greater than the interval on which $-c_{12}(\mathbf{x}^*) + \xi_{12}$ is defined. Thus:

$$\begin{aligned}
P_{11}(\mathbf{c}(\mathbf{x}^*)) &= \Pr(-c_{11}(\mathbf{x}^*) + \xi_{11} \geq -c_{12}(\mathbf{x}^*) + \xi_{12}) \\
&= \Pr(-c_{11}(\mathbf{x}^*) + \xi_{11} \geq -c_{12}(\mathbf{x}^*) + \xi_{12} \,|\, -c_{12}(\mathbf{x}^*) + \xi_{12} \leq -15) \cdot \Pr(-c_{12}(\mathbf{x}^*) + \xi_{12} \leq -15) \\
&\quad + \Pr(-c_{11}(\mathbf{x}^*) + \xi_{11} \geq -c_{12}(\mathbf{x}^*) + \xi_{12} \,|\, -c_{12}(\mathbf{x}^*) + \xi_{12} \geq -15) \cdot \Pr(-c_{12}(\mathbf{x}^*) + \xi_{12} \geq -15) \\
&= 1 \cdot 0.5 + 0.5 \cdot 0.5 = 0.75
\end{aligned}$$

exploiting the uniformity of the distributions.

Following the same logic:

$$\Pr(-c_{12}(\mathbf{x}^*)+\xi_{12} \geq -c_{11}(\mathbf{x}^*)+\xi_{11}) = 1-0.75 = 0.25 = P_{12}(\mathbf{c}(\mathbf{x}^*)) \text{ and}$$

$$P_{13}(\mathbf{c}(\mathbf{x}^*)) = 0$$

thus confirming that **x*** = (75, 25, 0) is a SUEGE solution, and it notably consists of an SUE over an equilibrated but non-universal choice set. We have searched numerically (by grid search) for other SUEGE solutions on the network, but did not find any. Therefore we believe the solution **x*** above to be a unique solution in the example.

In conclusion, we have defined in the present section a formulation of SUE that permits both discrete and continuous, bounded and unbounded error distributions. We have shown that includes both Wardrop's conditions/DUE and traditional SUE as special cases, and with simple illustrative examples have shown how it may indeed lead to solutions with equilibrated but non-universal choice sets.

4 STOCHASTIC USER EQUILIBRIUM WITH RESTRICTIONS

4.1 SPECIFICATION AND DEFINITION

In section 3, we presented formulations of SUE that, by moving away from the continuous/unbounded error distributions traditionally associated with RUM theory, were able to capture equilibrated, non-universal choice sets within an SUE framework. A disadvantage of this approach, however, is precisely the fact that it departs from the well-understood and well-researched range of choice models that incorporate continuous, unbounded distributions, such as logit, probit, path-size logit, and so forth. Therefore, as an alternative to the SUEGE models introduced in section 3, in the present section we explore the formulation of models that on the one hand retain the connection to these well-researched methods of representing choice across the used routes, but on the other hand includes equilibrium conditions that distinguish used and unused routes.

The inspiration for these conditions derives from the Wardrop conditions; in order to understand this connection, it is helpful to first write the Wardrop conditions in a slightly alternative, but equivalent form:

Definition 7: Wardrop conditions (alternative form)

For each OD movement:

(i) *the generalised travel costs on all paths actually used are equal;*

(ii) *the 'reference cost' is equal to the cost on any used path;*

(iii) *the cost which would be experienced by a traveller on any unused path is greater than or equal to the reference cost as defined in ii).*

This alternative definition introduces the notion of a 'reference cost' as a single value representing all used paths, as the benchmark against which to judge unused alternatives. For the Wardrop conditions, it is very clear what this reference cost must be, since all used paths have the same travel cost, and so introducing it seems an unnecessary complication. However, this turns out to be a key element to defining our new conditions, in a situation where the used paths have unequal travel costs. These conditions are a combination of (Def. 4) and (Def. 7):

Definition 8: Φ-Restricted Stochastic User Conditions

For each OD movement:

(i) *the proportion of travellers on any used path is equal to the probability that that path has a perceived utility greater than or equal to the perceived utility of all alternative used paths;*

(ii) *the 'reference cost' is a value uniquely defined by some relationship Φ to the travel costs on the used paths;*

(iii) *the travel cost which would be experienced by a traveller on any unused path is greater than or equal to the reference cost as defined in ii).*

In comparison with the Stochastic User conditions (Def. 4), it can be seen that condition (i) above overcomes one of the main limitations, in that they may only apply to a sub-network of the paths available. This is also true of SUE models applied to a pre-defined Master Choice set, but the key difference in (Def. 8) is that, at equilibrium, conditions (ii)/(iii) must be simultaneously satisfied alongside condition (i). That is to say, given that perceived utility is an affine function of travel cost, plus random errors, the three conditions above must be *consistently* satisfied, at the *same* travel cost levels. Thus, they do indeed yield an alternative mechanism for defining *equilibrated*, non-universal choice sets in an SUE framework. It is also worth remarking that the Φ-Restricted Stochastic User conditions, though owing their inspiration partly to the Wardrop conditions (Def. 7), are not as tightly defined, since there exist several alternative, plausible ways for defining the reference costs in condition (ii). That is to say, (Def. 8) defines a *class* of conditions that is as wide as the ways in which the relationship Φ may be defined.

A final remark on (Def. 8) is that these criteria may, in principle, be applied to a flow allocation produced by any method, and reported as a plausibility measure of the resulting flow pattern. For example, they could be applied to the iterations of a conventional path-based SUE solution algorithm, in which at any iteration typically only a subset of the available paths are used, as a sensible guide to whether some important plausible paths may still need to be included. Alternatively, they could be applied to an estimated SUE solution based on a pre-defined Master Choice Set, in this case as a measure of the extent to which the estimated equilibrium costs on the network support the assumed Master Choice Set. However, henceforth in this section we explore the properties of an equilibrium model in its own right, based on the Φ-Restricted Stochastic User Conditions:

Definition 9: Φ -Restricted Stochastic User Equilibrium (RSUE(Φ))

Suppose that we are given a collection of continuous, unbounded random variables $\{\xi_{mr} : r \in R_m, m = 1,2,...,M\}$ *defined over the whole choice set* R_m, *and that for any* non-empty *subsets* $\tilde{R}_m$ *of* R_m (*m* = 1,2,…,*M*), *probability relations* $P_{mr}(\mathbf{c}|\tilde{R}_m)$ *are given* $\tilde{R}_m$ *by considering the relevant marginal joint distributions from* $\{\xi_{mr} : r \in R_m, m = 1,2,...,M\}$. *The route flow* x ∈ *G is a RSUE(* Φ *) if and only if for all r* ∈ *R*$_m$ *and m* = 1,2,…,*M:*

$$x_{mr} > 0 \quad \Rightarrow \quad r \in \tilde{R}_m \quad \wedge \quad x_{mr} = d_m \cdot P_{mr}(\mathbf{c}(\mathbf{x})|\tilde{R}_m) \tag{13}$$

$$x_{mr} = 0 \quad \Rightarrow \quad r \notin \tilde{R}_m \quad \wedge \quad c_{mr}(\mathbf{x}) \geq \Phi\left(\left\{c_{ms}(\mathbf{x}) : s \in \tilde{R}_m\right\}\right) \tag{14}$$

The RSUE(Φ) conditions ensure that the restricted choice set contains only the used paths and that the Φ-Restricted Stochastic User conditions hold. Comparing the SUE formulation (Def. 9) to that for DUE (Def. 2), it can be seen that there are similarities: they both have one statement concerning utilised paths and one statement concerning non-utilised paths, but present several important differences.

Firstly, comparing the conditions on used paths, there is the use of *perceived* utility in RSUE(Φ) rather than actual travel costs in DUE; in this way, RSUE overcomes the main limitation of DUE, as it accounts for perception errors of path attributes by allowing traffic to be distributed to non-minimum cost paths, in order that the SUE conditions are satisfied on the restricted choice set of used paths.

Secondly, in the RSUE conditions we have a choice of how to define the operator Φ, whereas in the DUE model we do not. In fact, in the DUE model the variable π_m, even if defined as a free variable, must at equilibrium equal the travel cost on any used path for OD movement *m*, and we do not need to add any additional constraint to ensure that π_m is related to the path costs in this way. In the RSUE model, no such condition emerges, and we then need an explicit definition of how the reference OD travel cost is related to the path travel costs of used paths. In addition, since in RSUE used paths will typically not have the same travel cost, there is some leeway in how precisely to define Φ.

It should be noted that, in the RSUE definition (Def. 9), we consider only RUM models with continuous and unbounded error distributions. As noted in section 3, under such an assumption all alternatives in the RUM (in this case, those in $\tilde{R}_m$) will have a non-zero probability of being chosen. Thus, condition (14) will never be relevant for a path that is subject to the RUM, i.e. in $\tilde{R}_m$, since such a path will always attract a positive flow. This makes the separation of used/unused paths coincide with the separation of those paths subject to the RUM and not subject to it.

A final remark is on the relation of the RSUE(Φ) model to conventional notions of equilibrium in networks. Unlike the SUEGE model, the RSUE(Φ) model does not contain DUE as a special case, in spite of the similarities in the specification of RSUE(Φ) and DUE. This is due to the fact that we restrict the attention in RSUE(Φ) to choice models which have *continuous* random utilities on the used paths, and thus the probability of two paths being exactly equal in terms of perceived utility is zero, whatever continuous distribution is adopted for the error terms. RSUE(Φ) does, however, contain SUE as a special case (regardless of the specification of Φ). This may be seen by setting $\tilde{R}_m = R_m$ in the RSUE definition (Def. 9), meaning that there are no paths for which condition (14) is tested, and condition (13) is simply an SUE condition on the universal choice set. This is true for any problem, and therefore we can guarantee existence of at least one RSUE(Φ) solution by exactly the same conditions as those that guarantee existence of a SUE solution. In particular, Cantarella (1997) proposed a Fixed-Point formulation of SUE, and using Brouwer's theorem he showed that a solution exists if the choice function and the cost-flow functions are continuous, the link flow feasible set F is non-empty (i.e., at least 1 path exists between OD-pairs m for which d_m>0), compact and convex, and the link flows resulting from the flow network loading map (expressing link flows in terms of link costs) are always feasible.

4.2 INSTANCES OF RSUE(Φ) MODELS

A key question that appears is the definition of Φ. Since in condition (14) the actual travel cost on an *unused* alternative must be compared with the actual travel costs on *used* alternatives, and since these unused alternatives are not subject to the random utility specification, it seems reasonable that Φ must map to something that makes sense in terms of the actual travel costs (rather than the randomly perceived utilities). Thus, while it might seem a possibility, it is not so sensible that Φ is a satisfaction function (expected maximum perceived utility, such as logsum for multinomial logit) over the used alternatives, as then we are in the 'scale' of perceived utility as opposed to actual travel cost. An alternative, then, might be to define Φ as the average or median travel cost of the used alternatives, but there are surely many possibilities that might be explored. In our case, we focus on two example possibilities (without wishing to rule out others), each seemingly having its own attractive features.

The two particular examples are the RSUE(min) model, obtained by defining for any non-empty set B:

$$\Phi(B) = \min\{b : b \in B\} \tag{15}$$

and the RSUE(max) model, obtained by defining:

$$\Phi(B) = \max\{b : b \in B\} \ . \tag{16}$$

An attraction of the RSUE(min) model is that, apparently, it leads in the direction of a computationally tractable method. The reason for believing this is as follows. With the min operator in equation (14), then if we are given some candidate flow pattern, and wish to verify whether it satisfies condition (14), then we have a form which is relatively easy to verify using standard computational tools for networks. In particular, given some path flow allocation and resulting network link costs, one can use some standard shortest path algorithm (for each OD movement) to identify the minimum cost path of any kind on the network. If the cost on this is (strictly) less than the cost on the currently minimum cost used route (for the corresponding OD-pair), then condition (14) is not satisfied.

Thus, the RSUE(min) model is in some sense the logical combination of DUE and SUE. However, it has a disadvantage in that it allows for traffic to be assigned to paths with actual travel costs greater than the actual travel costs of paths which are not utilised. From a behavioural point of view, one might question the plausibility of this, and in this respect the RSUE(max) model has an advantage.

The RSUE(max) model requires that no path is unutilised if it has an actual travel cost that is lower than or equal to the actual travel cost on the longest utilised path. While this seems behaviourally more defensible, it may lead to a less tractable computational model. Certainly, property (18) is more difficult to verify from a computational perspective for the RSUE(max) model than it is for RSUE(min), yet still there are standard network analysis tools for doing so. In particular, given some path flow allocation and the resulting network link costs, a standard tool can be used (for each OD movement) to identify the current k shortest paths (where k is the number of used paths). If there among these exists any currently unused path on which the cost is (strictly) less than the cost on currently maximum cost used route (for the corresponding OD movement), then condition (14) is violated. Clearly, the computational effort involved in solving k-shortest path problems and identifying any unused paths among these is significantly greater than that required for solving standard shortest path problems, and so verifying that the RSUE(max) conditions are satisfied is much more demanding than the verification of the RSUE(min) conditions.

Proposition 3

Any RSUE(max) solution is also a RSUE(min) solution. An RSUE(min) solution may not, however, necessarily fulfil the RSUE(max) conditions.

Proof

Suppose a flow allocation satisfies the RSUE(max) conditions. Then from conditions (14) when Φ is the max operator, any unused path must have a travel cost greater than or equal to the maximum cost used path. By definition, the maximum cost used path must have cost at least as great as the minimum cost used path, and so property (14) is also satisfied when instead Φ is the min operator. Property (13) of RSUE(max) is the same regardless of the choice of Φ, and so we have shown that the flow allocation must also satisfy the RSUE(min) conditions. For the converse situation, suppose that a flow allocation satisfies the RSUE(min) conditions, and in addition has an unused path which has a cost less than the maximum cost of any used path. Then the RSUE(max) conditions are violated as illustrated in the following Example 3. □

Example 3

In this example we explore the multiplicity of solutions in a simple example in which we can exhaustively check the conditions for all non-empty subsets $\tilde{R}_m$ of the universal choice set R_m. We illustrate that RSUE solutions do indeed exist with an equilibrated but non-universal choice set and that RSUE(min) solutions may violate the RSUE(max) conditions.

Consider the network also considered in Example 1 and Example 2, serving an OD demand of d_1=100 and consisting of three parallel links/paths, now with link cost functions $t_1(\mathbf{f}) = 8 + f_1/10 \quad t_2(\mathbf{f}) = 13 + f_2/15 \quad t_3(\mathbf{f}) = 15 + f_3/50$,

Suppose that the choice model for used routes is a multinomial logit model with $\theta = 1$. With such a small network, it is possible to identify all 7 possible choice sets, and for each choice set to find an SUE solution by some traditional path-based solution method. We may then subsequently check each of these 7 possibilities with respect to the RSUE conditions. Clearly such an exhaustive search of possible choice sets would be infeasible for large-scale networks, but this example allows investigating the existence and multiplicity of RSUE solutions. The solution method is a path-based MSA (Sheffi and Powell, 1982) with 10,000 iterations and the solutions are shown in Table 1.

Table 1 – SUE solutions for all possible choice sets. Note: Hatch defines the choice set: Hatch/no hatch if path excluded from/included in the choice set

	Conf. 1 $[c_{1r}/x_{1r}]$	Conf. 2 $[c_{1r}/x_{1r}]$	Conf. 3 $[c_{1r}/x_{1r}]$	Conf. 4 $[c_{1r}/x_{1r}]$	Conf. 5 $[c_{1r}/x_{1r}]$	Conf. 6 $[c_{1r}/x_{1r}]$	Conf. 7 $[c_{1r}/x_{1r}]$
Path 1	13.9/59.1	18.0/100.0	8.0/0	8.0/0	14.6/66.0	14.9/68.5	8.0/0
Path 2	14.7/26.0	13.0/0	19.7/100.0	13.0/0	15.3/34.0	13.0/0	16.2/47.4
Path 3	15.5/14.8	15.0/0	15.0/0	17.0/100.0	15.0/0	15.6/31.5	16.1/52.6
RSUE(min)	YES (=SUE)	NO	NO	NO	YES	NO	NO
RSUE(max)	YES (=SUE)	NO	NO	NO	NO	NO	NO

For all cases, SUE has been found among utilised paths. This means that the first condition (13) of the RSUE(min) as well as the RSUE(max) definition is fulfilled in all cases, conditional on $\tilde{R}_m$ being the set of utilised paths. The second condition is fulfilled if the *actual* travel cost of paths not in the choice set is not shorter than the *actual* travel cost on the shortest (longest) utilised path for the RSUE(min) (RSUE(max)). Note that this is always fulfilled in the case where all paths are in the choice set, and the traditional SUE will always be a RSUE solution. From Table 1 we see that there exist unused paths which are shorter than the shortest (longest) used path for configurations 2-4 and 6-7, and these do thus not fulfil the second RSUE(min) (RSUE(max)) condition and do therefore not constitute RSUE(min) (nor RSUE(max)) solutions. The violation of the RSUE(max) conditions could have also been realised by using (Prop. 3) and the knowledge that the RSUE(min) conditions are violated.

However, since $\min\left\{c_{1s}(\mathbf{x}) : \mathrm{s} \in \tilde{R}_1\right\} = \min(14.6, 15.3) = 14.6$ for configuration 5 with paths 1 and 2 used (i.e. $\tilde{R}_1 = \{1,2\}$), and since 15.0≥14.6 then the second RSUE(min) condition is fulfilled for configuration 5 that consequently gives a RSUE(min) solution. Assuming instead a max operator for Φ, then the second condition (14) requires that any unused paths have cost at least as great as the *maximum* cost of a used path (= 15.3 in this case), and since 15.0 < 15.3 the flow solution where paths 1 and 2 are used is not a RSUE(max) solution.

From this example we can see that RSUE solutions exist with equilibrated but non-universal choice sets, and that solutions that satisfy RSUE(min) may not satisfy RSUE(max) for a given problem. In the example we did not find any RSUE(max) solutions using a non-universal choice set. We could however imagine such a solution by adding a fourth non-overlapping route with free-flow travel cost of e.g. 20. In such a case configuration 1 (the current full SUE solution) would be a RSUE(max) with equilibrated but non-universal choice set, as a cost of 20 on the unused route would be higher than the most expensive used route (=15.5) causing the second RSUE(max) condition to be satisfied.

In the example we have focused on the RSUE(min) and RSUE(max), but surely other formulations of Φ could be investigated. One such could be the RSUE(avg) discussed earlier, where the operator is average travel cost of the used alternatives. This seems to 'mediate' between the RSUE(min) and RSUE(max) by putting a stricter condition on the cost on unused paths than the RSUE(min), however not as strict as the RSUE(max). In the example, configurations 1 and 5 satisfies the RSUE(avg) conditions, thus coinciding with the possible RSUE(min) solutions. It is important to note that RSUE(min) solutions does not always fulfil the RSUE(avg) conditions; Imagine adding a fourth alternative with free-flow travel cost of 14.8. In such a case configuration 5 using paths 1 and 2 would still be a RSUE(min) solution but not a RSUE(avg) solution, as the operator specifies a threshold cost of 14.95.

5 ANALYSIS OF SUEGE AND RSUE MODELS

After presenting two alternative frameworks and models for representing route choice based on RUM theory, both of which lead to an equilibrated but potentially non-universal choice set, we analyse and compare these approaches in terms of three key areas: (i) applicability and potential for calibration, (ii) theoretical issues, and (iii) potential for devising solution methods for large-scale networks.

5.1 APPLICABILITY AND POTENTIAL FOR CALIBRATION

The applicability and calibration potential of SUEGE and RSUE models requires considering first what the 'stochastic' terms might be useful for representing. In particular, we shall make a case that each model may be suitable for representing quite different kinds of variation.

Considering the SUEGE models, the pertinent cases are those in which the error distribution in the random utilities is bounded (either discrete or continuous). In such cases, the lower and upper bounds of the random utility distribution, and hence those of the error term, play the key role in distinguishing used from unused routes. The important question then arises: in what circumstances might it be (a) reasonable to assume such bounds, and (b) possible to estimate the bounds. This connects directly to what the error term may be capturing, for which there are several possibilities (see Watling et al. (2013) for a further discussion of this issue in a somewhat different context).

Two possibilities are that the random error is capturing (i) unobserved factors that might affect route choice, other than those measured in the generalised travel cost, or (ii) unobserved heterogeneity across the population of travellers. In these cases, we cannot *directly* estimate the bounds on the distributions of the error term, but supposing for example that we had data on routes actually chosen by OD movement, an *indirect* method could be (i) specify the error distribution as a bounded parameteric family of distributions, and (ii) estimate the parameters by finding the best fit between the observed route distributions and that distribution of routes that would occur under the SUEGE model with those parameters. Such a problem could, for example, be addressed by using some least squares metric as a goodness-of-fit measure, and then maximising the fit subject to a SUEGE constraint (a kind of Mathematical Program with Equilibrium Constraints, MPEC).

Although this method (for indirectly calibrating unobserved phenomena) sounds potentially feasible, there are several significant difficulties with it. It is not clear what goodness-of-fit measure would truly focus on the boundary of distinguishing lightly used from unused routes. If our focus is on identifying which routes are not used at all, it seems we would need to observe *all* trips, rather than estimating route proportions from a sample. Moreover, it is still rather unusual to have *route-level* information available for network problems; with only link flow information, we are one stage further removed from the 'direct observation' described above.

A third possible interpretation of the random error in the utilities would be that they are a representation of perception errors of the travellers. The SUEGE model with discrete bounded distributions seems particularly attractive to represent this, as travellers will typically think of travel times or other measures in whole units (e.g. minutes), or even further rounded. It seems possible to determine the distributions to use through experiments where a sample of travellers are asked to estimate travel times (both pre- and post-trip), which can then be compared with independent measurements of the actual trip times.

Considering the RSUE models, there are two quite distinct mechanisms to consider for calibration: the RUM for dispersing traffic among used routes, and the operator Φ for distinguishing used from unused routes. The RUM element is effectively the same as for conventional SUE models, and so this element effectively offers no new challenge over-and-above calibrating a conventional SUE model based on RUM models. The new challenge for RSUE is in specifying a sensible Φ operator that allows the modeller to restrict the assumed choice set to a manageable size, and is not about traveller perception or unobserved factors. In this respect, we can make comparisons with actual travel costs (e.g. travel times) observed on the network, separately from the issue of perception.

For example, if we suppose that the only element of travel cost were travel time, and that we have observations of link travel times across a network, then we could assume and apply an operator Φ to the observed travel times, and compare the ranking of routes with the distinction between used and unused routes as predicted by a RSUE(Φ) solution. If we also have observations of actual routes chosen by travellers, then we might verify whether the actually chosen routes are all used routes in an RSUE solution, what is the distribution of travel costs across these chosen routes, and how does it compare with the distribution of travel costs on used routes in the RSUE solution for a given problem. The answers to these questions vary under different assumed Φ functions, and under different assumed choice models for a given problem.

Summarising, there is potential for calibrating both SUEGE and RSUE models, especially when more route-level data become available in the future through increasingly popular and ever more precise tracking devices. In both cases, this tracking needs to penetrate to quite a large fraction of travellers, if we are truly to distinguish lightly used from completely unused routes. The SUEGE models seem particularly appropriate for modelling traveller perception errors, whereas RSUE is probably more suited to capturing unobserved factors and unobserved heterogeneity. The RSUE models seem more straightforward to apply in the short term, as extensions of existing and calibrated SUE models, especially if supplemented with some information on routes actually chosen.

5.2 THEORETICAL CONSIDERATIONS

To the best of the authors' knowledge no work exists, establishing the existence and uniqueness of SUEGE solutions, other than that for the traditional SUE case of unbounded and continuous distributions (e.g., Cantarella, 1997). In the case of RSUE models, we can guarantee existence of at least one solution under the same condition as continuous, unbounded SUE solutions exist (Cantarella, 1997), but what we are really interested in is the existence of other RSUE solutions which do not use the full choice set.

Thus while it seems behaviourally implausible that travellers are error-free and identical in their perceptions of travel cost (as in DUE), or use all available paths (as in SUE), the price we pay for the additional plausibility in SUEGE or RSUE is a model with non-unique solutions, which may not be so convenient for cost-benefit analysis. In defence of these new models we would offer several arguments. Firstly, uniqueness of DUE and SUE solutions is only known under quite limited circumstances, which break down when we have problems with non-additive path costs, within-day dynamics, junction interactions, multiple vehicle types, responsive control or non-separable/monotone variable demand models. In fact examples of multiple solutions are known to exist in many such cases (e.g., Watling, 1996; Iryo, 2011). Secondly, there has been recent work that has deliberately sought to generate non-unique solutions, through multi-objective route choice modelling (Wang and Ehrgott, 2013; Wang et al., 2014). While the intention of the present study is to postulate more realistic models, rather than non-unique solutions, we may see the models as a way of generating reasonable candidate solutions. Thirdly, the SUEGE and RSUE models contain SUE or DUE as special or limit cases and hence, in a calibration process, it is legitimate to consider whether SUE or DUE offer a better fit to observations. If they do, they may be preferred, and so we do not rule out their use. Fourthly, traditional equilibrium models are heavily calibrated on link flow data (even the OD matrix). When the non-uniqueness we are referring to leads to different link flow solutions, then we may (at the calibration stage) choose between alternative candidate SUEGE/RSUE solutions based on such conventional link data. Fifthly, in the future, as it becomes more typical to have access to data from mobile/GPS devices, then the focus of calibration of equilibrium models may switch to a more route-based one. In such a case, there is an even better chance to resolve the non-uniqueness at the calibration stage, given several candidate solutions with different equilibrated choice sets – see the discussion of section 5.1.

A wider issue that has had great influence on the possibility to establish theoretical properties of network equilibrium models (as well as to devise efficient solution methods) is the kind of mathematical formulation adopted (e.g., convex optimisation problem, variational inequality, fixed point problem). It is therefore appropriate to examine the formulations adopted for the models presented. In the case of SUEGE with a *continuous* bounded error distribution, the resulting problem is also an SUE problem, and so is a fixed point condition. Alternative formulations of SUE (e.g., optimisation problem) have been established under certain assumptions on the error terms, but these are specific to particular models with unbounded errors, and so do not transfer unless specifically proven. The case of a *discrete* error SUEGE distribution is more complex. It is certainly possible to convert the inequality constraints (12) into equalities, by adding appropriate slack variables (two per route), and thus define a fixed point equality condition. However, since we suspect there may be several solutions, an alternative approach would be to consider the inequalities (12) as *constraints* to an optimisation problem, namely to minimise $g(\mathbf{x})$ subject to $\mathbf{x}$ satisfying (12). With $g(\mathbf{x})$ =

constant, solutions to such a problem define the full solution set, while $g(\mathbf{x}) = x_r$ (or $g(\mathbf{x}) = -x_r$) would define for some r the lower (or upper) bound on the flow on route r in the solution set, and $g(\mathbf{x}) = \sum_{m=1}^{M} \sum_{r=1}^{R_m} \delta(x_r)$ (where $\delta(y) = 1$ if $y > 0$, 0 otherwise) would define a SUEGE solution using the fewest number of routes.

In the case of RSUE models, the definitions (13)/(14) appear rather complex, but a more parsimonious formulation can be gained using the δ(.) indicator function introduced above. We further denote $\boldsymbol{\delta}(\mathbf{y}) = (\delta(y_1), \delta(y_2), ..., \delta(y_n))$ and then re-write (14) as:

$$(1 - \delta(x_{mr})) \cdot (c_{mr}(\mathbf{x}) - \Phi(\mathbf{c}(\mathbf{x}), \boldsymbol{\delta}(\mathbf{x}))) \geq 0 \qquad \forall r \in R_m, m = 1, 2, ..., M$$

In vector notation:

$$(\mathbf{1} - \boldsymbol{\delta}(\mathbf{x})) \circ (\mathbf{c}(\mathbf{x}) - \boldsymbol{\Phi}(\mathbf{c}(\mathbf{x}), \boldsymbol{\delta}(\mathbf{x}))) \geq \mathbf{0}$$

where the symbol ∘ denotes the Hadamard product (element-wise multiplication):

$$a \circ b = \text{diag}(a)\ b\ .$$

Similarly we can re-write (13) as (with a different definition of P_{mr} note):

$$\delta(x_{mr}) \cdot (x_{mr} - d_m \cdot P_{mr}(\mathbf{c}(\mathbf{x}), \boldsymbol{\delta}(\mathbf{x}))) = 0 \qquad \forall r \in R_m, m = 1, 2, ..., M$$

where for all $r \in R_m$ (i.e. not just those that are subject to the RUM) and m=1, 2,..., M

$$P_{mr}(\mathbf{c}(\mathbf{x}), \boldsymbol{\delta}(\mathbf{x})) \equiv \delta(x_{mr}) \cdot \Pr(-\theta \cdot c_{mr}(\mathbf{x}) + \xi_{mr} \geq -\theta \cdot c_{ms}(\mathbf{x}) + \xi_{ms} \quad \forall s \in R_m \text{ such that } \delta(x_{ms}) = 1)$$

or in vector notation:

$$\boldsymbol{\delta}(\mathbf{x}) \circ (\mathbf{x} - \boldsymbol{\Gamma}\mathbf{d} \circ \mathbf{P}(\mathbf{c}(\mathbf{x}), \boldsymbol{\delta}(\mathbf{x}))) = \mathbf{0}$$

where $\boldsymbol{\Gamma}$ is the path-OD incidence matrix (i.e a N×M-dimension 0-1 matrix with a 1 only if a path is relevant to an OD movement). Overall, then, we can write RSUE($\boldsymbol{\Phi}$) as the two conditions:

$$\boldsymbol{\delta}(\mathbf{x}) \circ (\mathbf{x} - \boldsymbol{\Gamma}\mathbf{d} \circ \mathbf{P}(\mathbf{c}(\mathbf{x}), \boldsymbol{\delta}(\mathbf{x}))) = \mathbf{0} \tag{17}$$

$$(\mathbf{1} - \boldsymbol{\delta}(\mathbf{x})) \circ (\mathbf{c}(\mathbf{x}) - \boldsymbol{\Phi}(\mathbf{c}(\mathbf{x}), \boldsymbol{\delta}(\mathbf{x}))) \geq \mathbf{0} \tag{18}$$

In this formulation, we have a combination of a complementarity kind of condition (as in DUE) and a fixed point condition (as in SUE). We can see special cases as follows:

- If $\delta(\mathbf{x}) = \mathbf{1}$ then the second constraint is redundant, and the first reduces to the SUE condition on the universal path set ($\mathbf{x} = \mathbf{\Gamma d} \circ \mathbf{P}(\mathbf{c}(\mathbf{x}),\mathbf{1})$).
- If $\delta(\mathbf{x}) = \delta_{\mathbf{0}} \neq \mathbf{1}$ then the first condition above represents a pre-defined (non-equilibrated) restricted choice set.
- If we terminate an SUE solution algorithm (on the universal choice set) after a finite number of steps, at some point $\mathbf{x}_{est}$, then in most practical cases not all routes will be used, which means $\delta(\mathbf{x}_{est}) \neq \mathbf{1}$, i.e. not an actual SUE solution is found. In this case, similarly to the case of a pre-defined choice set, we only satisfy (approximately) the first condition, there is no analogous condition to the second for those routes not used.

It should be noted that this is a fixed point problem in $\mathbf{x}$, but a rather difficult one to solve given that $\delta(\mathbf{x})$ maps onto integers ($\mathbf{x}$ is real, but the functions are non-smooth, unlike traditional SUE).

5.3 COMPUTATIONAL CONSIDERATIONS

A key question is to what extent the proposed models could be applied in large-scale networks. In the case of SUEGE with a continuous, bounded error distribution, an obvious general candidate is the Method of Successive Averages (MSA) algorithm (Sheffi and Powell, 1982). However, there appears to be no existing proof of convergence of such an algorithm that covers the case of bounded error distributions, and so the approach would be heuristic. An alternative possibility arises by noting that the set of used routes in a continuous, bounded SUE problem is affected only by the bounds, and these bounds will change only in response to the flows through the travel cost functions; thus, for fixed flows, the set of used routes is fixed. Therefore, it would seem possible to devise an algorithm in which, at each iteration, updates the choice set by dropping or appending new options based on a column generation method (if needed based on the latest bounds), and then a path-based SUE problem is solved on the fixed choice set. However, this latter element would require some developments to avoid Monte Carlo methods, entailing large computational cost, in the case of general bounded distributions.

In the case of SUEGE models with a discrete error distribution, an MSA would again be a potential candidate algorithm. However, given the combinatorial nature of the discrete problem, it would seem more sensible to explore the possibility to employ deterministic algorithms to exploit such a structure, such as those specifically developed for solving shortest path algorithms with discrete distributions (e.g., Mirchandani, 1976; Fajardo and Waller, 2012) or problems associated with connectivity reliability (e.g., Bell and Iida, 1997).

In the case of RSUE models, the reformulation (17)/(18) shows that the problem contains elements of not only SUE but also DUE (complementarity), and for this reason it would seem sensible to consider the possible transfer to the RSUE case of developments in path-based DUE algorithms (e.g., Larsson and Patriksson, 1992; Chen et al., 2002; Carey and Ge, 2012). On the other hand, the structure of the RSUE problem seems to lend itself well too to methods that align with its two separate conditions, which define (a) the dispersion among used paths and (b) the question of which paths are used. In this respect, we would note algorithms for solving path-based SUE problems (e.g., Xu et al., 2012), and especially those that decompose path generation and path loading (e.g., Damberg et al., 1996; Bell et al., 1997). This is an element where the RSUE model is particularly attractive (relative to SUEGE with bounded distributions), in that any sub-problems of allocating flow on restricted choice sets with continuous *unbounded* stochastic error distributions are problems with which the traffic assignment field has considerable familiarity.

6 Conclusions

The commonly used models within traffic assignment, namely DUE and SUE, have some known limitations by either allowing only routes with the minimum cost to be used (DUE) or requiring all routes to be used regardless of their costs (SUE). The paper shows how we might overcome these limitations by *consistently integrating* the problem of distinguishing used and unused paths within the concept of SUE. This has led to the proposal of two distinct, alternative methodological approaches of potential interest to the transportation research community. The two approaches define not only an equilibrated flow solution but also an equilibrated choice set in which the equilibrium conditions (and not the solution algorithm adopted) specify that some available routes could be unused at perfect equilibrium. The potential benefits of such approaches are greatest, it would seem, in large-scale regional and trans-national studies, meaning that we no longer have the choice only between DUE (which will tend to assign all-or-nothing to congested parts of such networks) and SUE (which can be computationally demanding and rather implausible, in attempting to assign some traffic to all routes).

The present study justifies, defines and illustrates both approaches as well as discusses their similarities, differences, capabilities and potentials. Also, we have outlined possible approaches for calibration and application of both methods. The RSUE seems more straightforward to apply in short term, as extensions of existing, calibrated SUE models, especially if supplemented with some information on routes actually chosen to aid in the determination of the Φ operator. In a companion paper (Rasmussen et al., 2014), we develop solution methods for generating RSUE solutions for large-scale networks, and explore the characteristics of the solutions produced. Beyond these two papers, what is required next, we believe, is a further development and study of the capabilities and potential of RSUE and SUEGE. One issue to consider lies within the possible non-uniqueness of solutions. We have outlined and discussed different possibilities for tackling this challenge in real-life applications, and believe that especially the increased availability of observed route choices can be utilised in doing so. Additional future extentions could be to multi-criteria traffic assignment (Dial, 1997; Nagurney, 2000; Nagurney and Dong 2002; Wang and Ehrgott, 2013), and the fact that RSUE and SUEGE has a path-based formulation that fits well with non-additive problems such as reliability (e.g. Chen and Zhou, 2010; Chen et al., 2011), non-linear transformations of path travel cost (e.g. Gabriel and Bernstein, 1997), and problems requiring to know turning flows for asymmetric junction interactions (Watling, 1996; Nielsen et al., 1998).

Our research began with a re-thinking and formulation of 'user conditions' analogous to Wardrop's, and we found this especially helpful in the development of our model. Indeed, in doing so we found that the oft-quoted informal description of the behaviour underlying SUE as 'minimising perceived cost' was not especially helpful, yet a more formal articulation did not apparently exist (analogous to Wardrop's very clear conditions). We believe such an approach, beginning with a development of the user conditions, is especially suitable for the traffic assignment community to exploit the insights from empirical work and behavioural studies. This is especially relevant as nowadays we have the potential to track routes through GPS or mobile phone devices, and as a result there is a rapidly growing body of evidence. While many phenomena may be location-specific, it is also interesting to look across such data sets for transferable phenomena which may be included as (potentially adjustable) rules within a new set of user conditions. The original developers of the route choice conditions underlying DUE or SUE could not have envisaged the wealth of explicit route-based data to which we now have access, and so it seems timely to reconsider these conditions in the light of such a new evidence-base.

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APPENDIX 5: RASMUSSEN ET AL. (2014C)

Stochastic User Equilibrium with Equilibrated Choice Sets: Part II – Solving the Restricted SUE for the Logit Family

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Abstract: We propose a new class of solution algorithms to solve the Restricted Stochastic User Equilibrium (RSUE) that was introduced in the companion paper. The class is path-based and allows a flexible specification of how the choice sets are 'systematically' grown by considering congestion effects and how the flows are allocated among routes. The specification allows adapting traditional path-based stochastic user equilibrium flow allocation methods (designed for pre-specified choice sets) to the generic solution algorithm. We also propose a cost transformation function and show that by using this we can, for certain Logit-type choice models, modify existing path-based Deterministic User Equilibrium solution methods to fit the RSUE solution algorithm. The transformation function also leads to a two-part relative gap measure for consistently monitoring convergence to a RSUE solution. Numerical tests are reported on two real-life cases, in which we explore convergence patterns and choice set composition and size for alternative specifications of the RSUE solution algorithm.

Keywords: Restricted Stochastic User Equilibrium; Solution Methods; Path-swapping; Convergence criteria; Gap function; Stochastic User Equilibrium.

1 INTRODUCTION

Within the Stochastic User Equilibrium traffic assignment model (SUE, Daganzo and Sheffi, 1977), the most commonly applied choice models are based on the assumption that the error terms are distributed according to some continuous distribution with unbounded support (e.g., probit and logit-type models). Such an assumption however induces, in the equilibrium state, that flow is assigned to all routes no matter how expensive they might be. This is practically infeasible for large-scale problems, and the problem 'scales' as the network grows. In practical applications, the algorithms does not generate and assign traffic to all routes, but rather finds an *estimated* SUE on a sub-set of routes. As mentioned, the SUE does not provide any help in the sub-set selection, and the route selection is thus left for the algorithm.

This issue led us to propose two models which consistently integrate the problem of distinguishing used and unused paths within the SUE framework allowing the use of Random Utility Models (RUMs) among used paths (Watling et al., 2014). On the one hand, the Stochastic User Equilibrium with General Error distribution (SUEGE) model facilitates the use of a range of probability distributions and, assuming a (discrete or continuous) bounded distribution, at equilibrium allows used and unused routes. The most commonly used RUM however apply continuous distributions with unbounded support, and in such cases the SUEGE collapses to the ordinary SUE (Watling et al., 2014). On the other hand, the Restricted Stochastic User Equilibrium (RSUE) model does not pose restrictions on the distribution applied, but equilibrates the choice set composition via additional constraints on the cost on unused routes. This facilitates the use of a range of RUMs for the distribution of flow among the routes in the choice set, while still allowing equilibrated choice sets not being the full universal choice set. The RSUE is formulated by two conditions, one concerned with the distribution of flow and one posing a cost restriction on the unused routes, thus making it reasonable to devise a solution algorithm which decomposes path generation and path loading. This decomposition and the connection to existing RUMs causes the RSUE to be more straight forward to apply in the short term than the SUEGE with bounded distributions. The path loading constitutes a sub-problem of loading traffic among a restricted set of routes according to a RUM, and this is a problem well-known in practical SUE applications. It would thus seem evident to propose RSUE as extensions of existing and calibrated SUE models, especially when data on actually chosen routes are available to validate the operator used in the condition posing a cost restriction on the unused routes (see Watling et al., 2014).

Solution algorithms have yet to be proposed for the RSUE model on a non-predefined choice set in large-scale applications. In this paper, we tackle this challenge by proposing a generic solution algorithm to the RSUE model. The solution algorithm is very flexible with regards to the strategy adopted to the path generation as well as flow distribution. Paths are generated based on column-generation, which 'systematically' grows the choice sets without the need for simulation. Flows are distributed based on the adaptation of existing path-based SUE solution algorithms. Alternatively, we propose a class of cost transformation functions allowing to fit path-based Deterministic User Equilibrium (DUE, Wardrop, 1952) solution algorithms into the generic solution algorithm to find RSUE solutions. We focus on logit-type choice models for which closed-form expressions are available for the choice probabilities, thereby obviating the need for computationally expensive simulation. These choice models include a large part of the models most commonly adopted in practical implementations and/or considered to be state-of-the-art (e.g., Path Size Logit, Ben-Akiva and Bierlaire, 1999). Moreover, in this paper we tackle the issue of measuring convergence of RSUE solution algorithms by proposing a two-part convergence measure, which *consistently* measures the convergence to equilibrated choice sets as well as the convergence of the distribution of flow among the used paths to fulfil the underlying choice model.

The remainder of the paper is structured as follows. Section 2 presents the notation and the definition of the RSUE model in its two variants. Section 3 introduces the class of cost transformation functions and shows how it allows transforming closed-form Logit-type RSUE problems into equivalent corresponding DUE-like problems. Section 4 proposes the new measure for monitoring convergence to an RSUE solution. Section 5 presents the generic solution algorithm and gives an example of how methods developed for DUE assignment can be adopted. Section 6 shows numerical results for two real-life networks. Section 7 discusses the implications of the findings, and finally section 8 draws the main conclusions from the study.

2 NOTATION AND DEFINITIONS

We introduce the notation and definitions with the objective of summarising the Restricted Stochastic User Conditions and the RSUE model formulation presented in the companion paper (Watling et al., 2014) as well as presenting the frameworks at the foundation of the equivalent DUE-like formulation and the solution method.

2.1 NOTATION

Consider a network as a directed graph composed of links a (a = 1, …, A) with non-negative flow f_a, and let $\mathbf{f}$ be the A-dimensional vector of link flows. We assume the *actual* flow-dependent (generalised) travel cost on link a to be a continuous function of the flow, and denote it by $t_a(\mathbf{f})$.

The network consists of M OD-pairs, and the demand d_m for each OD-pair m composes a non-negative M-dimensional vector $\mathbf{d}$. For each OD-pair m, R_m is the set of all simple acyclic paths (routes) connecting origin and destination, and N_m is the number of paths in R_m. R refers to the joint set of all simple paths across OD-pairs, with dimension $N = \sum_{m \in M} N_m$.

We denote the flow on path r between OD-pair m as x_{mr} and the N-dimensional flow-vector on the universal choice set across all M OD-pairs as $\mathbf{x}$. The convex set G of demand-feasible non-negative path flow solutions G is given by:

$$G = \left\{ \mathbf{x} \in \mathbb{R}_+^N : \sum_{r=1}^{N_m} x_{mr} = d_m, m = 1,2,...,M \right\} \tag{1}$$

where $\mathbb{R}_+^N$ denotes the N-dimensional non-negative Eucledian space. It should be noted that x_{mr} refers to element number $r + \sum_{k=1}^{m-1} N_m$ in the vector $\mathbf{x}$. The corresponding convex set of demand-feasible link flows is:

$$F = \left\{ \mathbf{f} \in \mathbb{R}_+^A : \mathrm{f}_a = \sum_{m=1}^{M} \sum_{r=1}^{N_m} \delta_{amr} \cdot x_{mr}, a = 1,2,...,A, \mathbf{x} \in G \right\} \tag{2}$$

where δ_{amr} =1 if link a belongs to path r for OD-pair m and zero otherwise.

Let $c_{mr}(\mathbf{x})$ be the *actual* (generalised) cost on path r for OD-pair m. As links may be used by several paths within and across the OD-pairs, $c_{mr}(\mathbf{x})$ depends on the flow vector $\mathbf{x}$. Additionally, the cost $c_{mr}(\mathbf{x})$ is a positive value and may be a weighted sum of several attributes, such as e.g. travel time, travel distance, and congestion charge.

In vector/matrix notation, let $\mathbf{x}$ and $\mathbf{f}$ be column vectors, and define $\boldsymbol{\Delta}$ as the $A \times N$-dimensional link-path incidence matrix. Then the relationship between link and path flows may be written as $\mathbf{f} = \boldsymbol{\Delta}\mathbf{x}$. We suppose that the travel cost on path r for OD-pair m is additive in the link travel costs of the utilised links:

$$c_{mr}(\mathbf{x}) = \sum_{a=1}^{A} \delta_{amr} \cdot t_a(\Delta \mathbf{x}) \qquad (r \in R_m;\ m = 1,2,\ldots,M;\ \mathbf{x} \in G) \tag{3}$$

Let $\mathbf{c}(\mathbf{x})$ be the vector of costs on all paths, $\tilde{c}_{mr}(\mathbf{x})$ be a *transformed* cost of path r for OD-pair m, and hence $\tilde{\mathbf{c}}(\mathbf{x})$ be the vector of transformed costs on all paths. The RSUE model distinguishes between used and unused paths, and consequently we let $\tilde{R}_m$ be the subset of R_m consisting of all utilised paths (non-zero flow) for OD-pair m (i.e. $\tilde{R}_m \subseteq R_m$).

The following notation is also used in the paper:

- l_a is the length of link a.
- L_{mr} is the length of path r for OD-pair m.
- Γ_{mr} is the set of links constituting path r for OD-pair m.
- $\Phi\left(\{c_{mr}(\mathbf{x}) : r \in \tilde{R}_m\}\right)$ is the mapping function used in the RSUE definition, specifies criterion to be fulfilled by unused paths.
- π_m is a free variable used in the DUE formulation, which equals the cost on the cheapest path between OD-pair m.
- z is the number of zones.
- V is the number of vertices in the network.
- $P_{mr}(\mathbf{c}(\mathbf{x}) | \tilde{R}_m)$ is the proportion of flow on OD-movement m that uses path r among the alternatives in the restricted set of utilised paths $\tilde{R}_m$ for OD-pair m.

2.2 RESTRICTED STOCHASTIC USER EQUILIBRIUM

The choice probability function $P_{mr}(\mathbf{c}(\mathbf{x})|\tilde{R}_m)$ is supposed given by a RUM separately for each OD-pair *m*:

$$P_{mr}(\mathbf{c}(\mathbf{x})|\tilde{R}_m) = \Pr\left(-\theta \cdot c_{mr}(\mathbf{x}) + \xi_{mr} \geq -\theta \cdot c_{ms}(\mathbf{x}) + \xi_{ms}, \forall s \neq r, s \in \tilde{R}_m\right) \qquad \forall r \in \tilde{R}_m \tag{4}$$

where $\theta \geq 0$. The choice model hereby holds for any proper subset of the universal choice set. The flow on any *used* path is then the total demand multiplied by the choice proportion (4), whereas the flow on any *unused* path is zero by definition. The equilibrium path flows must then satisfy:

$$x_{mr} = \begin{cases} d_m \cdot P_{mr}(\mathbf{c}(\mathbf{x})|\tilde{R}_m) & \text{if } r \in \tilde{R}_m \\ 0 & \text{otherwise} \end{cases} \qquad (r \in R_m;\ m \in M) \tag{5}$$

where

$$\tilde{R}_m = \{r : r \in R_m \text{ and } x_{mr} > 0\} \qquad (m \in M) \tag{6}$$

The conditions (5) and (6) are necessary but not sufficient, as the restricted choice set is 'internally defined' and not necessarily the universal choice set. It should be noted that the RUM is supposed to be such that for any non-empty restricted choice set $\tilde{R}_m \subseteq R_m$ and for any cost vector $\mathbf{c}$, the probability function has the properties:

$$\sum_{r \in \tilde{R}_m} P_{mr}(\mathbf{c}(\mathbf{x})|\tilde{R}_m) = 1 \tag{7}$$

and

$$P_{mr}(\mathbf{c}(\mathbf{x})|\tilde{R}_m) > 0 \qquad \forall r \in \tilde{R}_m \tag{8}$$

These properties imply that the OD demands are automatically satisfied (from (7)) and the path flows are non-negative (from (8)). Since we consider only RUMs in which these properties hold, then we shall not explicitly state them below as necessary conditions.

We introduce the mapping $\Phi(\cdot)$ that, for each OD-pair m, acts upon the costs of used paths, i.e. on $\{c_{mr}(\mathbf{x}) : r \in \tilde{R}_m\}$. We require that any unused path on OD-pair m has a cost greater than or equal to $\Phi\left(\{c_{mr}(\mathbf{x}) : r \in \tilde{R}_m\}\right)$, i.e. for any unused path the following has to hold:

$$x_{mr} = 0 \quad \Rightarrow \quad c_{mr}(\mathbf{x}) \geq \Phi\left(\{c_{ms}(\mathbf{x}) : s \in \tilde{R}_m\}\right) \qquad (r \in R_m; \;\; m = 1,\ldots,M) \qquad (9)$$

Bringing together these elements, the equilibrium conditions are defined as (for details, see Watling et al., 2014):

Definition: Restricted Stochastic User Equilibrium (RSUE(Φ))

Suppose that we are given a collection of continuous, unbounded random variables

$\{\xi_{mr} : r \in R_m, m = 1,2,\ldots,M\}$ *defined over the whole choice set* R_m; *and that for any* non-empty *subsets* $\tilde{R}_m$ *of* R_m (*m* = 1,2,…,*M*), *probability relations* $P_{mr}(\mathbf{c}(\mathbf{x}) | \tilde{R}_m)$ *are given over* $\tilde{R}_m$ (*m* = 1,2,…,*M*) *by considering the relevant marginal joint distributions from*

$\{\xi_{mr} : r \in R_m, m = 1,2,\ldots,M\}$. *The route flow* x ∈ *G is a RSUE(*Φ*) if and only if for all r* ∈ R_m *and m* = 1,2,…,*M:*

$$x_{mr} > 0 \quad \Rightarrow \quad r \in \tilde{R}_m \quad \wedge \quad x_{mr} = d_m \cdot P_{mr}(\mathbf{c}(\mathbf{x}) | \tilde{R}_m) \qquad (10)$$

$$x_{mr} = 0 \quad \Rightarrow \quad r \notin \tilde{R}_m \quad \wedge \quad c_{mr}(\mathbf{x}) \geq \Phi\left(\{c_{ms}(\mathbf{x}) : s \in \tilde{R}_m\}\right) \qquad (11)$$

Note that the set of utilised paths $\tilde{R}_m$ is *implicitly* defined by the restrictions. In the companion paper (Watling et al., 2014) we introduced the RSUE(min) and RSUE(max) by letting $\Phi\left(\{c_{ms}(\mathbf{x}) : s \in \tilde{R}_m\}\right)$ be $\min\{c_{ms}(\mathbf{x}) : s \in \tilde{R}_m\}$ and $\max\{c_{ms}(\mathbf{x}) : s \in \tilde{R}_m\}$, respectively. Watling et al. (2014) also provided an alternative formulation of the RSUE(Φ), namely a formal mathematical formulation which is a combination of a complementarity kind of condition and a fixed point condition.

2.3 EXTENSION TO MULTIPLE USER CLASSES AND VEHICLE TYPES

The notation may be readily modified to include the more general case of (i) multiple user classes differing in their definition of travel cost and in the OD matrix, and (ii) multiple vehicle types differing in the contribution they make to the total traffic flow. In this case, *m* denotes a *commodity* which is a combination of OD movement, user class and vehicle type, so that M is the product of the number of OD movements, user classes and vehicle types. In order to reflect different contributions to traffic flow, we suppose that the demand d_m for commodity m is measured in equivalent passenger car units. The only modification required to the notation is then that $t_{am}(\mathbf{\Delta x})$ now denotes the travel cost on link a as perceived by commodity m when the total pcu route flows are $\mathbf{x}$. Thus, the route cost-flow functions are defined by:

$$c_{mr}(\mathbf{x}) = \sum_{a=1}^{A} \delta_{amr} \cdot t_{am}(\mathbf{\Delta x}) \qquad (r \in R_m;\ m = 1,2,\ldots,M)\ . \tag{12}$$

Under these changes, all the subsequent models and methods presented may be applied to this more general case. In the following we will refer to OD-pair m, but this may as well, in the case of multiple vehicle and/or user classes, refer to commodity m. In section 6.2 we perform tests using 19 user classes and 2 vehicle-types.

3 EQUIVALENT USER EQUILIBRIUM TRANSFORMATION

We introduce a function transforming the *actual* path costs into *transformed* costs in order to reformulate the RSUE problem as an equivalent problem for certain Logit-type choice models. This reformulated problem has a formulation which is very similar to a traditional DUE problem by containing one condition equalising costs on utilised paths and one condition specifying the criteria to be fulfilled by unused paths. The similarity allows modifying and using efficient path-based DUE solution algorithms to solve for solutions satisfying the RSUE conditions.

3.1 MNL RSUE

The MNL RSUE(Φ) can be transformed into an equivalent problem which is very similar to the traditional DUE formulation. The DUE is defined for all OD-pairs *m* (Patriksson, 1994):

$$x_{mr} > 0 \Rightarrow c_{mr}(\mathbf{x}) = \pi_m \qquad \forall r \in R_m \tag{13}$$

$$x_{mr} = 0 \Rightarrow c_{mr}(\mathbf{x}) \geq \pi_m \qquad \forall r \in R_m \tag{14}$$

The definition of the DUE *implicitly* distinguishes between used and unused paths, and implies that the costs on all utilised paths are the same, namely equal to the cost of the lowest-cost alternative for OD-pair *m*. The DUE conditions can thus be rewritten as:

$$x_{mr} > 0 \Rightarrow r \in \tilde{R}_m \wedge c_{mr}(\mathbf{x}) = c_{ms}(\mathbf{x}) \qquad \forall s \in \tilde{R}_m \tag{15}$$

$$x_{mr} = 0 \Rightarrow r \notin \tilde{R}_m \wedge c_{mr}(\mathbf{x}) \geq \left(\pi_m = \min_{s \in \tilde{R}_m} c_{ms}(\mathbf{x}) = \max_{s \in \tilde{R}_m} c_{ms}(\mathbf{x})\right) \tag{16}$$

For the transformation of the MNL RSUE(Φ) into an equivalent set of conditions similar to conditions (15) and (16), we now introduce the *transformed* cost $\tilde{c}_{mr}(\mathbf{x})$ of path *r* as a function of the *actual* generalised cost on path *r,* the flow on path *r* and a parameter $\theta \geq 0$[1]:

$$\tilde{c}_{mr}(\mathbf{x}) = x_{mr} \cdot \exp\left(\theta \cdot c_{mr}(\mathbf{x})\right) \tag{17}$$

[1] A similar transformation was used to equilibrate routes in the gradient projection algorithm using a reference path cost proposed in Bekhor and Toledo (2005)

We note that there is no physical meaning of the transformed route cost $\tilde{c}_{mr}(\mathbf{x})$ as a purely mathematical construct to be used in the equilibration. However, equalising the transformed cost among the *used* paths (e.g. through a DUE algorithm equalising the transformed costs rather than the actual costs) induces the first MNL RSUE condition (10) to be fulfilled:

$$\tilde{c}_{mr}(\mathbf{x}) = \tilde{c}_{ms}(\mathbf{x}) \qquad \forall (r,s) \in \tilde{R}_m \tag{18}$$

Equation (17) can be rewritten to express the flow on path *r*:

$$x_{mr} = \frac{\tilde{c}_{mr}(\mathbf{x})}{\exp\left(\theta \cdot c_{mr}(\mathbf{x})\right)} \tag{19}$$

Furthermore, the sum of flows on all paths for OD-pair *m* is equal to the total demand, and considering that $P_{mr}(\mathbf{c}(\mathbf{x})|\tilde{R}_m)$ is equal to the flow on path *r* divided by the demand d_m for OD-pair *m* ($P_{mr}(\mathbf{c}(\mathbf{x})|\tilde{R}_m) = x_{mr}/d_m$) yields the following:

$$\begin{aligned} P_{mr}(\mathbf{c}(\mathbf{x})|\tilde{R}_m) &= \frac{x_{mr}}{d_m} = \frac{x_{mr}}{\sum_{s\in\tilde{R}_m} x_{ms}} = \frac{\dfrac{\tilde{c}_{mr}(\mathbf{x})}{\exp\left(\theta \cdot c_{mr}(\mathbf{x})\right)}}{\sum_{s\in\tilde{R}_m} \dfrac{\tilde{c}_{ms}(\mathbf{x})}{\exp\left(\theta \cdot c_{ms}(\mathbf{x})\right)}} = \frac{\dfrac{\tilde{c}_{mr}(\mathbf{x})}{\exp\left(\theta \cdot c_{mr}(\mathbf{x})\right)}}{\sum_{s\in\tilde{R}_m} \dfrac{\tilde{c}_{mr}(\mathbf{x})}{\exp\left(\theta \cdot c_{ms}(\mathbf{x})\right)}} \\ &= \frac{\tilde{c}_{mr}(\mathbf{x}) \cdot \dfrac{1}{\exp\left(\theta \cdot c_{mr}(\mathbf{x})\right)}}{\tilde{c}_{mr}(\mathbf{x}) \cdot \sum_{s\in\tilde{R}_m} \dfrac{1}{\exp\left(\theta \cdot c_{ms}(\mathbf{x})\right)}} = \frac{\exp\left(-\theta \cdot c_{mr}(\mathbf{x})\right)}{\sum_{s\in\tilde{R}_m} \exp\left(-\theta \cdot c_{ms}(\mathbf{x})\right)} \end{aligned} \tag{20}$$

This corresponds to the choice probability formulation of the MNL model with scale parameter θ, and shows that the solution to the problem (18) on the set of utilised paths is equal to the solution to the corresponding MNL SUE problem on the original costs $\mathbf{c}(\mathbf{x})$. The opposite implication can also be shown: starting from the MNL choice probabilities and using the definition of the transformation function, (20) can be utilised to show that the system (18) arises from using the transformed costs $\tilde{\mathbf{c}}(\mathbf{x})$.

Using the second RSUE condition as in definition (11), the equivalence shown above yields the following transformed RSUE conditions, which are equivalent to the original RSUE conditions if for all $r \in R_m$ and $m \in M$:

$$x_{mr} > 0 \quad \Rightarrow \quad r \in \tilde{R}_m \quad \wedge \quad \tilde{c}_{mr}(\mathbf{x}) = \tilde{c}_{ms}(\mathbf{x}) \qquad \forall s \in \tilde{R}_m \tag{21}$$

$$x_{mr} = 0 \quad \Rightarrow \quad r \notin \tilde{R}_m \quad \wedge \quad c_{mr}(\mathbf{x}) \geq \Phi\left(\{c_{ms}(\mathbf{x}) : s \in \tilde{R}_m\}\right) \tag{22}$$

These transformed RSUE conditions have some similarities with the DUE conditions (15) and (16): both contain a statement for used paths and a statement concerning non-used paths. Comparing equations (15) and (21), it can be seen that they both equalise costs on utilised paths. However, one operates on *actual* costs, while the other operates on the *transformed* costs when distributing traffic between utilised paths. The second condition is also quite similar, especially for the RSUE(max) and RSUE(min). The difference, however, is that while the criteria value to be fulfilled by the unused paths is defined *implicitly* by the DUE formulation, the RSUE needs an explicit definition of how the reference OD travel cost is related to the path travel costs of used paths. Consequently, though not equivalent, the similarity between the transformed RSUE problem and the DUE has led us to propose a generic RSUE(Φ) solution algorithm which facilitates the use of path-based DUE algorithms using the transformed costs.

3.2 EXTENSION TO CLOSED-FORM LOGIT-TYPE RSUE MODELS

The above transformation of the RSUE(Φ) was based on the MNL model, and hence it does not account for correlations across alternatives. This disadvantage is critical in a route choice application, as paths typically overlap with other considered paths on segments. However a similar transformation of the RSUE(Φ) problem into an equivalent DUE-like problem can also be applied (and solved via efficient modified DUE solution algorithms) for certain other choice models by altering the deterministic term of the utility function. This is done through a cost transformation function similar to the one proposed for the MNL, however allowing $c_{mr}(\mathbf{x})$ to be composed, in addition to traditional elements such as travel time and direct travel cost, by (deterministic) elements taking correlations into account.

Many MNL modifications accounting for correlations have been proposed, for example the C-Logit (Cascetta et al., 1996) and the Path Size Logit (PSL) model (Ben-Akiva and Bierlaire, 1999). The PSL model has been applied with success in various route choice studies (e.g., Bekhor and Prato, 2009; Frejinger et al., 2009; Ramming, 2001; Ben-Akiva et al., 2012), and in the following it is shown that by applying the transformed cost-function (23) we can write the PSL RSUE as a DUE-like system similar to equations (21) and (22). A similar approach can be applied to show equivalence for other closed-form Logit-type models where the modification from the MNL model consists of altering the deterministic term of the utility function (by replacing the expression $c_{mr}(\mathbf{x})+\beta_{PS}\cdot\ln\left(PS_{mr}\right)$ in (23) with the corresponding expression for the selected choice model).

Consider the following cost transformation:

$$\tilde{c}_{mr}(\mathbf{x})=x_{mr}\cdot\exp\left(\theta\cdot\left(c_{mr}(\mathbf{x})+\beta_{PS}\cdot\ln\left(PS_{mr}\right)\right)\right) \tag{23}$$

where $\theta\geq0$, $\beta_{PS}\leq0$ and $c_{mr}(\mathbf{x})$ is the general MNL cost-function applied in section 3.1. Let PS_{mr} be defined as in Ben-Akiva and Bierlaire (1999):

$$PS_{mr}=\sum\nolimits_{a\in\Gamma_{mr}}\frac{l_a}{L_{mr}}\cdot\frac{1}{\sum\nolimits_{k\in\tilde{R}_m}\delta_{amk}} \tag{24}$$

where L_{mr} and l_a are measures of impedance, and can either be measured as distance or cost ($l_a=t_a(\mathbf{f})$ and $L_{mr}=c_{mr}(\mathbf{x})$). Using cost makes PS_{mr} dependent not only on the composition of the choice set, but also on the flow on the paths in the choice set. Choosing cost as measure of impedance thus implies that the PS_{mr}-factors have to be updated in every iteration of a solution algorithm, even if no additional paths are added to the choice set. It should be also noted that the allocation of flow at equilibrium may also vary between using cost or length as measure of impedance (Zhou et al., 2012). In the following, it is assumed that the impedance is equal to the flow-dependent cost, but the derivation is the same if using distance.

Expression (23) can be rewritten to express the flow on path *r:*

$$x_{mr} = \frac{\tilde{c}_{mr}(\mathbf{x})}{\exp\left(\theta \cdot \left(c_{mr}(\mathbf{x}) + \beta_{PS} \cdot \ln\left(PS_{mr}\right)\right)\right)} \tag{25}$$

The condition on the transformed costs (18) should hold in equilibrium (as in the MNL case), and combining (18) and (25) with the choice probability for OD-pair *m* yields the following:

$$P_{mr}(\mathbf{c}(\mathbf{x}) | \tilde{R}_m) = \frac{x_{mr}}{d_m} = \frac{x_{mr}}{\sum_{s \in \tilde{R}_m} x_{ms}} = \frac{\dfrac{\tilde{c}_{mr}(\mathbf{x})}{\exp\left(\theta \cdot \left(c_{mr}(\mathbf{x}) + \beta_{PS} \cdot \ln\left(PS_{mr}\right)\right)\right)}}{\sum_{s \in \tilde{R}_m} \dfrac{\tilde{c}_{ms}(\mathbf{x})}{\exp\left(\theta \cdot \left(c_{ms}(\mathbf{x}) + \beta_{PS} \cdot \ln\left(PS_{ms}\right)\right)\right)}} =$$
$$\frac{\tilde{c}_{mr}(\mathbf{x}) \cdot \dfrac{1}{\exp\left(\theta \cdot \left(c_{mr}(\mathbf{x}) + \beta_{PS} \cdot \ln\left(PS_{mr}\right)\right)\right)}}{\tilde{c}_{mr}(\mathbf{x}) \cdot \sum_{s \in \tilde{R}_m} \dfrac{1}{\exp\left(\theta \cdot \left(c_{ms}(\mathbf{x}) + \beta_{PS} \cdot \ln\left(PS_{ms}\right)\right)\right)}} = \frac{\exp\left(-\theta \cdot \left(c_{mr}(\mathbf{x}) + \beta_{PS} \cdot \ln\left(PS_{mr}\right)\right)\right)}{\sum_{s \in \tilde{R}_m} \exp\left(-\theta \cdot \left(c_{ms}(\mathbf{x}) + \beta_{PS} \cdot \ln\left(PS_{ms}\right)\right)\right)} \tag{26}$$

This corresponds to the PSL choice probabilities with scale parameter θ and path size parameter β_{PS}. The term $\ln(PS_{mr})$ ranges from $-\infty$ to 0, where 0 arises when path *r* is unique and the value then decreases with increasing overlap with other paths in the choice set. As $\beta_{PS}<0$ the cost (disutility) of path *r* thus increases with decreasing uniqueness of path *r*.

The equivalence shown above yields equivalence between the PSL RSUE on the PSL costs $c_{mr}(\mathbf{x}) + \beta_{PS} \cdot \ln\left(PS_{mr}\right)$ ((10) and (11)) and the DUE-like system (21) and (22) when letting $\tilde{c}_{mr}(\mathbf{x})$ be defined by (23).

4 MEASURING CONVERGENCE TO A RSUE SOLUTION

Traditionally, it has been a challenge to measure convergence for SUE models, and in practice most applications of SUE use measures of stability, such as change in link flows between iterations (e.g., Liu et al., 2009; Zhou et al., 2012), rather than directly measuring proximity to equilibrium. We utilise the introduced transformation functions (17) and (23), as well as the derived knowledge that these transformed costs are equal on used paths at equilibrium, in order to propose a novel two-part *consistent* convergence measure for the proximity to a RSUE(min) or RSUE(max) solution. Since we utilise the transformation functions introduced, the underlying choice model needs to be the MNL or PSL model. The first part of the measure concerns the convergence to fulfil the SUE conditions among utilised paths (RSUE condition (10)), whereas the second part measures to what degree the criteria on unutilised paths are fulfilled (RSUE condition (11)). The first part is thus 'conditional' on the choice set, and the second part measures the 'convergence' of the composition of the choice sets. It is important to ensure convergence of *both* measures – obtaining convergence among the used routes does not imply overall convergence if there are additional attractive routes not in the choice set.

The first part is based on the relative gap measure of the DUE (Rose et al., 1988) and approaches zero as the flow on utilised paths approach the SUE flow solution. For iteration *n,* the measure can be computed as:

$$Rel.gap_n^{Used} = \frac{\sum_{m=1}^{M}\sum_{r\in\tilde{R}_m} x_{mr,n}\cdot\left(\tilde{c}_{mr}(\mathbf{x}_n)-\tilde{c}_{m,\min}(\mathbf{x}_n)\right)}{\sum_{m=1}^{M}\sum_{r\in\tilde{R}_m} x_{mr,n}\cdot\tilde{c}_{mr}(\mathbf{x}_n)} =$$

$$\frac{\sum_{m=1}^{M}\sum_{r\in\tilde{R}_m} x_{mr,n}\cdot\left(x_{mr,n}\cdot\exp\left(\theta\cdot c_{mr}(\mathbf{x}_n)\right)-x_{\min\left(x_{mr,n}\cdot\exp\left(\theta\cdot c_{mr}(\mathbf{x}_n)\right)\right)}\cdot\exp\left(\theta\cdot c_{m,\min}(\mathbf{x}_n)\right)\right)}{\sum_{m=1}^{M}\sum_{r\in\tilde{R}_m} x_{mr,n}\cdot x_{mr,n}\cdot\exp\left(\theta\cdot c_{mr}(\mathbf{x}_n)\right)} \tag{27}$$

where $\tilde{c}_{m,\min}(\mathbf{x}_n)$ is the minimum of the transformed cost on paths utilised between OD-pair *m* for iteration *n*, and $\tilde{c}_{mr}(\mathbf{x}_n)$ and $c_{mr}(\mathbf{x}_n)$ depend on the selected choice model.

As the transformed costs will become equal in the RSUE solution, the proposed relative gap measure will approach zero as the algorithm converges and will become zero at full convergence.

The second part of the convergence measure captures that there may exist unused paths which violate the second RSUE condition. The measure computes how close the costs on the cheapest unused path are to fulfilling the criteria describing unused paths in the RSUE definition. This corresponds to investigating for the RSUE(min) whether any unused path is cheaper than the cheapest used path, whereas for the RSUE(max) whether there exists any unused path which is cheaper than the most expensive used path. The measure becomes zero at convergence and it is based on actual costs rather than transformed costs. For the RSUE(min) it is defined as:

$$Rel.Gap_n^{Unused} = \frac{\sum_{m=1}^{M} d_m \cdot \left(\min_{\forall r \in R_m, x_{mr}>0} \left(c_{mr}(\mathbf{x}_n) \right) - \min_{\forall r \in R_m} \left(c_{mr}(\mathbf{x}_n) \right) \right)}{\sum_{m=1}^{M} d_m \cdot \min_{\forall r \in R_m, x_{mr}>0} \left(c_{mr}(\mathbf{x}_n) \right)} \tag{28}$$

For the RSUE(max) model it is expressed as:

$$Rel.Gap_n^{Unused} = \frac{\sum_{m=1}^{M} d_m \cdot \left(\max_{\forall r \in R_m, x_{mr}>0} \left(c_{mr}(\mathbf{x}_n) \right) - c_{mr,k}(\mathbf{x}_n) \right)}{\sum_{m=1}^{M} d_m \cdot \max_{\forall r \in R_m, x_{mr}>0} \left(c_{mr}(\mathbf{x}_n) \right)} \tag{29}$$

where k refers to the amount of used paths for the corresponding OD-pair m in iteration n and $c_{mr,k}(\mathbf{x}_n)$ is the cost on the current k-th shortest path on the network between OD-pair m.

The computation of the second part of the convergence measure requires for the RSUE(min) the result of a shortest path search based on updated travel costs, and for the RSUE(max) the results of a k-th shortest path search based on updated costs. These searches are quite computationally demanding, but the computation of the convergence measure does not induce significant extra calculation time, as these searches are part of the next iteration of the solution algorithm anyhow (see section 5.2).

The first part of the convergence measure is zero if the choice set consists of only one route for a given OD-pair. This is the case at the end of the first iteration of many traditional SUE solution algorithms based on column generation (e.g., Sheffi and Powell, 1982). The algorithm has (probably) not converged at the end of iteration 1 though, as the distribution of flow to this one route will cause other routes to be attractive. This will be caught by a non-zero value of the second part of the measure, which highlights the need to evaluate both parts of the gap measure. It should be noted that the proposed measures of convergence are not only valid for the solution methods proposed in the present study, but they can also be applied to all other solution methods solving for a closed-form Logit-type RSUE(min) or RSUE(max).

5 RSUE SOLUTION METHODS

We motivate and propose a generic path-based solution algorithm for solving RSUE(Φ) problems. Subsequently we discuss possible approaches to adopt in two important components of the generic algorithm as well as the convergence and computational attractiveness of different variants of the algorithm.

5.1 SOLVING IN THE SPACE OF LINK- OR PATH FLOWS

Traditional SUE and DUE solution methods are formulated and solved in the space of either the path flows or the link flows. Link-based formulations are attractive by not having to enumerate the paths, as path enumeration is computationally demanding and requires large computer memory. Path-based formulations allow for a flexible formulation of the set of alternatives considered (especially for the SUE which is often solved among a fixed pre-defined choice set in practical implementations). Behaviourally unrealistic paths such as paths repeatedly going on and off highways can thereby be omitted. For the RSUE, a solution method based on a path-based formulation is pursued as the obvious approach. This is motivated by investigating various properties for DUE, SUE as well as solutions satisfying the RSUE conditions.

Various link- as well as path-based algorithms have been formulated to solve for solutions satisfying the DUE conditions (e.g., LeBlanc et al., 1975; Jayakrishnan et al., 1994; Han, 2007; Dafermos and Sparrow, 1969; Larsson and Patriksson, 1992; Kupiszewska and van

Vliet, 1998). However, it is important to note that DUE flow solutions are unique in link flows, but not path flows; there is not a one-to-one mapping between link- and path-flows in DUE problems. This is especially important to bear in mind when applying a path-based solution algorithm, as the found flow solution provides uniqueness in link flows but not path flows.

Unlike DUE, SUE flow solutions provide uniqueness in path and link flows. This is true if the choice function and the cost-flow function are continuous and if *(i)* the link flow feasible set F is non-empty (i.e., at least one path exists between OD-pairs m for which $d_m>0$), compact and convex, and *(ii)* the link flows resulting from the flow network loading map (expressing link flows in terms of link costs) are always feasible (Cantarella, 1997). Consequently, a one-to-one mapping exists between path and link flows for SUE solutions, and both link- and path-based solution methods have been proposed for the SUE (e.g., Sheffi and Powell, 1982; Damberg et al., 1996; Bekhor and Toledo, 2005; Zhou et al., 2012; Akamatsu, 1996; Bell et al., 1997; Leurent, 1997; Maher and Hughes, 1997).

The RSUE conditions, unlike the DUE and SUE, do not provide uniqueness in link flows (nor in path flows), as the set of used paths is not uniquely defined by the RSUE conditions (see Watling et al., 2014). Consequently, a found link-flow solution cannot be mapped to a path-flow solution via a one-to-one relation, and additionally, this link-flow solution may not be unique. Furthermore, the definition of the RSUE conditions clearly distinguished between used and unused paths, and specifies a criterion to be fulfilled among used paths. As a consequence, the definition of the conditions does not specify unique link flows and necessitates the consideration of paths, which has led us to pursue the proposal of a path-based solution algorithm.

5.2 RSUE(Φ) SOLUTION ALGORITHM

We propose an iterative approach to solve for RSUE(Φ) solutions. One iteration of the proposed generic solution algorithm consists of four steps, namely the *Column generation phase*, the *Restricted master problem phase*, the *Network loading phase* and the *Convergence evaluation phase*.

<table>
<tr><th>Algorithm</th><th colspan="3"></th></tr>
<tr><td>Step 0</td><td colspan="3">Initialisation. Iteration n=1. Perform deterministic all-or-nothing assignment for all $m \in M$ OD-pairs and obtain the flow vector for all utilised paths $\mathbf{X}_n$.
Perform network loading, compute link travel costs $t_a\left(\mathbf{f}_n\right)$ on all network links $a \in A$, and compute generalised path travel costs $c_{mr}(\mathbf{X}_n)$. Set n=2.</td></tr>
<tr><td rowspan="2">Step 1</td><td colspan="3">Column generation phase. Let $k_{m,n-1}$ denote the current number of unique paths in the choice set of used paths for OD-pair $m=1, 2,..., M$ in iteration n-1.</td></tr>
<tr><td>For RSUE(min):
For each origin, perform a shortest path search to all destinations based on actual link travel costs $t_a(\mathbf{f}_{n-1})$. If for any OD-pair $m=1, 2,..., M$ a new unique path i is generated, add it to the choice set $\tilde{R}_{m,n}$ with flow $x_{mi,n-1}$=0.</td><td>For RSUE(Φ):
For each OD-pair $m \in M$, based on actual link travel costs $t_a(\mathbf{f}_{n-1})$, check for a new route to add to the choice set $\tilde{R}_{m,n}$ by applying some path generation method which supports the fulfilment of the Φ operator. If for any OD- pair $m=1, 2,..., M$ a new unique path i is generated, add it to the choice set $\tilde{R}_{m,n}$ with flow $x_{mi,n-1}$=0; if several routes possible, add only the shortest one.</td><td>For RSUE(max):
Perform $k_{m,\,n-1}$-shortest path search for each OD-pair $m=1, 2,..., M$ based on actual link travel costs $t_a(\mathbf{f}_{n-1})$.
If for any OD-pair $m=1, 2,..., M$ a new unique path i is generated among the k generated paths, add it to the choice set $\tilde{R}_{m,n}$ with flow $x_{mi,n-1}$=0; if several new unique paths are generated, add only the shortest one.</td></tr>
</table>

Step 2	*Restricted master problem phase*. Given the choice sets $\tilde{R}_{m,n}$ for all *m=1, 2,..., M*, apply the selected *inner assignment component* and *averaging scheme* to find the new flow solution $\mathbf{X_n}$.
Step 3	*Network loading phase.* Perform the network loading to obtain $\mathbf{f_n}$ from $\mathbf{X_n}$, compute the link travel costs $t_a(\mathbf{f}_n)$, the generalised path travel costs $\mathbf{C}(\mathbf{X}_n)$ and (if relevant/included) the Path Size factors.
Step 4	*Convergence evaluation phase.* If the gap measure consisting of the sum of $Rel.Gap_n^{Used}$ and $Rel.Gap_n^{Unused}$ is below a pre-specified threshold ξ, Stop. Else, set n=n+1 and return to *Step 1*

It should be noted that the path flow vector is denoted by **X** rather than **x.** This is to emphasise that in practical implementations it is not possible/practical to operate with the vector **x**, as this requires enumerating the universal choice set for all OD-pairs to obtain its dimension. Rather, the dimension of the flow vector is not pre-specified in practical implementations, but is allowed to increase as the algorithm progresses. The same occurs for the path cost vector **c**(**x**), which we have denoted **C**(**X**) to highlight that this might grow as the algorithm progresses. The elements x_{mr} and c_{mr} thus refer to the vectors **X** and **C**, respectively.

5.3 COLUMN GENERATION PHASE

The choice set is 'systematically' grown (as in the DUE case) based on built-in rules for the generation of new alternatives. The search for new alternatives may be performed in various ways, but in the solution algorithm we have proposed a single shortest path search for the RSUE(min) and a *k*-shortest path search for the RSUE(max). Basing the search for new paths to introduce to the choice set on the *actual* costs, induces the condition (11) on unused paths to be fulfilled.

For other formulations of the operator Φ, alternative path generation techniques may be applied, such as variations of shortest path algorithms (e.g., Akgün et al., 2000; Hunt and Kornhauser, 1997; Lombard and Church, 1993; Van der Zijpp and Fiorenzo-Catalano, 2005), application of heuristic rules (e.g., Ben-Akiva et al., 1984; Azevedo et al., 1993; De la Barra et al., 1993), branch and bound algorithms (Hoogendoorn-Lanser et al., 2006; Prato and Bekhor, 2006), biased random walk algorithm (Frejinger et al., 2009), and breadth first search with network reduction (Rieser-Schüssler et al., 2013). Some of these alternative approaches may also be attractive to apply for the RSUE(min) and RSUE(max). It is however essential that the column generation approach adopted should ensure that the condition (10) on unused paths are fulfilled upon termination of the algorithm.

It should be noted that the proposed approach for the *Column generation phase* for the RSUE(min) and RSUE(max) induces the RSUE condition (11) to be fulfilled; For the RSUE(min) there is no non-included path which is deterministically shorter than the once already included in the choice set when the algorithm above terminates. The condition (11) is fulfilled for the RSUE(max) model, since there is no non-included path that is deterministically shorter than the longest path already included in the choice set when the algorithm terminates.

5.4 THE RESTRICTED MASTER PROBLEM PHASE

The allocation of flow between the alternatives in the choice sets (the *Restricted master problem phase*) can be performed by deploying either DUE allocation methods using the transformed costs or SUE allocation methods.

Numerous path-based DUE solution algorithms which equilibrate path costs on used routes are available, such as the method of successive averages All-or-Nothing (MSA AoN, applied in e.g. Bekhor and Toledo (2005)), the Path Equilibrator (Dafermos and Sparrow, 1969), the Disaggregate Simplicial Decomposition (DSD, Larsson and Patriksson, 1992), the Gradient Projection (GP, Jayakrishnan et al., 1994; Chen et al., 2002), the Social Pressure (Kupiszewska and van Vliet, 1998), the Projected Gradient (Florian et al., 2009) and the slope-based multipath flow update (Kumar and Peeta, 2010) methods. Components of these DUE solution algorithms could be modified to fit into *Step 2* by applying the transformed costs rather than the actual costs in the *inner direction finding step*. Thereby the transformed costs are equilibrated, which corresponds to the first RSUE condition (10) being fulfilled.

Another branch of path-based DUE solution algorithms are the algorithms based on path-swapping, which usually swap traffic to/from paths based on cost differences. The swapping has a direct and a plausible behavioural interpretation, namely flow should be swapped to cheap paths and away from costly paths. The algorithms proposed in the literature are differentiated by swapping between pairs of paths (Han, 2007; Carey and Ge, 2012), all paths (Mounce and Carey, 2011), or to only one path (Mounce and Carey, 2011; Nie, 2003).

Among the algorithms tested in Carey and Ge (2012), an algorithm that is swapping flows between pairs of paths was found providing stable and fast convergence. The algorithm swaps flow from the most expensive path to the cheapest path, the second-most expensive path to the second-cheapest path, etc., and performs one network loading per iteration while not requiring any simulation. We utilise the introduced cost transformation function to adapt this approach to fit the generic RSUE solution algorithm proposed above by letting *Step 2* be comprised of the following.

Step 2	
Step 2.1	For each path r in the choice set for OD-pair m for iteration n, compute the transformed cost $\tilde{c}_{mr}(\mathbf{X}_{n-1})$ according to equation (17) or (23) (dependent on selection of choice model).
Step 2.2	Rank all paths for OD-pair m in ascending order of $\tilde{c}_{mr}(\mathbf{X}_{n-1})$: $\tilde{c}_{mp_1}(\mathbf{X}_{n-1}) \le \tilde{c}_{mp_2}(\mathbf{X}_{n-1}) \le \cdots \le \tilde{c}_{m\overline{p}}(\mathbf{X}_{n-1})$. Pair the paths as $(p_1, \overline{p}), (p_2, \overline{p}-1)$, ...: for odd number of paths, the path $(\overline{p}+1)/2$ will not be paired.
Step 2.3	For each pair (p_i, p_j), compute the swapping-factor: $\Gamma(p_i, p_j) = \frac{\tilde{c}_{mp_j}(\mathbf{X}_{n-1}) - \tilde{c}_{mp_i}(\mathbf{X}_{n-1})}{\sqrt{\left(\tilde{c}_{mp_j}(\mathbf{X}_{n-1})\right)^2 + \left(\tilde{c}_{mp_i}(\mathbf{X}_{n-1})\right)^2}}$ For each pair (p_i, p_j), perform the swap: $x_{mp_i,n} = x_{mp_i,n-1} + \gamma_n \cdot \Gamma(p_i, p_j) \cdot x_{mp_j,n-1}$ $x_{mp_j,n} = x_{mp_j,n-1} - \gamma_n \cdot \Gamma(p_i, p_j) \cdot x_{mp_j,n-1}$ For odd number of paths, set $x_{m,(\overline{p}+1)/2,n} = x_{m,(\overline{p}+1)/2,n-1}$

As the path-swapping algorithm equalizes the transformed costs, we know from section 3.2 that the RSUE condition (10) is also fulfilled upon termination. We note that the pairwise path-swapping algorithm by Carey and Ge (2012) is especially attractive for Logit-type choice models. This is due to a special characteristic of the Logit-type choice models, namely the Independence from Irrelevant Alternatives (IIA) property that induces the ratio of the choice probabilities between two paths to not depend on any other path. Basing the solution methods on pairwise path-swapping makes good sense in this framework by only considering the two paths between which flow is swapped in the swapping process.

Using the cost transformation in combination with a path-based DUE cost equilibration is however not the only option. Components of path-based SUE solution algorithms can also be adapted fit *Step 2* of the proposed generic solution algorithm. Among these are modified versions of two of the most promising path-based DUE solution methods, namely the DSD and GP methods (for MNL choice models, see Damberg et al., 1996; Bekhor and Toledo, 2005); for Cross Nested Logit, see Zhou et al., 2012; Bekhor et al., 2008). While the latter three applications apply a pre-defined choice set (which in the adaptation to the RSUE solution algorithm would be defined by the routes generated by the *Column generation phase*), it is worth noting that the algorithm proposed by Damberg et al. (1996) allows augmenting the choice set. Damberg et al. (1996) noted that this augmentation can be done following many strategies, and suggested one strategy that generates a solution fulfilling the MNL RSUE(min) conditions (and fits actually as a version of the generic RSUE(min) solution algorithm proposed). Regarding the computational attractiveness of the SUE adaptations of the GP and DSD algorithms, Bekhor and Toledo (2005) found that they perform similarly when applied to a small grid network and the Sioux Falls network. Bell et al. (1997) also formulated a path-based SUE solution algorithm which augment the choice set and actually produces a MNL RSUE(min) solution as well.

The *averaging scheme* of *Step 2* involves weighing the current solution with the found auxiliary solution by using a step-size γ_n. To avoid potentially obtaining negative path flows, the step-size should be chosen such that $0 \leq \gamma_n \leq 1$. The step-size can be determined in various ways, for example pre-determined as well as various versions of the line search method proposed by Armijo (1966) (see Chen et al., 2012, 2013; Xu et al., 2012; Zhou et al., 2012).

Bekhor et al. (2007) compare the MSA, Armijo's approximation method and the computation of the exact optimal step-size, and find that Armijo's approximation performs best in terms of computation time until convergence. MSA is known for requiring many iterations before convergence because the auxiliary flow pattern generated at each iteration contributes equally to the final solution (e.g., Bekhor et al., 2007). Liu et al. (2009) test different alternative pre-defined averaging schemes, and introduce the method of successive weighted averages (MSWA). While being pre-defined, the MSWA allows giving higher weigh to auxiliary flow patterns from later iterations, and the step-size γ_n at iteration n is defined as the following.

$$\gamma_n = \frac{n^d}{1^d + 2^d + \ldots + n^d} \tag{30}$$

where $d \geq 0$ is a real number. Increasing the value of d moves more flow towards the auxiliary solution. The MSA is a special case of the MSWA, namely when d=0.

5.5 CONVERGENCE OF PROPOSED SOLUTION METHODS

To obtain convergence, both RSUE conditions have to be fulfilled. The condition (11) on unused routes will always be fulfilled upon termination of the algorithm, as *Step 1* induces that if additional attractive routes (violating condition (11)) exists, these will be added to the choice set. The other RSUE condition (10) is fulfilled if the flow is allocated among the paths in the restricted choice sets so that the flow solution fulfil the 'SUE' condition among these paths. To our knowledge, no proof of convergence has been given for the standard DUE version of the pairwise path-swapping algorithm. The proposed solution algorithm based on pairwise path-swapping may thus be seen as heuristics only. However, Carey and Ge (2012) provide numerical evidence of promising convergence in the DUE case, and our numeric results also indicate nice convergence behaviour of the path-swapping algorithm when using the transformed costs (section 6). Some of the other path-based DUE and SUE solution algorithms (such as the SUE DSD proposed by Damberg et al. (1996)) has been shown to converge under certain assumptions, and thus applying e.g. the Damberg et al. (1996) algorithm for the *Restricted master problem phase (Step 2)* will induce convergence to a RSUE solution.

5.6 COMPUTATIONAL PERFORMANCE OF THE ALGORITHMS

For large-scale applications, it is important that the algorithms adopted are computationally attractive. The path searches (*Column generation phase)* and the network loading are usually by far the most expensive components in practical implementations. The number of path searches and network loadings needed per iteration, as well as the complexity of the path searches, varies across alternative algorithms. Regarding the *Column generation phase*, the RSUE(min) is far more computationally attractive (per iteration) than the RSUE(max). Firstly, searching for only one shortest path is far less computationally demanding than searching for *k* shortest paths. Secondly, single shortest path algorithms are available to identify the shortest path from an origin to all destinations, whereas the available *k*-shortest path algorithms find only the *k* shortest paths between two points. Moreover, *per iteration* of the *Column generation phase* in general *z* searches, each with calculation complexity $O(V \cdot \log(V) + A)$, are needed for the RSUE(min), while z^2searches, each with calculation complexity $O(k_m \cdot V \cdot (V \cdot \log(V) + A))$, are needed for the RSUE(max) (Cormen et al., 2009).

Regarding the number of network loadings, the proposed solution algorithm performs one network loading as part of *Step 3*. However, it should be noted that some of the proposed approaches for the *Restricted master problem phase* may also require one or several network loadings. The pairwise path-swapping algorithm with pre-determined step-size is attractive by not requiring any network loadings in *Restricted master problem phase*, whereas algorithms such as Han (2007) require an additional network loading *per OD-pair* per iteration to determine the step-size. The DSD algorithm proposed by Damberg et al. (1996) may be very time consuming per iteration as it iterates the inner assignment step until full convergence among the choice set before the *Column generation phase* is re-evaluated. This requires numerous time consuming network loadings (as well as Armijo line searches).

The calculation complexity thus varies greatly between specifications of the generic solution algorithm (and solution algorithms in general). Comparison of convergence speed across algorithms should thus be based on computation time rather than number of iterations. It should be noted that while simulation is also computationally demanding, none of the proposed variants of the generic solution algorithm requires simulation for the MNL or PSL choice models.

6 NUMERICAL RESULTS

We have evaluated several different versions of the proposed generic solution algorithm numerically on two case-studies, namely the well-known Sioux Falls network and the large-scale Zealand network.

The case-studies also serve to demonstrate that the relative gap measures proposed are attractive to apply in the evaluation of the convergence of certain Logit-type RSUE(min) and RSUE(max) solution algorithms.

6.1 SIOUX FALLS NETWORK

The Sioux Falls network contains 76 links and 528 OD-pairs between which there is a non-zero demand[2]. We present the application of several instances of the solution algorithm with the focus on the MNL model. Firstly, the RSUE(min) as well as the RSUE(max) problem are addressed by applying the cost transformation to solve the *Restricted master problem phase* by using the pairwise path-swapping algorithm introduced in section 5.4 All assignments used θ=0.1 and were done using MATLAB.

6.1.1 MNL RSUE(MIN) AND MNL RSUE(MAX)

Figure 1 reports the proposed two-part convergence measure for the path-swapping MNL RSUE(min) and the MNL RSUE(max)[3] solution algorithms with d=0. As can be seen, the MNL RSUE(min) as well as the MNL RSUE(max) solution algorithms both provided nice convergence patterns. The MNL RSUE(min) in general converged fastest, requiring only 7 iterations for the equilibrium choice sets to (with a few exceptions) be generated, and by iteration 75 the distribution of flow among used paths also had converged to the same low level as reached at iteration 250 of the MNL RSUE(max).

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[2] See Bar-Gera (2013) for a detailed description of the network structure, performance and demand. Note that the travel cost function are constituted solely by the travel time.

[3] It is important to note that the gap for unused paths of the RSUE(max) cannot be directly compared to that of the RSUE(min) as the measures are computed differently.

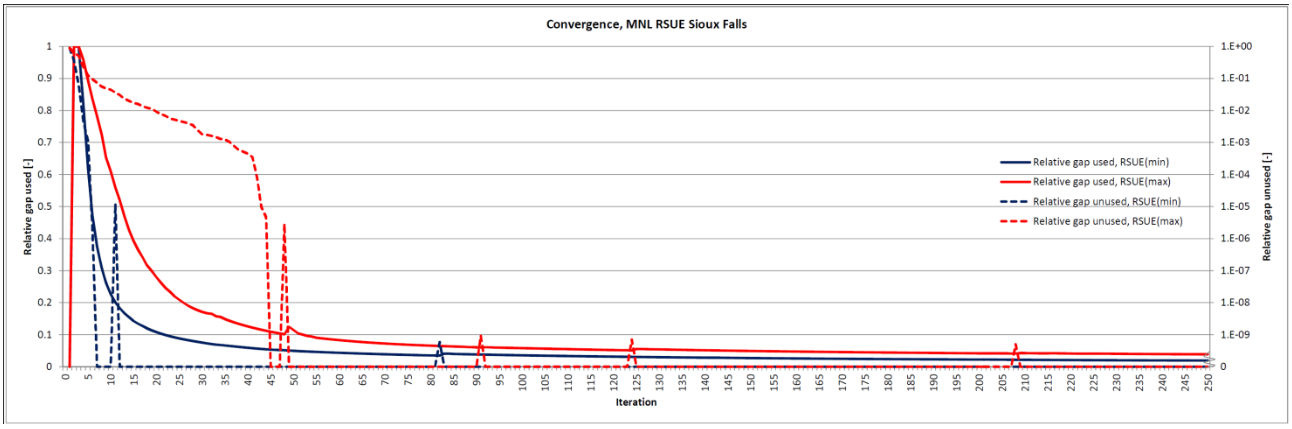

Figure 1 – Convergence of MNL RSUE(min) and MNL RSUE(max), Sioux Falls network

Figure 2 illustrates the development of the minimum, average and maximum size of the path choice sets as the two solution algorithms progress4. The condition on unused paths is stronger in the RSUE(max) than in the RSUE(min), which expectedly resulted in larger path choice sets. The average choice set size of the MNL RSUE(max) was approximately 7.4 from iteration 50 and onwards, which is larger than the largest choice set generated by the MNL RSUE(min) algorithm (7 paths). The choice sets had an average size of 2.5 from iteration 5 onwards when solving for the MNL RSUE(min) flow solution.

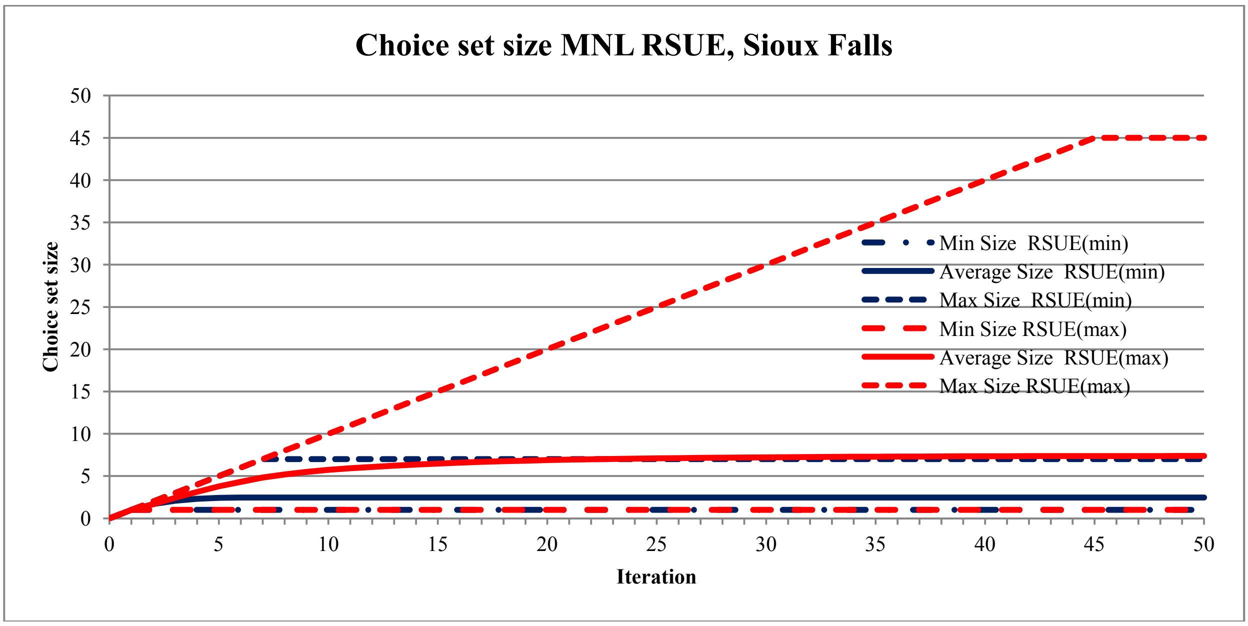

Figure 2 – Choice set size, MNL RSUE(min) and MNL RSUE(max), Sioux Falls

[4] Iterations 1-50 reported. Minimum and maximum choice set size does not change after iteration 50, and the mean choice set size only changes marginally.

Figure 3 shows an example of the generated choice set for one OD-pair. The choice set of the MNL RSUE(min) consisted of three paths, whereas the choice set of the MNL RSUE(max) contained 10 paths (3 of which are the paths of the MNL RSUE(min)).

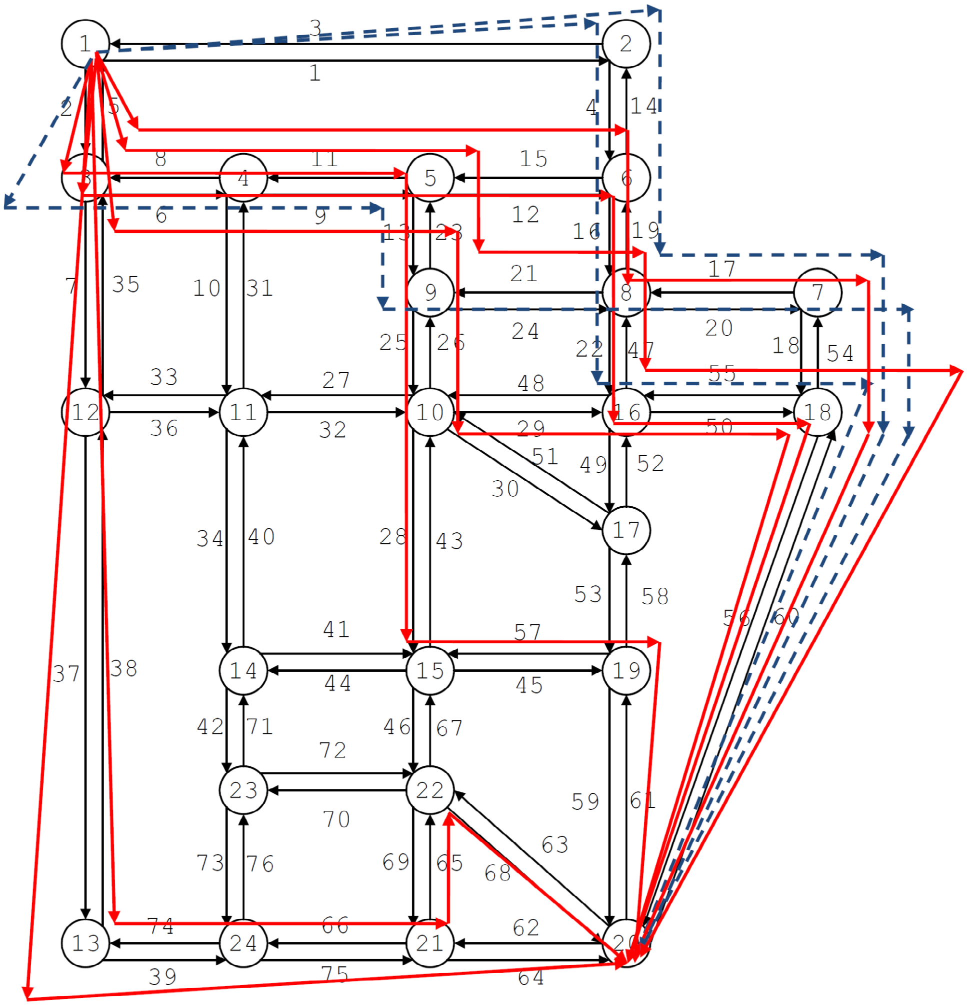

Figure 3 – Example of utilised path choice set 1 OD-pair, Sioux Falls. Note: Dashed paths: Paths used in RSUE(min). Dashed + continuous paths: Paths used in RSUE(max)

This example of used paths highlighted a main issue arising when pre-specifying the choice set based on free-flow travel time, as often done in traditional SUE assignment algorithms, but which are avoided by the RSUE. The path 1-3-4-5-9-8-7-18-20 carried 7% and 16% of the total demand between the selected OD-pair in the MNL RSUE(max) and MNL RSUE(min) solutions, respectively. This path was however the 41st shortest path when based on free-flow travel time (34 minutes as opposed to 22 minutes of travel time for the shortest). Consequently, this obviously attractive path would not have been generated to the choice set if a *k*-shortest path approach with *k*<41 had been used for pre-generating the path choice sets.

6.1.2 *ALTERNATIVE APPROACHES TO THE RESTRICTED MASTER PROBLEM PHASE*

This section presents the results of an evaluation of four alternative possible approaches for the *Restricted master problem phase* for the MNL RSUE(min) problem. The first two alternatives exploited the proposed RSUE model transformation proposed and applied path-based DUE solution algorithms: (i) a pairwise path-swapping algorithm (MNL Path Swap) as described in section 5, and (ii) an all-or-nothing algorithm assigning all traffic to the path with the lowest transformed cost (MNL AoN). The third alternative used the MNL probability formula to obtain an auxiliary solution (MNL Inner Logit). Each of these three alternatives were tested with two MSWA averaging strategies, namely the MSA (d=0) and one which 'trusted' the auxiliary solution more (d=2). The fourth alternative was the SUE DSD solution algorithm (MNL DSD, Damberg et al., 1996). As aforementioned, the MNL DSD iterates within the *Restricted master problem phase* until a converged solution is found for the 'conditional' choice set. In the application of the MNL DSD algorithm, the *Restricted master problem phase* was terminated (and new possible attractive paths were identified) when the relative gap on used routes reached 0.01%.

Figure 4 and Figure 5 illustrate the convergence of the algorithms. The relative gap for the choice set composition converged within the first 4-15 iterations (the MNL AoN with d=2 required 30 iterations), indicating that the final choice sets were generated within the first iterations. The convergence of the distribution of flow (which is conditional on the choice set) also converged fast for most algorithms. The MNL DSD algorithm required only a very few iterations to converge to a very low value of the relative gap measures. The MNL Inner Logit with d=2 was the second fastest (in terms of number of iterations), followed by MNL Path Swap with d=2. However, both required considerably more iterations than the MNL DSD algorithm to converge. The MNL AoN, the MNL Inner Logit and MNL Path Swap path-swapping algorithms with d=0 required additional iterations to reach convergence. The worst performance was seen by the MNL AoN approach with d=2.

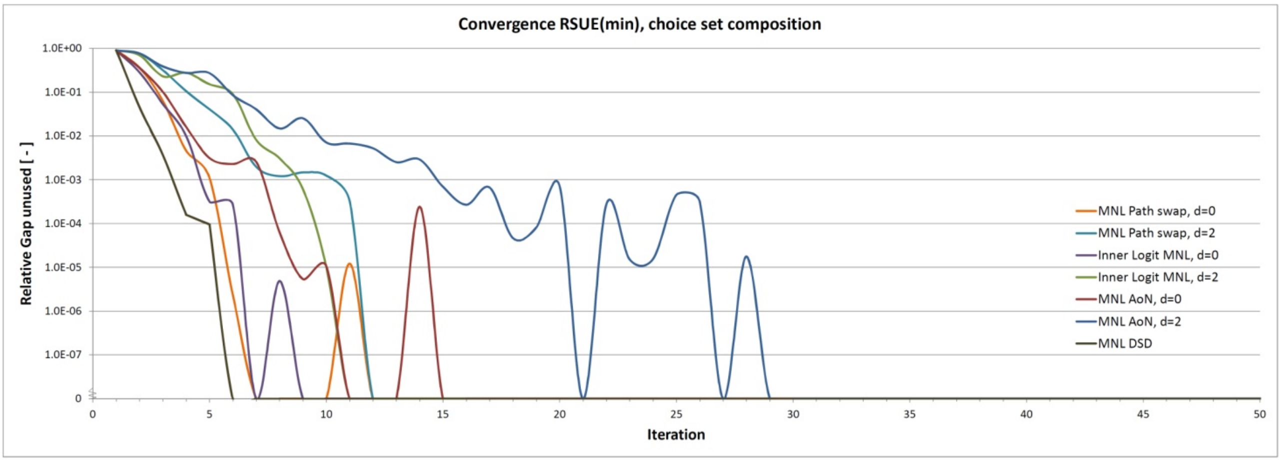

Figure 4 – Relative gap measure for convergence of choice set composition as function of iteration number, Sioux Falls application. Notice the log-scale on the vertical axis

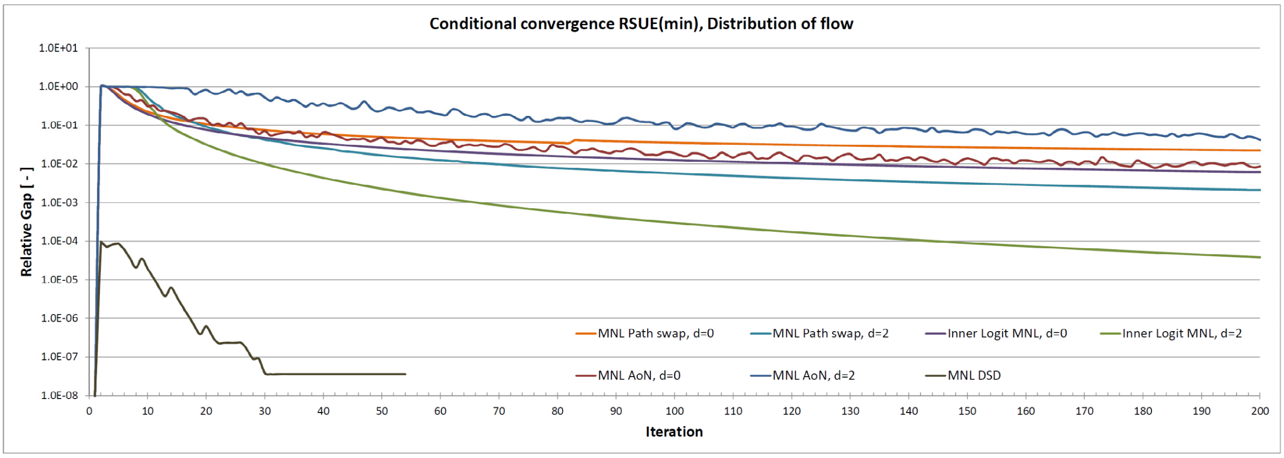

Figure 5 – Relative gap measure for convergence of flow distribution among routes in the choice set as function of iteration number, Sioux Falls application. Notice the log-scale on the vertical axis

The calculation complexity however differed between algorithms, and especially the MNL DSD algorithm was notably more demanding in requiring iterations in the *Restricted master problem phase* until near-convergence for the 'conditional' choice set. This aspect has also to be considered in the evaluation of algorithms, and Figure 6-Figure 7 show the convergence as a function of the computation time rather than the number of iterations.

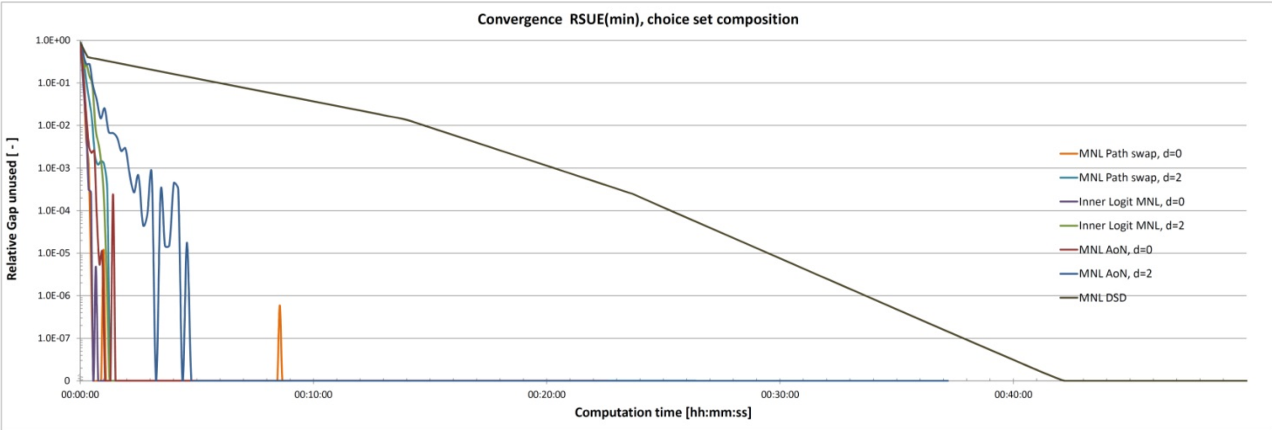

Figure 6 – Relative gap measure for convergence of choice set composition as function of computation time, Sioux Falls application. Notice the log-scale on the vertical axis

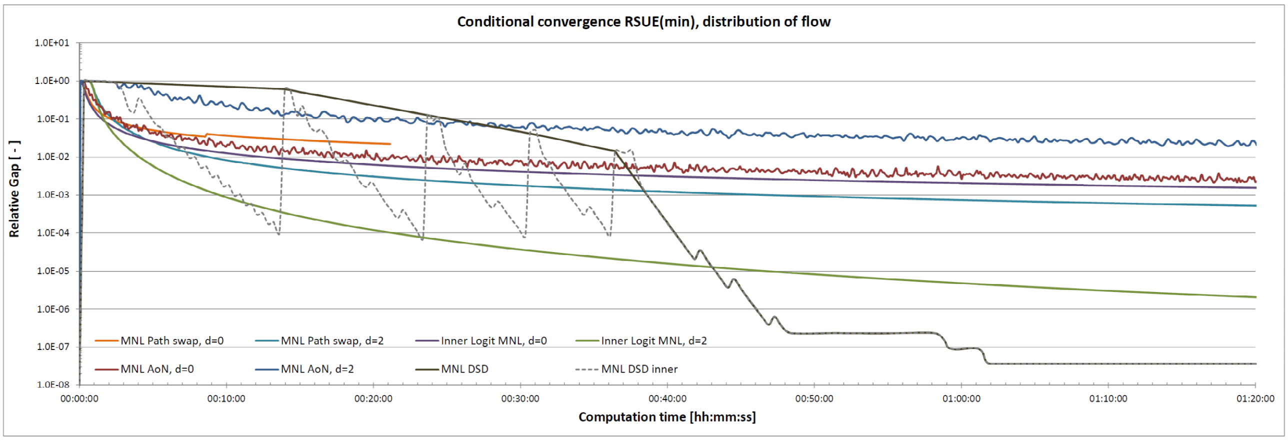

Figure 7 – Relative gap measure for convergence of flow distribution among routes in the choice set as function of computation time, Sioux Falls application. Notice the log-scale on the vertical axis

The MNL DSD was not so attractive when considering computation time, as the final choice sets were identified after 41 minutes, whereas these were determined within 2-9 minutes for the remaining algorithms. This means that while the distribution of flow among the available choice set converged fast to an MNL solution (due to the line search, see the dotted line in Figure 7), the overall convergence of the MNL DSD was slow. Looking at the overall convergence, the MNL Inner Logit with d=2 thus converged the fastest to the final choice sets with a relative gap among the used routes of less than 0.01%.

None of the four alternative algorithms required simulation, and each of these thus converged to the same solution for repeated applications, given the same initial conditions. The RSUE conditions however do not induce uniqueness, and the algorithms tested may therefore converge to different solutions which are all MNL RSUE(min) solutions. The non-uniqueness lies within the generation of the choice sets, which is also illustrated by the varying average and maximum choice set size across algorithms (Table 1). The DUE and SUE both provide theoretical uniqueness (only on link-level for the DUE), but this is however typically not seen in practical real-life applications anyhow, as the choice sets have to be pre-specified or simulated (Watling et al., 2014). Actually, using simulation for path generation in SUE additionally induces difficulties in the reproduction of solutions for repeated applications.

Table 1 – Choice set size characteristics, Sioux Falls

Method	Step size	Choice set size		
	d	Min.	Avg.	Max.
AoN	0	1	2.59	7
	2	1	4.01	10
Path Swap	0	1	2.46	7
	2	1	3.03	7
Inner Logit	0	1	2.39	6
	2	1	3.37	9
DSD	-	1	2.05	4

Using d=0 generated quite similar average choice set sizes across the algorithms (2.39-2.59), whereas the average choice set size varied more when d=2 (3.03-4.01). The larger choice sets when d=2 were due to the larger oscillations of flow in the initial iterations, which were a consequence of the larger step-size moving a larger share of the flow to the auxiliary solution. This caused larger changes in the costs of the links, which induced more unique routes to be generated. The MNL DSD solution algorithm generated small choice sets. This was (also) a consequence of the flow solution found in the *Restricted master problem phase* – the 'inner equilibration' before considering adding additional routes to the choice sets induced less flow fluctuations between iterations.

6.2 ZEALAND NETWORK

Two versions of the proposed RSUE(min) solution algorithm were applied to the large-scale Zealand network. Approximately 2.5 million people live in the area covering 9200 km^2, and the digitised road network representation consists of 12,015 links and 429 zones. The demand matrix applied covered a 24 hour period and contained a total of 3.2 million trips across 19 different user classes and two vehicle types (car and lorry) to be assigned to the road network[5]. The study area consists of urban as well as rural areas, and the congestion level is spatially distributed as well as distributed across road type classifications (see Figure 8).

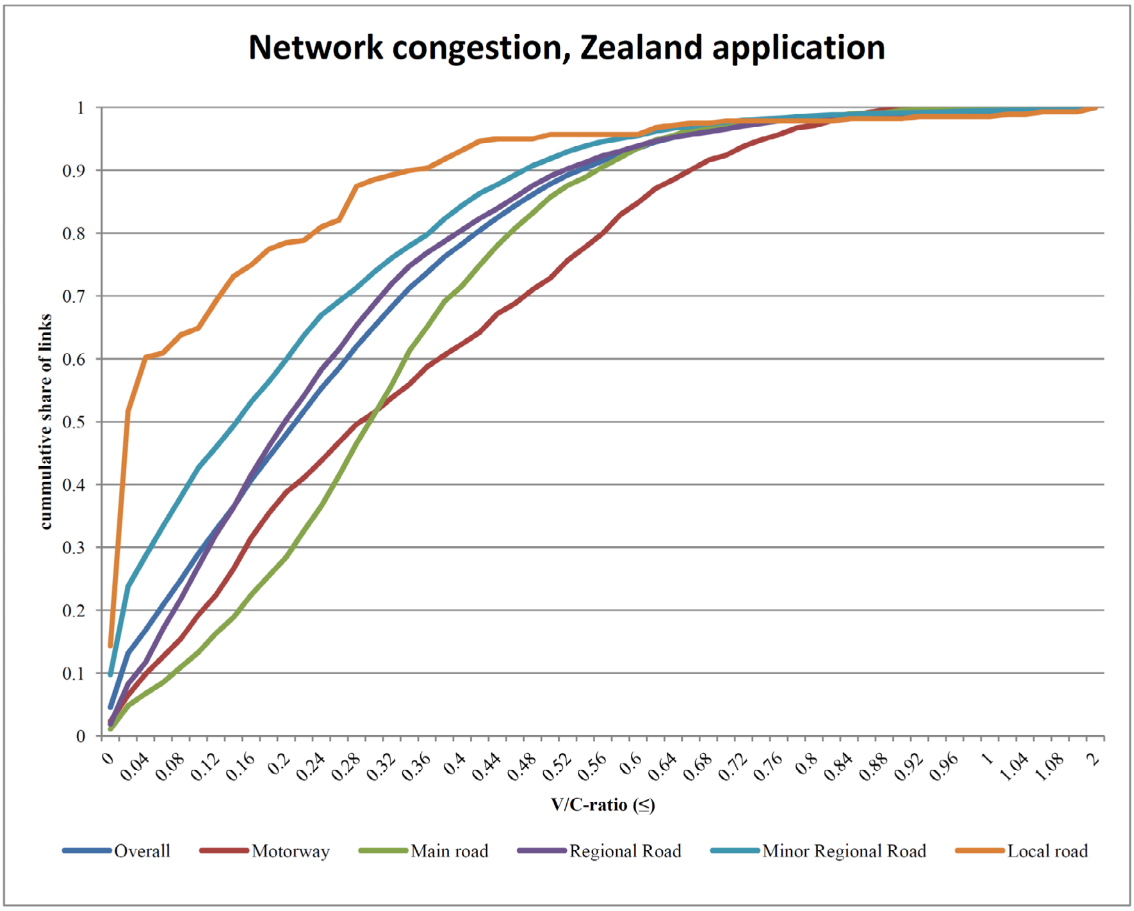

Figure 8 – Network congestion by road type classifications. Cumulative share of links as function of Volume/Capacity-ratio

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[5] The Zealand network is a subset of the network to be used in the Danish National Model, currently under development at DTU Transport.

The two tested versions of the algorithm were differentiated by the approach to the *Restricted master problem phase*. One used the pairwise path-swapping approach presented in section 5.4 (Path Swap), while the other deployed an approach where the auxiliary solution is found by applying the corresponding logit choice probability formula directly (Inner Logit). The algorithms have been implemented in the Traffic Analyst traffic assignment module for ArcGIS (Rapidis, 2013), and both accommodated the use of the MNL and the PSL models.

In the application we specified θ=0.2 and the step-size constant d=2, as an initial test comparing d=0 to d=2 found best performance in terms of convergence speed when d=2 (as also found in the Sioux Falls application). The generalised travel costs were constituted by a weighted sum of free-flow travel time, congested travel time, travel distance, and travel (monetary) cost.

The calculation time per iteration was approximately one minute for both solution algorithms when using a computer with a 3.2 GHz Intel Zeon CPU and 32 GB RAM. Figure 9 illustrates the convergence pattern as a function of the iteration number for the MNL choice model.

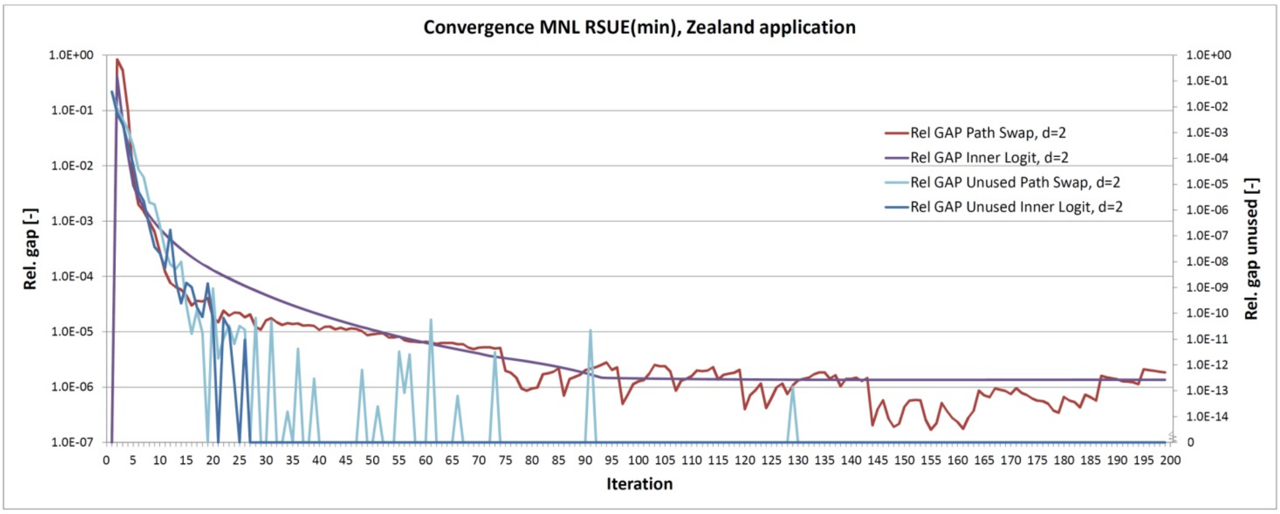

Figure 9 – Relative gap measures, Zealand network application. Notice the log-scale on the vertical axis

The figure indicates that the applied algorithms seem to provide fast convergence behaviour when applied to the large-scale network. The distribution of flow among used routes seemed to converge better for the Path Swap algorithm in the first 50 iterations, but the relative gap on used routes reached a very low level of 0.001% at iteration 50 for both algorithms. From this iteration the relative gap among used routes was at the same level for the two algorithms, although with some fluctuations for the Path Swap algorithm. It is however important to remember that this measure of convergence is conditional on the choice set composition. The corresponding gap measure converged fast for both algorithms. The measure quickly reached zero, but for the Path Swap algorithm it repeatedly increased slightly from zero as the algorithm iterated (the redistribution of the flow caused new routes to be attractive). It was not until iteration 129 that no extra routes were added to any of the choice sets for the algorithm based on path-swapping. The relative gap did however never grow large in these increases from zero, indicating that it was only for a few OD-pairs that new routes were attractive. This is also indicated by the development of the average choice set size (Figure 10), which was almost constant from iteration 10 onwards. The choice sets were thus (generally) generated within the first few iterations.

We applied the algorithms to the original OD-matrices scaled by different factors ranging from 1 to 2. The number of iterations needed to obtain convergence as well as the size of the choice sets generated showed expectedly to increase with increasing scale-factor. However, both algorithms converged within a reasonable number of iterations for all demand levels tested, and overall both algorithms seems to be robust towards the general congestion level in the network it is applied to.

The converged solution generated was not the same for the two algorithms, as the composition of the choice sets varied between them. The Inner Logit generated, on average, slightly smaller choice sets (Figure 10). This was probably a consequence of a more ‘equal’ distribution of flow between the paths in the initial iterations (smaller oscillations), as indicated by a lower relative gap on used routes during the first few iterations. An average choice set size of 2.5-3 routes may seem small. However, this should be seen in light of the network composition; the case-study area includes, in addition to urban areas, large rural areas in which there is no congestion and only one or two relevant alternatives. An analysis of the spatial distribution of the choice set size showed that the choice sets generated for trips conducted in rural areas were considerably smaller than those generated for urban trips. Finally, we note that while identifying a flow solution that fulfils the original SUE conditions would require (infeasible) enumeration of millions of routes for each OD-pair, the solution algorithms proposed found converged solutions satisfying the RSUE(min) conditions with a computational feasible maximum size of the generated choice sets of 11 routes.

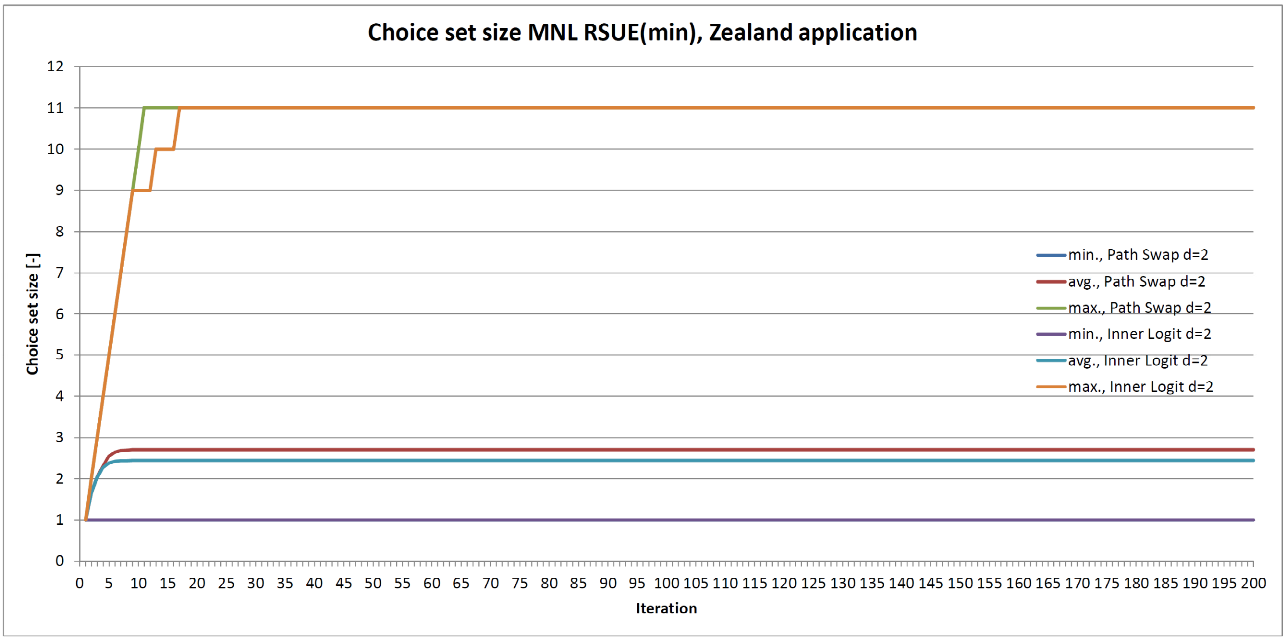

Figure 10 – Minimum, average and maximum choice set size, Zealand application

We also implemented and tested the corresponding algorithms for the PSL choice model. An analysis of the results showed that, in general, the convergence pattern as well as choice set composition and size were similar to the corresponding results obtained when using the MNL model. In the PSL application we tested different values of β_{PS} (ranging from –25 to 0) and evaluated the results by comparing the link flows obtained with corresponding real life observed link flow counts (for 1169 links distributed across the network). The evaluation was done using the coefficient of determination (R^2) obtained from a linear regression of the modelled flows as a function of the observed flows (using the Path Swap algorithm). In general very high correspondence were found, with R^2=0.9444 when β_{PS} =0 (MNL case), declining R^2 with increasing negative value of β_{PS} until R^2=0.9404 when β_{PS} =-25.

Last, we performed a disaggregate evaluation of the choice set composition and flow distribution for one OD-pair within the study area. Both algorithms generated the same five unique routes shown in Figure 11 for the MNL as well as the PSL choice models. The trip was a commuting trip and the size and composition of the choice sets seems reasonable; one alternative (Path 3) used motorway as far as possible, one alternative avoided motorway but rather used uncongested minor local roads (Path 2) and three alternatives were versions of the lowest cost route using a combination of motorway and minor local roads.

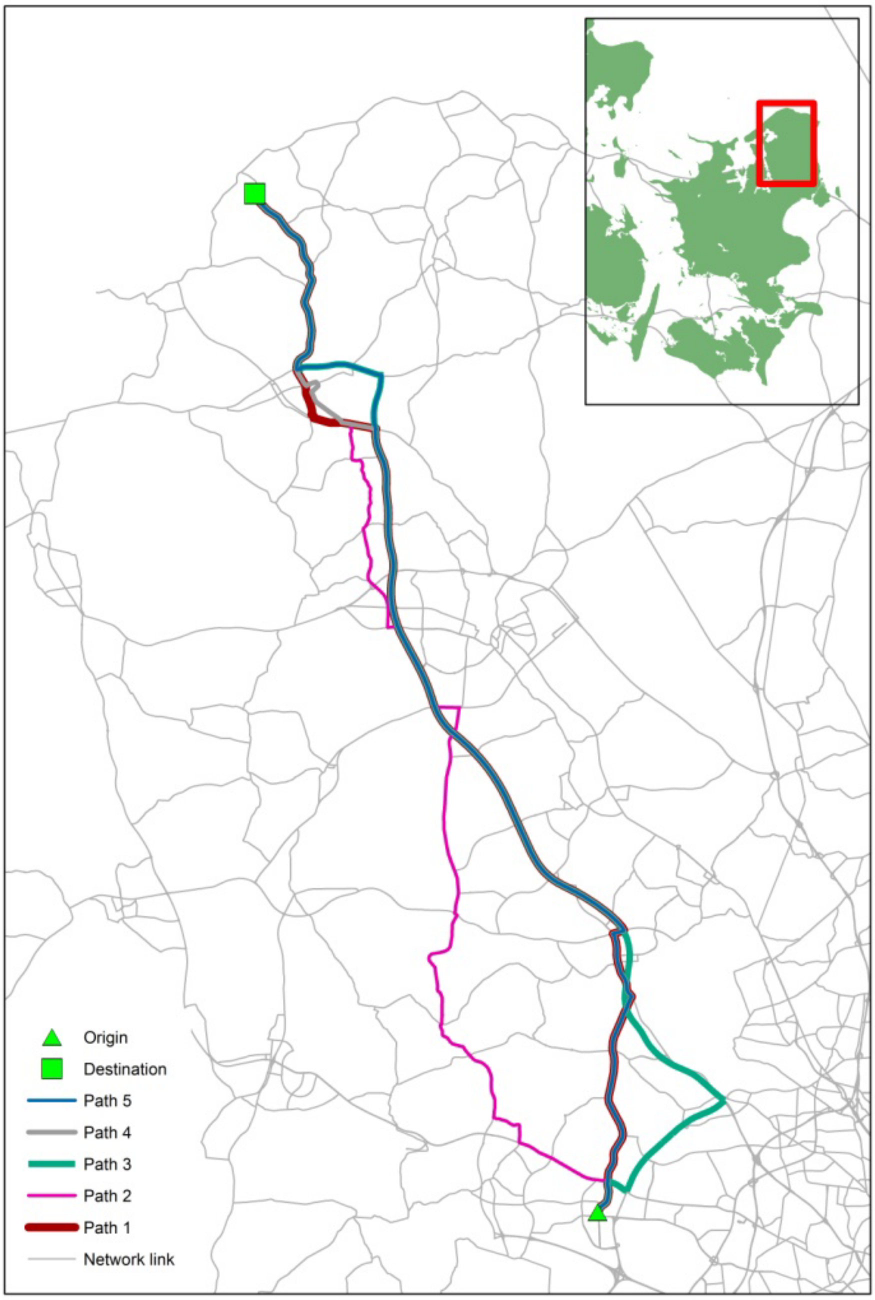

Figure 11 – Illustration of generated choice set, 1 OD relation Zealand application

The generalised route costs and flow shares for the MNL and the PSL choice models can be seen in Table 2 (results from two different β_{PS} values reported).

Table 2 – Generalised costs and flow distribution for various choice models, 1 OD relation Zealand application

	MNL		PSL β_{PS} =-3			PSL β_{PS} =-8.5		
PathID	Gen. Cost [-]	Flow [%]	Gen. Cost [-]	PS term [-]	Flow [%]	Gen. Cost [-]	PS term [-]	Flow [%]
1	126.16	22.7	126.07	0.30	21.0	125.90	0.30	16.2
2	130.40	9.7	130.49	0.78	15.2	130.65	0.78	30.9
3	129.19	12.4	129.18	0.48	14.9	129.18	0.48	18.4
4	125.28	27.1	125.20	0.28	23.7	125.05	0.28	16.6
5	125.08	28.1	125.00	0.29	25.1	124.87	0.29	18.0

In the MNL case, the three routes with the lowest generalised costs (paths 1, 4 and 5) attracted 78% of the traffic, whereas the almost unique path 2 only attracted 9.7% of the flow despite being only 4% more expensive than the cheapest path. Accounting for path overlap changed the distribution of flow between the path as well as the path costs (through redistribution of flows for all OD-pairs). While paths 1, 3 and 5 highly overlap, path 2 is the most unique path, and thus was the one that attracted flow from the other paths when compared to the MNL case. When using β_{PS} =-3 the share on path 2 was a reasonable 15.2%, whereas the share on this path was an unreasonable 30.9% when using β_{PS} =-8.5. This highlights the need for aggregate as well as disaggregate analysis when evaluating the models; The MNL performed best in terms of reproducing link counts, but accounting for path overlapping (using a well suited β_{PS} –value) produced the most reasonable distribution of flow among the paths.

7 DISCUSSION

We demonstrated that the *k-shortest path* approach in the *Column generation phase* for the RSUE(max) was feasible for the Sioux Falls network, but its calculation complexity let us to not pursue to apply it in the iterative algorithm on the Zealand network. We have supported this by performing some initial large-scale tests of the *k*-shortest path search algorithm of Yen (1971), for which we found computation times per OD-pair that would cause infeasibly long calculation times in an iterative algorithm on large-scale networks. Consequently, while the *k*-shortest path approach in principle is possible to apply, more research is needed to deduce a sufficiently efficient choice set generation approach to the RSUE (max). Alternatively, it could also be interesting to investigate other formulations of the operator $\Phi\left(\{c_{ms} : s \in \tilde{R}_m\}\right)$, which allows application of efficient choice set generation algorithms while posing stricter requirements to the costs on unused paths than the RSUE(min).

Another approach could be to reduce the 'solution space' through network aggregation. Aggregating the network to only include main roads while keeping the network disaggregate in the areas surrounding the origin and destination of a trip may not be 'behaviourally' unrealistic: travellers may actually consider different alternatives with small deviations from each other (and having enough network knowledge to do so) around the origin and destination, but may only consider (and know of) some main alternatives on the intermediate part of a trip.

The efficiency of a *step-size strategy* varies across networks and demands, and more research could be put into how to (dependent on network characteristics) choose a suitable step-size strategy. While the DSD algorithm of Damberg et al. (1996) provided slow overall convergence, the line search strategy caused the inner assignment problem to be solved very fast. Therefore it could be interesting to implement the line search strategy and use this in the Inner Logit and Path Swap algorithms (i.e. only one line search per OD-pair per iteration rather than an equilibration as in the DSD algorithm).

Consistent *evaluation of convergence* has traditionally been a challenge for conventional SUE solution algorithms. The first part of the proposed consistent RSUE convergence measure is however also applicable and thus highly attractive to adopt to such SUE algorithms solving on a fixed choice set. The user can freely define the thresholds for both parts of the convergence measure to reach before the algorithm is terminated. Of course they should depend on the network, parameter specification and desired level of convergence. We have found that using 0.01% for the sum of the two parts seems to produce a fairly converged solution. Boyce et al. (2004) also proposed a threshold of 0.01% for the DUE relative gap measure.

We *compared* the *modelled* link flows to real-life *observed link flows* for the Zealand application, and found remarkably high correspondence. This was despite the parameters not having been properly estimated to the RSUE, which would possibly generate even higher correspondence. An interesting finding is that the correspondence decreases with increasing weight on the path size correction term. This does however not mean that no correction for path overlapping should be done, as this might cause the distribution of flow between the paths and the choice set composition to be more reasonable (as highlighted by the disaggregate Zealand example). A further analysis of this could e.g. consist of a comparison between generated paths and corresponding observed real-life route choices.

8 CONCLUSIONS

The paper tackles the challenge of applying the RSUE(Φ) model framework to large-scale networks and is the first to propose an algorithm to solve for RSUE flow solutions. We have demonstrated the applicability, convergence and scalability of different variants of the RSUE(min) and RSUE(max) solution algorithm on the well-known Sioux Falls network as well as the large-scale Zealand network.

The Sioux Falls application compared several different approaches to allocate flow between routes, and found that the algorithms, in general, converged to the corresponding RSUE solution. The application furthermore highlighted *(i)* the need to evaluate convergence as a function of computation time rather than number of iterations and *(ii)* that the step-size highly influences the convergence speed.

Two promising algorithms were tested for the RSUE(min) on the Zealand network, one utilising a cost transformation function to apply a DUE solution algorithm based on pairwise path-swapping and the other a traditional SUE algorithm utilising the closed-form choice probabilities directly to find the auxiliary solution. These converged fast to fulfil the underlying RSUE conditions and were efficient in generating the choice sets within the first few iterations. The equilibrated non-universal choice sets were reasonable and computationally attractive in size, and we found that the algorithms managed to reproduce real-life observed link flows very well for the MNL and the PSL choice models.

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APPENDIX 6: RASMUSSEN ET AL. (2014D)

STOCHASTIC USER EQUILIBRIUM WITH EQUILIBRATED CHOICE SETS: PART III – MODEL REFORMULATION TO INCLUDE THRESHOLDS ON COSTS AND LARGE-SCALE APPLICATION

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Abstract: The Deterministic User Equilibrium (DUE) and the Stochastic User Equilibrium (SUE) have limitations by allowing only minimum cost routes to be used in the DUE and requiring all routes to be used in SUE. The Restricted Stochastic User Equilibrium (RSUE) was proposed to remove these limitations and facilitate large-scale application. The RSUE use random utility maximisation models for flow allocation among equilibrated non-universal choice sets. While no attractive paths are allowed unused, unattractive paths may be used at equilibrium. We address this issue by proposing the Restricted Stochastic User Equilibrium with Threshold (RSUET) as a model generic to the RSUE. To ensure that no unattractive paths are used, the RSUET adds a behaviourally realistic 'threshold' condition on the costs which needs to be fulfilled among used paths.

We propose a corresponding generic solution algorithm and tested several variants of this on the large-scale Zealand network. These showed very attractive computation times and that the modification supports an improvement in behavioural realism, especially for high-congestion cases. Extremely fast and well-behaved convergence to equilibrated solutions among non-universal choice sets was seen (across different congestion levels, scale parameters and step-sizes). We validated the choice set composition by comparisons to 16,618 observed route choices and found the RSUE and RSUET to perform equally well. Both models were also very successful in reproducing observed link counts.

Keywords: Restricted Stochastic User Equilibrium; Restricted Stochastic User Equilibrium with Threshold; Choice Set; Solution Methods; Traffic Assignment; Large-Scale Application.

1 INTRODUCTION

The most well-researched and commonly applied discrete choice models based on random utility maximisation (RUM) represent random error structures through the use of error terms which are assumed to follow a distribution with unbounded support (e.g. normal distribution leading to the probit model or gumbel distribution leading to the logit-family of models). However, choosing a distribution with unbounded support leads to the assignment of a non-zero choice probability to all available alternatives no matter how costly they may be (Watling et al., 2014). Whereas this may lead to behaviourally questionable choices in many applications, typically it does not pose computational challenges for forecasting or parameter estimation in most discrete choice model applications, e.g. to the choice of mode of transport. It however poses a significant challenge in the application to route choice modelling because the number of alternatives are so large, and may not be perceived by all travellers.

In route choice modelling the (universal) set of available alternatives is typically infeasible to enumerate and allocate traffic to, even for moderate-sized networks. This issue is further complicated by the 'scalability' of the problem, meaning that even small changes (e.g. the addition of a few links) in one part of the network 'scales' to require also updating choice sets and redistributing flows in other parts of the network. Consequently, in most practical route choice modelling applications to find the Stochastic User Equilibrium (SUE, Daganzo and Sheffi, 1977), the solution algorithms either *(i)* terminate before the full universal choice set is generated (such as the simulation-based SUE algorithm of Sheffi (1985)), or *(ii)* find SUE among a fixed pre-specified choice set (e.g., Bekhor et al., 2008; Zhou et al., 2012).

1. In the first case it is difficult to specify when to terminate the algorithm in order to include a sufficient sample of all relevant routes, since there could be almost infinitely many routes to which traffic would be allocated. In practice the approach also poses convergence problems, since both the route set generation and the choice model use sampling, typically via Monte Carlo simulation. This may also raise a question of sample-correction methods, which is usually overlooked in applied models (e.g., Frejinger et al., 2009; Guevara and Ben-Akiva, 2013).
2. In the case of pre-defined choice sets it is difficult to pre-specify the choice set to include all relevant alternatives without knowing in advance the equilibrium congestion-

dependent travel times in the network. This is especially the case when using the model to forecast future situations or effects of policies, since there is no chance to know the congested travel times from e.g. observations in such cases. The potential result of this is the generation non-equilibrated and inconsistent choice sets, which fails to include and exclude paths that are, respectively, attractive or unattractive at equilibrium..

Watling et al. (2014) addressed the issues raised above and formulated the Restricted Stochastic User Equilibrium model (RSUE). The RSUE differs from the SUE conditions by specifying an equilibrated flow solution according to the well-researched choice models, however among an equilibrated (possibly non-universal) choice set. In addition to some behavioural advantages (e.g., completely unrealistic routes may be omitted), the RSUE is very attractive in practical implementation by allowing the choice sets to be consistently formed to fulfil the underlying conditions at equilibrium, rather than to solely rely on the solution algorithm in the generation of these. In Rasmussen et al. (2014b) we proposed a corresponding solution algorithm obviating the need for simulation. We found nice convergent behaviour of various instances of this to solutions with equilibrated choice sets of reasonable sizes.

The RSUE was formulated to allow a non-universal choice set by implicitly posing a condition on the costs on unused paths. While ensuring that no attractive routes are left unused, the RSUE conditions do not implicitly ensure that the choice sets of used routes consist only of attractive routes leaving unattractive routes unused. We could imagine that a route which in some initial iteration of the solution algorithm is attractive and thus included into the choice set, becomes highly unattractive at the equilibrium state. For example, imagine an OD movement for which there are two attractive paths in the free-flow state. Path 1 is slightly cheaper than Path 2, but uses links on which the travel cost is highly sensitive to the flow. It is attractive to assign traffic to Path 1 in the initial iteration, with the result of a large increase in the travel cost. Path 1 thereby becomes highly unattractive, and flow will be moved towards Path 2. The RSUE will however not move all flow away from Path 1, no matter how unattractive it may be – it will still be assigned traffic at equilibrium since there is no mechanism to exclude paths once they are in the choice set. This may lead to behaviourally questionable solutions; imagine e.g. that the small amount of traffic on Path 1 causes this used path to cost 50% more than Path 2 at equilibrium. We believe that this issue could be especially evident in highly congested cases, and the RSUE has no built-in rules for the exclusion of such unattractive paths – this is left up to the solution algorithm adopted. In this paper, we aim to address this through the formulation of a model generic to the RSUE, namely the Restricted Stochastic User Equilibrium with Threshold model (RSUET). The RSUET has built-in rules providing aid for the solution algorithms to utilise in the exclusion of paths, ensuring that a RSUET flow solution is an equilibrated solution on equilibrated choice sets consisting only of the attractive paths. We propose a corresponding consistent solution algorithm to the RSUET model and validate several variants of this through large-scale application to the Zealand area. The validation consists of a comparison to the results and convergence of corresponding RSUE formulations. Comparisons are also done to a link- and simulation-based (possibly mixed) multinomial probit SUE solution algorithm, which is currently applied in the Danish National Model (Rich et al., 2010). We furthermore validate the distribution of the flow by comparison to real-life link counts and validate the choice sets generated by aggregate and disaggregate comparisons to corresponding observed route choices collected via GPS.

The remainder of the paper is organised as follows. Section 2 introduces the notation used, motivates the need for the new set of conditions and introduces the RSUET. In section 5 we present a RSUET solution algorithm and discuss different possible variants of this. Different variants of the solution algorithm are evaluated by application to the Zealand area, and section 4 introduces the case study along with the evaluation criteria and parameter-setup used etc. Section 5 presents and evaluates the results obtained, and in section 6 we discuss the findings draw the main conclusions.

2 RESTRICTED STOCHASTIC USER CONDITIONS WITH THRESHOLD

The notation used in the paper is introduced initially. Subsequently, we motivate the need to extend the RSU conditions and propose a set of conditions generic to the RSU conditions and a corresponding equilibrium model.

2.1 NOTATION

We consider a network as a directed graph composed of A links (arcs), indexed a = 1, …, A, denote the non-negative flow on link a as f_a, and let $\mathbf{f}$ be the A-dimensional vector of link flows. The *actual* flow-dependent (generalised) travel cost on link a is assumed to be a continuous function of the flow, and denote it by $t_a(\mathbf{f})$.

The network consists of M OD-pairs, and the demand is given by the non-negative M-dimensional vector $\mathbf{d}$. The m-th element d_m of $\mathbf{d}$ thus refers to the demand between OD-pair m. For each OD-pair m, let R_m be the set of all simple acyclic paths (routes) between the origin and destination of m, and let N_m refer to the number of paths in R_m. R refers to the joint set of all simple paths across OD-pairs, and this set has the dimension $N=\sum_{m=1}^{M} N_m$.

Denote the flow on path $r \in R_m$ between OD-pair m as x_{mr} and let $\mathbf{x}=\left(x_{11},x_{12},...,x_{1N_1},x_{21},x_{22},...,x_{2N_2},...,x_{M1},x_{M2},...,x_{MN_M}\right)$ be the N-dimensional flow-vector on the universal choice set across all M OD-pairs, so that the notation x_{mr} refers to element number $r+\sum_{k=1}^{m-1} N_m$ in the N-dimensional vector $\mathbf{x}$. The convex set G of demand-feasible non-negative path flow solutions G is given by:

$$G=\left\{\mathbf{x}\in\mathbb{R}_{+}^{N}:\sum_{r=1}^{N_m} x_{mr}=d_m \qquad ,m=1,2,...,M\right\} \tag{1}$$

where $\mathbb{R}_{+}^{N}$ denotes the N-dimensional non-negative Euclidian space. The corresponding convex set of demand-feasible link flows is:

$$F=\left\{\mathbf{f}\in\mathbb{R}_{+}^{A}:\mathrm{f}_a=\sum_{m=1}^{M}\sum_{r=1}^{N_m}\delta_{amr}\cdot x_{mr},a=1,2,...,A,\mathbf{x}\in G\right\} \tag{2}$$

where δ_{amr} =1 if link a is part of path r for OD-pair m and zero otherwise.

Let $c_{mr}(\mathbf{x})$ be the *actual* (generalised) cost on path r for OD-pair m. As links may be used by several paths within and across the OD-pairs, $c_{mr}(\mathbf{x})$ depends on the flow vector $\mathbf{x}$. Additionally, the cost $c_{mr}(\mathbf{x})$ is a positive value and may be a weighted sum of several attributes, such as e.g. travel time, travel distance, and congestion charge.

In vector/matrix notation, let $\mathbf{x}$ and $\mathbf{f}$ be column vectors, and define $\boldsymbol{\Delta}$ as the $A \times N$-dimensional link-path incidence matrix. Then the relationship between link and path flows may be written as $\mathbf{f} = \boldsymbol{\Delta}\mathbf{x}$. We suppose that the travel cost on path r for OD-pair m is additive in the link travel costs of the utilised links:

$$c_{mr}(\mathbf{x}) = \sum_{a=1}^{A} \delta_{amr} \cdot t_a(\Delta\mathbf{x}) \qquad (r \in R_m;\ m = 1,2,\ldots,M;\ \mathbf{x} \in G\,). \tag{3}$$

Let $\mathbf{c}(\mathbf{x})$ be the vector of costs on all paths. The RSUET model distinguishes between used and unused paths, and consequently we let $\tilde{R}_m$ be the subset of R_m consisting of all utilised paths (non-zero flow) for OD-pair m (i.e. $\tilde{R}_m \subseteq R_m$). Let $\tilde{N}_m$ denote the number of utilised paths for OD-pair m. $\tilde{R}$ refers to the joint set of all used paths across OD-pairs, and this set has the dimension $\tilde{N} = \sum_{m=1}^{M} \tilde{N}_m$.

In the evaluation of the solution algorithms we utilise observed route choices collected from a GPS-based travel survey. The origin and destination of each observation are included among the M OD-pairs in the network, and the observed paths are of course to find in the universal set R. The set of observed paths is denoted by $R_{obs} \subseteq R$.

The notation presented above is readily modified to include the more general case of (a) multiple user classes, where classes may differ in their definition of travel cost and in the OD matrix, and (b) multiple vehicle types, which may additionally differ in the contribution they make to total traffic flow. This was also done in Rasmussen et al. (2014b) and in this case we define m instead to denote a *commodity* which is a combination of OD movement, user class and vehicle type, so that M is the product of the number of OD movements, user classes and vehicle types. In order to reflect different contributions to traffic flow, we suppose that the demand d_m for commodity m is measured in equivalent passenger car units (pcu). The only modification required to the notation is then that $t_{am}(\mathbf{\Delta x})$ now denotes the travel cost on link a as perceived by commodity m when the total pcu route flows are $\mathbf{x}$. Thus, the route cost-flow functions are defined by:

$$c_{mr}(\mathbf{x}) = \sum_{a=1}^{A} \delta_{amr} \cdot t_{am}(\mathbf{\Delta x}) \qquad (r \in R_m;\ m = 1,2,\ldots,M). \tag{4}$$

Under these changes, all the subsequent models and methods presented may be applied to this more general case. In the following we shall continue to refer to OD-pair m, but this may as well, in the case of multiple vehicle and/or user classes, be a referral to commodity m. In section 5 we perform tests using 19 user classes and 2 vehicle-types.

The following notation is also used in the paper:

- l_a is the length of link a.
- L_{mr} is the length of path r for OD-pair m.
- Γ_{mr} is the set of links constituting path r for OD-pair m.
- $\Phi\left(\{c_{mr} : r \in \tilde{R}_m\}\right)$ is the mapping function used in the RSUET definition, specifying a criterion to be fulfilled by unused paths.
- $\Omega\left(\left\{c_{ms}(\mathbf{x}) : s \in \tilde{R}_m\right\}; \varsigma_m\right)$ is the mapping function used in the RSUET definition, specifying a criterion to be fulfilled by used paths.
- z is the number of zones.
- $P_{mr}\left(\mathbf{c}(\mathbf{x}) \middle| \tilde{R}_m\right)$ is the proportion of flow on OD movement m that uses path r among the alternatives in the restricted set of utilised paths $\tilde{R}_m$ for OD-pair m:

  $$P_{mr}(\mathbf{c}(\mathbf{x}) | \tilde{R}_m) = \Pr\left(-\theta \cdot c_{mr}(\mathbf{x}) + \xi_{mr} \geq -\theta \cdot c_{ms}(\mathbf{x}) + \xi_{ms}, \forall s \neq r, s \in \tilde{R}_m\right) \quad \forall r \in \tilde{R}_m$$
- $c_{m,min}(\mathbf{x})$ is the cost of the minimum cost route in the set $\tilde{R}_m$.
- $\tilde{N}_{min}$ is a constant used in the proposed solution algorithm, expressing the minimum choice set size required to allow the removal of routes.
- K_{min} is a constant used in the proposed solution algorithm, expressing the iteration number from which routes are allowed to be removed from the choice sets.

2.2 MOTIVATION

Rasmussen et al. (2014b) proposed a RSUE solution algorithm and tested various variants of this on the well-known Sioux Falls network (the cases of RSUE(min) and RSUE(max)) as well as the large-scale Zealand network (the case of RSUE(min)). While being computationally tractable for both applications, the tests highlighted an issue which potentially poses a behavioural as well as computational challenge. The issue arises in cases where routes which may have been attractive (and thus generated to the choice sets) in earlier iterations become highly unattractive at equilibrium. Such unattractive routes are not removed from the equilibrated choice sets.

The computational challenge of this is not so problematic for the RSUE(min) application, as the equilibrium conditions require that no unused routes are cheaper than the *cheapest* used route (i.e. the included unattractive path has no influence on the cost condition to be fulfilled by unused routes). The included but highly unattractive route will however potentially induce a large computational challenge for large-scale application of the RSUE(max); the conditions require all routes cheaper than this unattractive route to be enumerated. This causes the choice sets to continuously grow as the iterations of the proposed solution algorithm progress. The computational burden thereby increases considerably with the iterations by requiring to solve a k-shortest path problem for each OD-pair for increasingly large values of k. A total of z^2 k-shortest path problems thereby needs to be solved in each iteration, and each of these problems has a large calculation complexity of $O(k_m \cdot V \cdot (V \cdot \log(V) + A))$ (Cormen et al., 2009). Consequently, the proposed RSUE(max) solution algorithm will be very computationally demanding for large-scale applications. See Rasmussen et al. (2014b) for further details on this issue.

One could argue that the complication of the issue is restricted to the choice of solution algorithm, and that alternative algorithms could be sought. However, the issue may also lead to a behaviourally unrealistic distribution of costs on used routes. This is exemplified by Figure 1 and Figure 2, which report the relative generalised costs, at equilibrium, between each used route and the cheapest used route for the corresponding choice set on the Sioux Falls network (downloaded from Bar-Gera, 2013).

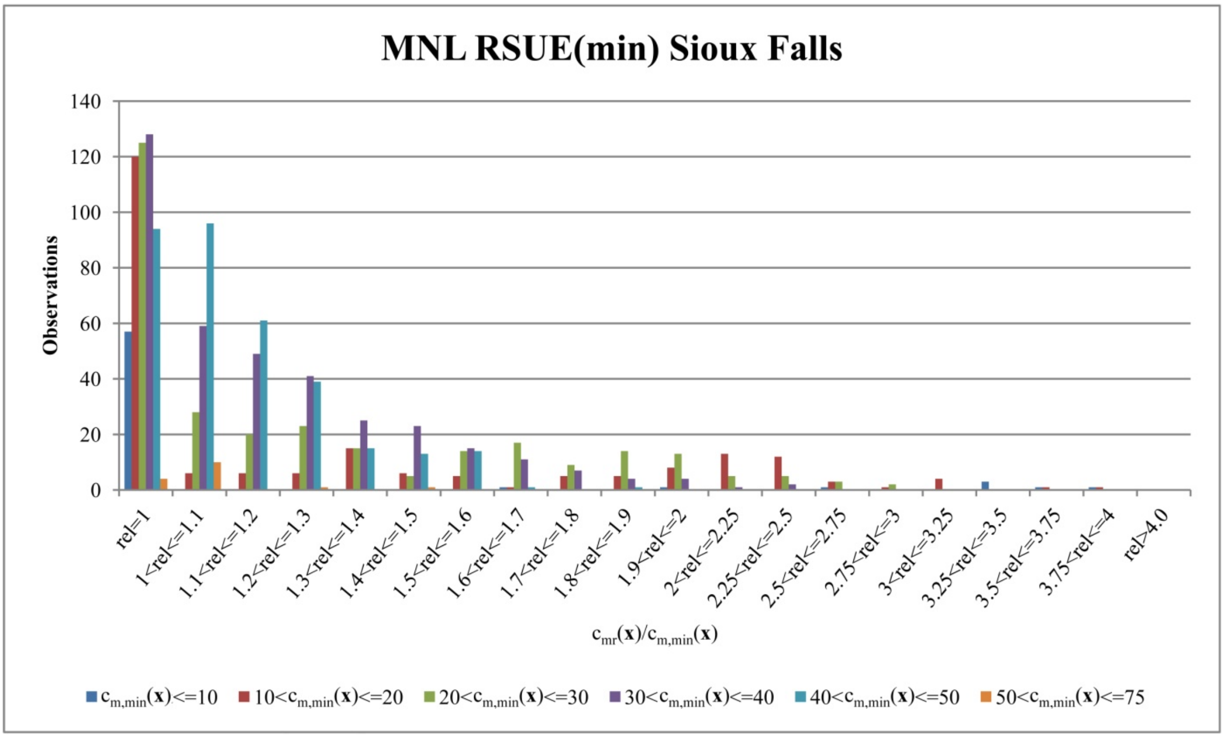

Figure 1 – Number of observations as a function of the ratio between the cost of a used route and the cost of the cheapest used route in the choice set for the MNL RSUE(min) application, Sioux Falls network. Grouped by the cost of the cheapest used route in the choice set

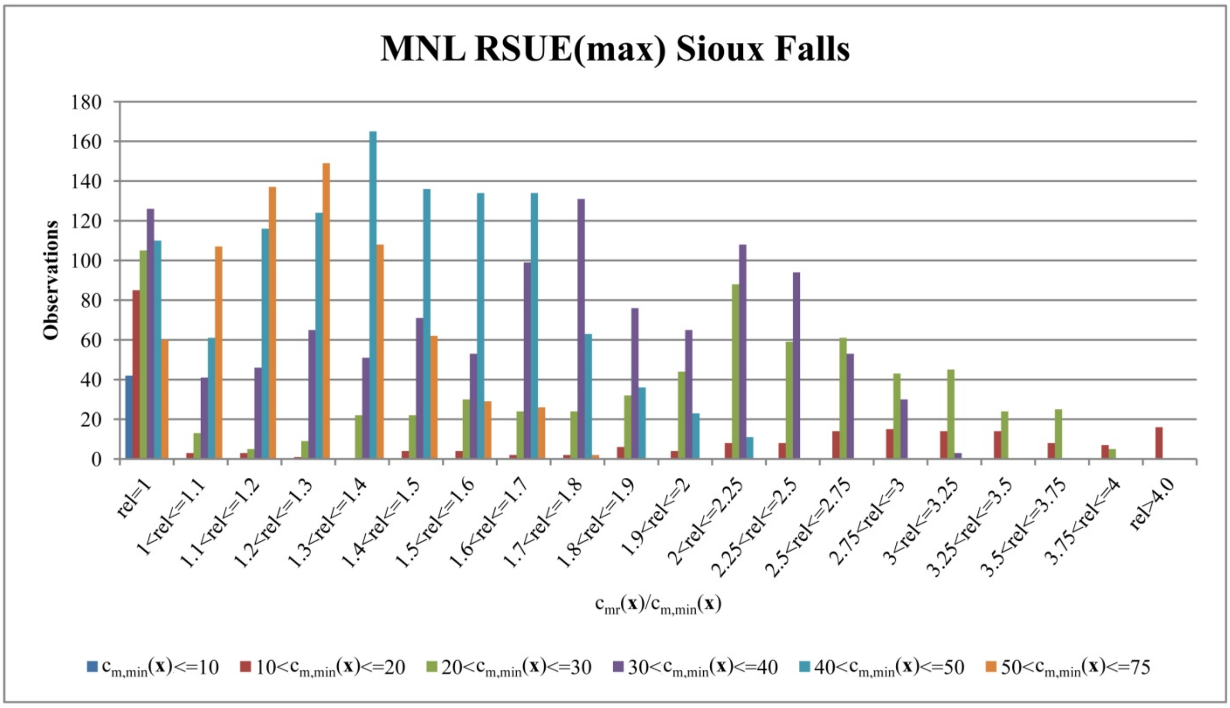

Figure 2 – Number of observations as a function of the ratio between the cost of a used route and the cost of the cheapest used route in the choice set for the MNL RSUE(max) application, Sioux Falls network. Grouped by the cost of the cheapest used route in the choice set

Some used routes, though not very much used, have a considerably higher generalised cost (e.g., 100-200% higher) than the cheapest route for the corresponding OD movement. This does not seem behaviourally justifiable, and we believe it arises from the fact that the conditions only pose a cost restriction on unused routes and not on the used routes; there are no conditions ensuring reasonability of the used routes, and there is no possibility to specify whether a small or large set of used routes is generally preferred. It is thus not an algorithmic problem only, but rather seems to stem from the lack of a mechanism in the underlying RSU conditions. Recall the definition of the RSU conditions (Watling et al., 2014):

Definition: Φ-Restricted Stochastic User Conditions (RSU(Φ))

For each OD movement m=1, 2,..., M:

i) *the proportion of travellers on any* <u>*used*</u> *path is equal to the probability that that path has a perceived utility greater than or equal to the perceived utilities on all alternative* <u>*used*</u> *paths;*

ii) *the 'reference cost' is a value uniquely defined by some relationship Φ to the actual travel costs on the* <u>*used*</u> *paths;*

iii) *the actual travel cost which would be experienced by a traveller on any* <u>*unused*</u> *path is greater than or equal to the reference cost as defined in ii).*

The definition specifies a reference cost to be applied to the *unused* routes (condition *iii)*). However, for used routes, the definition specifies a condition to be fulfilled regarding the distribution of flow, but does not specify a reference cost that can assist in determining whether a route should be used. This has the implication that there is no built-in condition to be used for dropping routes by the solution algorithms. A definition of such a condition to be fulfilled by *used* routes could be to use the same reference cost as specified for the *unused* routes by condition *iii)*. Such a specification would however be less useful; for example, for RSU(min) this would cause all but the shortest route(s) to be excluded, and for RSU(max) no effect would be seen. Rather than using the same reference cost, the next section proposes a new set of altered RSU conditions which applies two reference costs, one to be fulfilled among *used* routes and one to be fulfilled among *unused* routes.

2.3 DEFINITION OF THE RESTRICTED STOCHASTIC USER WITH THRESHOLD CONDITIONS

We extend the definition of RSU(Φ) by adding an additional function with a vector of OD-specific parameters, which defines a second type of reference cost, namely one which bounds the actual cost of used routes. After defining the general case and the corresponding equilibrium model below, we discuss some logical examples of this function.

Definition: Restricted Stochastic User with Threshold conditions (RSUT(Φ,Ω))

For each OD movement m=1, 2,..., M:

i) the proportion of travellers on any used path is equal to the probability that that path has a perceived utility greater than or equal to the perceived utilities on all alternative used paths;

ii) the 'external reference cost' is a value uniquely defined by some relationship Φ to the actual travel costs on the used paths;

iii) the actual travel cost which would be experienced by a traveller on any unused path is greater than or equal to the external reference cost as defined in ii);

iv) the 'internal reference cost' is a value uniquely defined by some relationship Ω to the actual travel costs on the used paths;

v) the actual travel cost on any used path is less than or equal to the internal reference cost as defined in iv).

Note: The words 'internal' and 'external' refers to the choice set, in that the internal costs are used to bound costs on route inside the choice set (chosen routes) and external costs are used for those routes outside the choice set.

Based on these conditions we can then define the corresponding equilibrium model:

Definition: Restricted Stochastic User Equilibrium with Threshold (RSUET(Φ,Ω))

Suppose that we are given a collection of continuous, unbounded random variables $\{\xi_{mr} : r \in R_m, m = 1,2,...,M\}$ *defined over the whole choice set* R_m; *and that for any* non-empty *subsets* $\tilde{R}_m$ *of* R_m *(m = 1,2,…,M), probability relations* $P_{mr}(\mathbf{c}(\mathbf{x})|\tilde{R}_m)$ *are given over* $\tilde{R}_m$ *(m = 1,2,…,M) by considering the relevant marginal joint distributions from* $\{\xi_{mr} : r \in R_m, m = 1,2,...,M\}$. *Given Φ and Ω then the route flow* $\mathbf{x} \in G$ *is a RSUET*(Φ,Ω) *if and only if for all r* ∈ R_m *and m* = 1,2,…,*M:*

$$x_{mr} > 0 \quad \Rightarrow \quad r \in \tilde{R}_m \quad \wedge \quad x_{mr} = d_m \cdot P_{mr}\left(\mathbf{c}(\mathbf{x})\middle|\tilde{R}_m\right) \quad \wedge \quad c_{mr}(\mathbf{x}) \le \Omega\left(\left\{c_{ms}(\mathbf{x}) : s \in \tilde{R}_m\right\};\varsigma_m\right) \tag{5}$$

$$x_{mr} = 0 \quad \Rightarrow \quad r \notin \tilde{R}_m \quad \wedge \quad c_{mr}(\mathbf{x}) \ge \Phi\left(\{c_{ms}(\mathbf{x}) : s \in \tilde{R}_m\};\xi_m\right) \tag{6}$$

where $\Phi\left(\{c_{ms}(\mathbf{x}) : s \in \tilde{R}_m\};\xi_m\right)$ and $P_{mr}\left(\mathbf{c}(\mathbf{x})\middle|\tilde{R}_m\right)$ are defined as in section 2.1. The function $\Omega\left(\left\{c_{ms}(\mathbf{x}) : s \in \tilde{R}_m\right\};\varsigma_m\right)$ is exogenously defined and specifies one threshold value (internal reference cost) per OD movement. In the definition above, the Ω-function is specified in a way that enables it to be formulated in numerous different ways. This could e.g. be an absolute non-negative threshold, a relative threshold relative to the minimum cost used route, or a combination. In the application we consider the following threshold function:

$$\Omega\left(\left\{c_{ms}(\mathbf{x}) : s \in \tilde{R}_m\right\};\tau_m\right) = \tau_m \cdot \min\left\{c_{ms}(\mathbf{x}) : s \in \tilde{R}_m\right\} \tag{7}$$

where $\tau_m \ge 1$. As $\tau_m \to \infty$ the condition v) in the RSUT(Φ,Ω) conditions becomes redundant, and so in this way the RSUT(Φ,Ω)) conditions are a generalisation of the RSU(Φ) conditions. When $\tau_m = 1$ for all *m*=1, 2,..., *M*, then a solution of the RSUET is also a DUE solution.

The choice of the thresholds in Ω causes these to have more or less influence on the solutions. We can either choose to have relatively low computational costs with relatively few used routes (and therefore a strong effect of the threshold), or to enable the inclusion of more used routes (and less effect of the threshold) at a higher computational cost. Choosing a high threshold, it poses not so much a behavioural parameter as much as a way of controlling the computation time of the algorithm by allowing it to drop routes that become highly costly (and little used). On the other hand, with a lower threshold the parameter is given more behavioural weight, since a low value will cause the exclusion of some routes with moderate cost (threshold will decide that these are unlikely to be used).

3 RSUET SOLUTION METHODS

3.1 RSUET(Φ,Ω) SOLUTION ALGORITHM

We introduce a generic solution algorithm for the RSUET by adapting the RSUE solution algorithm proposed in Rasmussen et al. (2014b). The adaptation is done in a way that allows it to account for the additional threshold condition. An additional step is added which checks for the fulfilment of the additional cost threshold and removes violating routes, if relevant. An iteration of the proposed solution algorithm consists of 4 steps, namely the *Column generation phase*, the *Restricted master problem phase,* the *Network loading phase* and the *Threshold condition phase*:

<table>
<tr><td>Algorithm</td><td colspan="3"></td></tr>
<tr><td>Step 0</td><td colspan="3">Initialisation. Iteration n=1. Perform deterministic all-or-nothing assignment for all m=1, 2,..., M OD-pairs and obtain the flow vector for all utilised paths $\mathbf{X}_n$.
Perform network loading, compute link travel costs $t_a\left(\mathbf{f}_n\right)$ on all network links $a \in A$, and compute generalised path travel costs $c_{mr}\left(\mathbf{X}_n\right)$. Set n=2.</td></tr>
<tr><td rowspan="2">Step 1</td><td colspan="3">Column generation phase. Let $k_{m,n-1}$ denote the current number of unique paths in the choice set of used paths for OD-pair m=1, 2,..., M in iteration n-1.</td></tr>
<tr><td>For RSUET(min, Ω):
For each origin, perform a shortest path search to all destinations based on actual link travel costs</td><td>For RSUET(Φ, Ω):
For each OD-pair m=1, 2,..., M, based on actual link travel costs $t_a\left(\mathbf{f}_{n-1}\right)$, check for a new route to add to the choice set $\tilde{R}_{m,n}$ by</td><td>For RSUET(max, Ω):
Perform $k_{m,n-1}$ - shortest path search for each OD-pair m=1, 2,..., M based on actual link travel costs</td></tr>
</table>

	$t_a(\mathbf{f}_{n-1})$. If for any OD-pair m=1, 2,..., M a new unique path r is generated, add it to the choice set with flow $x_{mr,n-1}$=0.	applying some path generation method which supports the fulfilment of the Φ criterion. If for any OD- pair m=1, 2,..., M a new unique path r is generated, add it to the choice set with flow $x_{mr,n-1}$=0; if several routes are possible, add only the shortest one.	$t_a(\mathbf{f}_{n-1})$. If for any OD-pair m=1, 2,..., M a new unique path r is generated among the k generated paths, add it to the choice set with flow $x_{mr,n-1}$=0; if several new unique paths are possible, add only the shortest one.
Step 2	*Restricted master problem phase*. Given the choice sets $\tilde{R}_{m,n}$ for all m=1, 2,..., M, apply the selected *inner assignment component* and *averaging scheme* to find the new flow solution $\mathbf{X}_n$.		
Step 3	*Network loading phase.* Perform the network loading to obtain $\mathbf{f}_n$ from $\mathbf{X}_n$. Compute the link travel costs $t_a(\mathbf{f}_n)$, the generalised path travel costs $\mathbf{C}(\mathbf{X}_n)$ and (if relevant/included) the path-size factors.		
Step 4	*Threshold condition phase*. Given the choice sets $\tilde{R}_{m,n}$ for all m=1, 2,..., M, check whether the threshold condition $c_{mr}(\mathbf{X}_n) \leq \Omega(\{c_{ms}(\mathbf{x}) : s \in \tilde{R}_m\}; \varsigma_m)$ is violated for any $r \in \tilde{R}_{m,n}$ for m= 1, 2,..., M. Remove relevant routes (maximum 1 route per OD-pair), redistribute the flow on routes removed among the remaining routes in the respective choice sets. If no routes have been removed for any of the M OD-pairs, continue. Else, perform the network loading, compute the link travel costs $t_a(\mathbf{f}_n)$, the generalised path travel costs $\mathbf{C}(\mathbf{X}_n)$ and (if relevant/included) the path-size factors.		
Step 5	*Convergence evaluation phase.* If the gap measure consisting of the sum of $Rel.Gap_n^{Stoch.}$ and $Rel.Gap_n^{Unused,Stoch.}$ is below a pre-specified threshold ξ, Stop[1]. Else, set n=n+1 and return to *Step 1.*		

[1] See section 4.3 for the computation of the two gap measures

Please note that the path flow vector is denoted by **X** rather than **x.** This is to emphasise that in practical implementations it is not possible/practical to operate with the vector **x**, as this requires enumerating the universal choice set for all OD-pairs to obtain the dimension of the **x** vector. Rather, in practical implementations, the dimension of the flow vector is not pre-specified, but is allowed to increase as the algorithm progresses. The same occurs for the path cost vector **c**(**x**), which we have denoted **C**(**X**) to highlight that this might grow as the algorithm progresses. The elements x_{mr} and c_{mr} thus refer to the vectors **X** and **C**, respectively.

3.2 THE *COLUMN GENERATION PHASE*, THE *RESTRICTED MASTER PROBLEM PHASE* AND *THE NETWORK LOADING PHASE*

The RSUET solution algorithm allows adapting the same procedures as the RSUE solution algorithm proposed in Rasmussen et al. (2014b). The algorithm is thus very flexible in its specification, allowing, for example, in the *Restricted master problem phase* the use of either standard path-based SUE solution methods or path-based DUE solution algorithms. The latter is only relevant for certain logit-type choice models, in which case the cost transformation functions introduced in Rasmussen et al. (2014b) should be used.

3.3 THE *THRESHOLD CONDITION PHASE*

The RSUET solution algorithm is flexible regarding the specification of how it might be ensured that the threshold conditions are fulfilled upon termination of the algorithm. There are several dimensions to consider in the specification, such as e.g. whether to (i) allow the exclusion of one or several routes per OD-pair per iteration, (ii) allow excluding paths from the initiation of the solution algorithm and for all iterations or e.g. only after 15 iterations and only every 5 iterations, (iii) require a minimum number of paths in the choice set to allow excluding routes, and (iv) redistribute flow among the remaining used paths according to the relative flows, the relative costs or using some other approach. One possible approach to the *Threshold condition phase* is introduced in the following:

Step 4.0	Set m=1
Step 4.1	For each route r in the choice set $\tilde{R}_{m,n}$, check whether the threshold condition $c_{mr}\left(\mathbf{X}_n\right) \leq$ $\Omega(\{c_{ms}(\mathbf{x}) : s \in \tilde{R}_m\}; \varsigma_m)$ is violated. If any route r violates this condition, and if $\tilde{N}_m \geq \tilde{N}_{min}$ and $n \geq K_{min}$, then flag the route that violates the threshold condition the most.
Step 4.2	If no route is flagged by *Step 4.1* and if $m<M$, set $m=m+1$ and return to *Step 4.1*. If no routes are flagged by *Step 4.1* and if $m=M$, continue to *Step 4.3*. If a route r is flagged by *Step 4.1*, remove the route from the choice set and redistribute flow $x_{mr,n}$ among the remaining currently-used routes s according to the following: $x_{ms,n} = x_{ms,n} + x_{mr,n} \cdot \dfrac{x_{ms,n}}{d_m - x_{mr,n}}$. If $m<M$, set $m=m+1$ and return to *Step 4.1*. If $m=M$, continue.
Step 4.3	If no routes have been removed for any of the M OD-pairs, continue. Else, perform the network loading, compute the link travel costs $t_a\left(\mathbf{f}_n\right)$, the generalised path travel costs $\mathbf{C}\left(\mathbf{X}_n\right)$ and (if relevant/included) the path-size factors.

We know that the link flows, and consequently also the route travel times, fluctuate much in the initial iterations, as algorithms move a large share of the flow towards the auxiliary solution in each iteration. This induces that a route just introduced to the choice set – and which potentially is highly attractive at convergence – may have a very high cost as this is assigned a lot of traffic. Applying the threshold condition at this stage would thus potentially remove this attractive route, with the result of having to add it again in the next iteration. In order to hinder this we allow the flow on routes to be 'smoothed' through conducting K_{min} iterations (e.g. 15 iterations) before the potential removal of any routes. A route which at iteration K_{min} is still unattractive will, due to the allowance of 'smoothing' the flows through K_{min} iterations and its high cost, carry a small share of the flow when removed. This leads to the additional benefit of the flow redistribution possibly not affecting the travel costs on the remaining routes so much (i.e. it will not 'destabilise' the solution algorithm). We also seek to avoid 'destabilisation' by additionally allowing a maximum of one route to be removed per choice set per iteration and by requiring that the choice set should contain a minimum of $\tilde{N}_{min}$ routes to allow the removal of routes.

4 LARGE-SCALE CASE STUDY: ZEALAND AREA

We used the large-scale case study of the Zealand Area to evaluate the RSUET model framework and demonstrate the applicability of several variants of the solution algorithm. This case study was also used in Rasmussen et al. (2014b). A main objective was to evaluate how large an impact the addition of the threshold condition has on the computation time and the equilibrium solution for different configurations of the model and for different network conditions (congestion levels). Among others, the evaluation used real life observed data to assess the realism of the solutions.

4.1 CASE STUDY

The case network covers the eastern part of Denmark (primarily Zealand). This area has approximately 2.5 million inhabitants. The network representation in the model consisted of 12,451 links (corresponding to 18,706 one-directional links in the network graph) being a geographically limited subset of the network used in the Danish National Model (Rich et al., 2010). The demand used stems from the Danish National Model, and the demand matrices included a total of 3.2 million daily trips done within, through, into and out from the study area. These trips were categorised in 19 different user classes and three vehicle types (car, van and lorry).

We also had access to a total of 16,618 GPS traces performed in car within the study area. These were utilised to perform a disaggregate evaluation of the algorithms' ability to reproduce observed route choices; The origin and destination of each trip were added to the network and corresponding trips were appended to the demand matrix (with zero demand to not cause additional congestion in the network). The GPS data stem from two data-sources; 554 observations were collected in a person-based data collection in which travellers carried the GPS unit with them during all their travels (across modes of transport, see Rasmussen et al., 2014a). The remaining 16,064 observations were collected in a vehicle-based data collection among a sample of employees of the Municipality of Copenhagen. While the second source is richer in the number trips, the first also contains information on the personal characteristics of the car drivers.

4.2 TESTED ALGORITHM

Several variants of the solution algorithm proposed in section 3 were implemented into the C#-based Traffic Analyst software package (Rapidis, 2014). This software package also contains a (possibly mixed) link-based multinomial probit MSA solution algorithm (denoted MNP SUE and mixed MNP SUE in the remainder of this paper).

The tests focused on the RSUET(min, $\mathbf{\Omega}$) formulation rather than the RSUET(max, $\mathbf{\Omega}$). The RSUET(max, $\mathbf{\Omega}$) is a lot more computationally demanding, as (i) z^2 k-shortest path searches are required to cover all OD-pairs, as opposed to z searches for the single shortest path method, and (ii) the k-shortest path search algorithm has a calculation complexity of $O(k_m \cdot V \cdot (V \cdot \log(V) + A))$ for each search, as opposed to $O(V \cdot \log(V) + A)$ for the single shortest path search method (see Rasmussen et al., 2014b). We implemented and did some initial tests of the k-shortest path algorithm. While managing to improve the computation time considerably compared to a first 'non-optimised' implementation, it still took approximately 2 seconds to compute the k=100 shortest paths between Rome and Copenhagen in the Transtools network (Rich et al., 2009). This has roughly the same size as the Danish National Model network, and the calculation time is about 100,000 times longer than the time required to compute the single shortest path between the same OD relations. Consequently, we did not manage to reach sufficiently low computation time levels to facilitate implementation in the iterative RSUET(max, $\mathbf{\Omega}$) algorithm, and believe that there is a need for further research to make the RSUET(max, $\mathbf{\Omega}$) operational for large-scale cases.

The tests of the RSUET(min, $\mathbf{\Omega}$) all utilise the threshold condition (7) for each m=1, 2,..., M, i.e. $\Omega\left(\left\{c_{ms}\left(\mathbf{x}\right): s \in \tilde{R}_m\right\}; \varsigma_m\right) = \Omega\left(\left\{c_{ms}\left(\mathbf{x}\right): s \in \tilde{R}_m\right\}; \tau_m\right) = \tau_m \cdot \min\left\{c_{ms} : s \in \tilde{R}_m\right\}$. We shall refer to this setup as the RSUET(min, $\boldsymbol{\tau}$·min) in the remainder of the paper.

The *Column generation phase* was thus based on single shortest path searches (see section 3.1). The implementation allowed the evaluation of two approaches in the *Restricted master problem phase*. One utilised the cost transformation functions introduced in Rasmussen et al. (2014b) (i.e. allows MNL and Path Size Logit (PSL) choice models) to identify an auxiliary solution using the pairwise path-swapping strategy described in Carey and Ge (2012) (referred to as the Path Swap variant). See Rasmussen et al. (2014b) for more information on the integration of the path-swapping strategy and the cost transformation functions. The second approach identified the auxiliary solution by directly using the closed-form MNL or PSL choice probability expressions (referred to as the Inner Logit variant).

The implementation facilitated the use of the method of successive weighted averages for the step-size (MSWA, Lin et al., 2009). While being pre-defined, the MSWA allows giving more weight to auxiliary flow patterns found in later iterations, defining the step-size γ_n at iteration n as:

$$\gamma_n = \frac{n^d}{1^d + 2^d + ... + n^d} \tag{8}$$

where $d \geq 0$ is a real number. The *Threshold condition phase* was conducted as outlined in section 3.3 using the threshold defined by (7).

4.3 EVALUATION CRITERIA

The MNL and PSL RSUET(min, τ·min) solution algorithms were evaluated in various ways. Firstly, convergence was evaluated using the two-part convergence measure proposed in Rasmussen et al. (2014b), consisting of a part measuring the convergence to satisfy the underlying choice model among the used routes and a part measuring the convergence to fulfil the criteria on unused routes:

$$Rel.gap_n^{Used} = \frac{\sum_{m=1}^{M} \sum_{r \in \tilde{R}_m} x_{mr,n} \cdot \left(x_{mr,n} \cdot \exp\left(\theta \cdot c_{mr}(\mathbf{x}_n)\right) - x_{\min\left(x_{mr,n} \cdot \exp\left(\theta \cdot c_{mr}(\mathbf{x}_n)\right)\right)} \cdot \exp\left(\theta \cdot c_{m,\min}(\mathbf{x}_n)\right) \right)}{\sum_{m=1}^{M} \sum_{r \in \tilde{R}_m} x_{mr,n} \cdot x_{mr,n} \cdot \exp\left(\theta \cdot c_{mr}(\mathbf{x}_n)\right)} \tag{9}$$

$$Rel.Gap_n^{Unused} = \frac{\sum_{m=1}^{M} d_m \cdot \left(\min_{\forall r \in R_m, x_{mr} > 0} \left(c_{mr}(\mathbf{x}_n) \right) - \min_{\forall r \in R_m} \left(c_{mr}(\mathbf{x}_n) \right) \right)}{\sum_{m=1}^{M} d_m \cdot \min_{\forall r \in R_m, x_{mr} > 0} \left(c_{mr}(\mathbf{x}_n) \right)} \tag{10}$$

It is important to note that the two gap-measures proposed above have been developed solely for closed-form logit-type choice models in the RSUE- and RSUET-model and can thus not be used to evaluate convergence of the link-based MNP SUE and mixed MNP SUE solution algorithms. There is not an equivalent *consistent* measure available for such algorithms, and most applications evaluate the convergence by using 'stability' measures. These measures do not evaluate the convergence to equilibrium directly, but rather the stability in solutions from iteration to iteration. One such measure is the link flow stability, weighed by flow and length:

$$Stability_n = \frac{\sum_{a=1}^{A} f_{a,n}(\mathbf{x}) \cdot l_a \cdot \frac{\left|f_{a,n}(\mathbf{x}) - f_{a,n-1}(\mathbf{x})\right|}{f_{a,n}(\mathbf{x})}}{\sum_{a=1}^{A} f_{a,n}(\mathbf{x}) \cdot l_a} = \frac{\sum_{a=1}^{A} l_a \cdot \left|f_{a,n}(\mathbf{x}) - f_{a,n-1}(\mathbf{x})\right|}{\sum_{a=1}^{A} l_a \cdot f_{a,n}(\mathbf{x})} \tag{11}$$

Secondly, it is also important to evaluate whether the different model setups generate route choice sets of reasonable sizes containing relevant routes leaving out non-sensible routes. We evaluated this by analysing the development of the choice set size and the ability to reproduce real-life observed route choices collected in the network. The latter is evaluated through a coverage measure, capturing the share of GPS observations for which the observed route is, at a certain overlap threshold, to be found among the routes in the corresponding generated choice set. The overlap between any observed route $r \in R_{obs}$ and any generated route $s \in \tilde{R}$ can be computed as (Ramming, 2002):

$$O_{rs} = \frac{L_{rs}}{L_r} = \frac{\sum_{a=1}^{A} \delta_{ar} \cdot \delta_{as} \cdot l_a}{\sum_{a=1}^{A} \delta_{ar} \cdot l_a} \tag{12}$$

where L_{rs} is the sum of length of overlapping elements between the observed path r and the generated path s. The overlap measure (12) can be computed for each generated path s for observation r, and let $O_{\max\{r\}}$ denote the highest overlap among the paths generated for observation r. Then the coverage using an overlap-threshold of λ (e.g. 80%) can be computed as (Ramming, 2002):

$$Cov(\lambda) = \frac{\sum_{r \in R_{obs}} I\left(O_{\max\{r\}} \geq \lambda\right)}{\left|R_{obs}\right|} \tag{13}$$

where $I(\cdot)$ is an indicator equal to 1 when the criteria is fulfilled and zero if not.

Combining the development in the choice set size and coverage, an efficient algorithm should generate a few routes inducing a high coverage level within the first few iterations. The size of the choice sets should then stabilise (at a small size), indicating that all relevant routes have been generated. Bekhor and Prato (2009) sought to combine these two components by proposing an efficiency index measure. This accounts for both the behavioural consistency (coverage) and computational efficiency (choice set size). The measure thus supplements the two-part analysis above, and the efficiency index (EI) of an algorithm can be computed as:

$$EI = \frac{1}{|R_{obs}|} \cdot \sum_{r \in R_{obs}} \left\{ \left[I\left(O_{\max\{r\}} \geq \lambda\right) + \left(1 - \frac{\tilde{N}_r - R_{rel,r}}{\tilde{N}_r}\right) \right] \Big/ 2 \right\} = \frac{1}{2} \cdot Cov(\lambda) + \frac{1}{2 \cdot |R_{obs}|} \cdot \sum_{r \in R_{obs}} \frac{R_{rel,r}}{\tilde{N}_r} \tag{14}$$

where $R_{rel,r}$ is the number of relevant routes of observation *r* and $\tilde{N}_r$ is the number of used routes for the OD-pair corresponding to observation *r*. The definition of the number of relevant routes is difficult to specify in real-life large-scale networks, but this study used $R_{rel,r} = 2$ for all observations as this was also used in Bekhor and Prato (2009).

1,169 counts were available of observed flow on links within the case-study area. We used these to analyse the realism of the flow distribution generated by the different algorithm variants.

Finally, the computational burden of the algorithms should also be evaluated. Other studies have found that the computational efforts required per iteration may vary greatly across different algorithm specifications and choice models (e.g., Rasmussen et al., 2014b). It is therefore important to not only evaluate the convergence as a function of the number of iterations, but to also consider the computational burden *per iteration* when evaluating the performance of an algorithm. We therefore also analysed the evolvement of the computation time per iteration across the algorithm variants and reported convergence etc. as a function of computation time rather than iteration number.

4.4 SPECIFICATION OF CHOICE FUNCTION AND PARAMETERS

The model was implemented as a multi-class model that allows distinguishing between different trip purposes and vehicle classes (categories). The utility (cost) function considered several variables, and the cost of alternative *r* on OD movement *m* was specified as:

$$c_{mr}(\mathbf{x}) = \beta_{FreeTT,m} \cdot t_{FreeTT,mr}(\mathbf{x}) + \beta_{CongTT,m} \cdot t_{CongTT,mr}(\mathbf{x}) + \beta_{l,m} \cdot l_{mr} + \varepsilon_{mr} \qquad (15)$$

where $\beta_{FreeTT,m}$, $\beta_{CongTT,m}$ and $\beta_{l,m}$ are the respective parameters associated with the free-flow travel time, congestion travel time and driving distance for the category associated with OD movement *m*. The distributed error term ε_{mr} expresses unobserved components and perception errors. The time-variables are measured in minutes, whereas all variables associated with length are measured in kilometres.

For the link-based MNP SUE and mixed MNP SUE, the error-term and (relevant only for the mixed MNP SUE) parameters associated to travel time were simulated from the gamma and the log-normal distribution, respectively. The parameters were simulated at OD-level to account for taste heterogeneity across individuals, whereas the error-term was simulated at link level per OD-pair. The mean of the error-term was zero, and the variance specified as proportional to the mean cost (using scale parameter $\beta_{m\varepsilon}$) to ensure consistent aggregation from link- to path-level (see Nielsen and Frederiksen, 2006).

The choice function consisted of an additional term for the PSL RSUET(min, τ·min)-application, seeking to account for the effect of path overlapping. The term $\beta_{PS,m} \cdot \ln\left(PS_{mr}\right)$ was added to the cost function (15), where $\beta_{PS,m}$ was a non-positive OD-specific parameter. PS_{mr} was defined as proposed by Ben-Akiva and Bierlaire (1999):

$$PS_{mr} = \sum\nolimits_{a \in \Gamma_{mr}} \frac{l_a}{L_{mr}} \cdot \frac{1}{\sum\nolimits_{k \in \tilde{R}_m} \delta_{amk}} \qquad (16)$$

where l_a and L_{mr} are measures of impedance on link *a* and on route *r* on OD movement *m*, and can either be measured as distance or cost ($l_a = t_a(\mathbf{f})$ and $L_{mr} = c_{mr}(\mathbf{x})$). Distance was used as a measure of impedance in the present application.

The parameter-values used were transferred directly from the multi-class link-based mixed MNP SUE model to be applied in the Danish National Model. No re-calibration was done to fit these to each of the RSUET solution algorithms tested, as the issue of parameter calibration and how this might be done in a consistent way for the RSUET framework is beyond the scope of this present work[2]. The multi-class assignment consisted of 19 user classes, and the parameters associated with each of these can be found in appendix 1.

Neither the parameters nor the error-terms are simulated in the RSUET(min, τ·min) application. This not only 'removes' the need for simulations, but also requires less parameters, as variances do not have to be specified. However, there was a need to specify the scale parameter θ_m, the threshold values τ_m as well as the algorithm step-size parameter d. The path-size parameter β_{PS} also needed to be specified when applying the PSL choice model. The selection of the parameters is a multidimensional optimisation problem, as e.g. the choice of the scale parameter may influence the optimal value of d. The case study applied a relative threshold value specifying that, for each OD-pair, no used route may be more than 20% more costly than the least costly used route (i.e. $\tau_m = \tau = 1.2$ for all m=1, 2, …, M). This value was not determined by addressing the multidimensional optimisation problem, but rather by using available data on observed path choices (section 5.1.1). Accordingly, we shall in the remainder of the paper refer to the RSUET(min, τ·min) as RSUET(min, 1.2·min). The study evaluated different values of the scale parameter θ_m and d, as shall be presented in section 5.2-5.3. The tests applying the PSL choice model used $\beta_{PS,m} = \beta_{PS} = -3$ for all m=1, 2,..., M. This value was adopted from Rasmussen et al. (2014b), who tested different values.

We only allowed to remove routes from a choice set if it contained at least $\tilde{N}_{min}$=2 routes. Subsequently, we verified that this did not give rise to unreasonably large fluctuations in flows when removing a route for any OD movement. In order for the flows to stabilise in the initial iterations before removing any routes, we additionally only allowed routes to be removed from iteration K_{min}=15 onwards.

[2] For a discussion on calibration issues for the RSUE, see Watling et al. (2014).

5 RESULTS

Several variants of the implemented algorithm were tested on different configurations of the network demand. We found that performing 100 iterations was sufficient to induce $Rel.gap_n^{Used}$ as well as $Rel.gap_n^{Unused}$ to reach a value below $1.3 \cdot 10^{-3}$ and $1.0 \cdot 10^{-12}$, respectively, for all applications. The analyses have been performed using both the Path Swap as well as the Inner Logit for the determination of the auxiliary flow solution in the *Restricted master problem phase* of the solution algorithm. In the paper, we only report the results obtained from the Inner Logit. Both approaches showed the same overall patterns, however with faster convergence of the variant for which the results are reported (as also found in Rasmussen et al. (2014b)).

The three parameters τ, d and θ are treated in a subsequent manner in the first three subsections. Section 5.1 sets off with the specification of the threshold based on observed data rather than based on the analysis of several alternative values. Section 5.2 addresses the specification of the step-size parameter, based on an analysis of the convergence patterns. The specification of the scale parameter was done based on several evaluation criteria, including the coverage, choice set size, efficiency index, link flow stability as well as the ability to reproduce observed link counts (section 5.3). Subsequently follows an analysis of the effect of correcting for path overlapping, and the section is ended by an analysis of the robustness towards the congestion level in the network.

5.1 THRESHOLD

5.1.1 DETERMINATION OF THRESHOLD FROM REVEALED CHOICES

The threshold specifies the maximum route cost, relative to the cheapest used path, on routes in order for them to be considered attractive by travellers. As mentioned, the threshold value is not determined from an optimisation routine, but rather from insights learned from analysing the choice of non-optimal paths in real-life observed route choices. Moreover, the threshold value was defined based on a comparison between costs on observed paths and costs on the corresponding minimum cost path. Figure 3 illustrates the cumulative share of observations as a function of the ratio between the cost on the observed path (path obtained from GPS data) and the cost on the minimum cost path between the corresponding locations. The observed paths were constituted by the 16,618 routes obtained from the GPS data. The cheapest path was found by, for each GPS trip, performing a shortest-path search in the congested network, between the origin and destination of the corresponding GPS trip. It is e.g. seen that 71% of the observed paths were less than 5% longer than the corresponding optimal path.

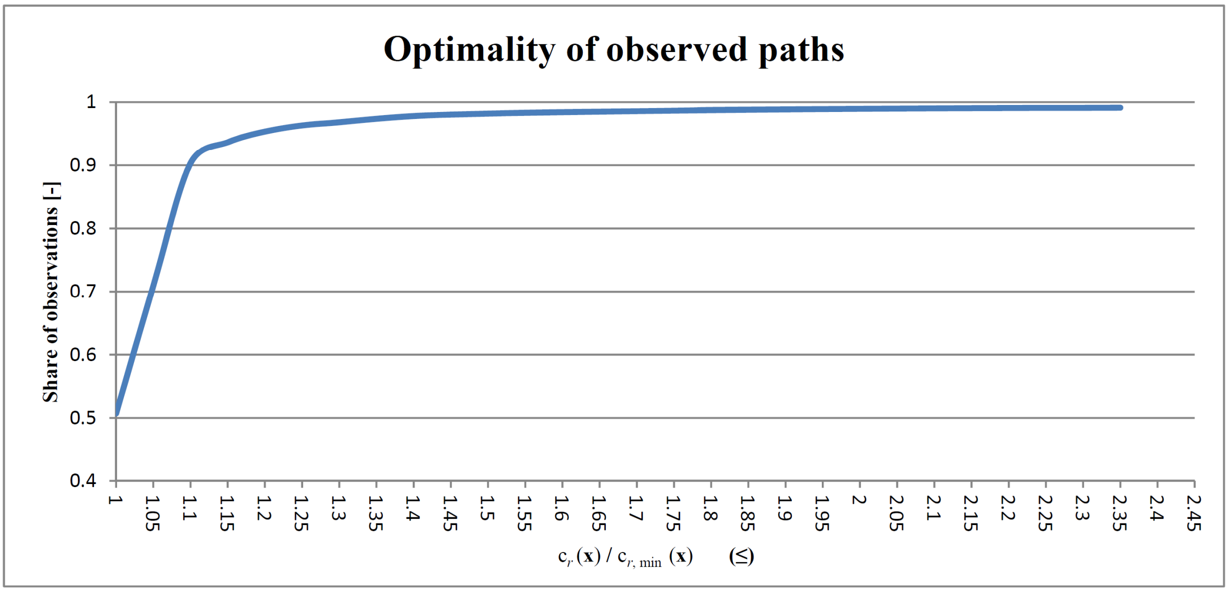

Figure 3 – Cumulative share of observations as a function of the ratio between the cost on the observed path r and the cost on the corresponding minimum cost path $c_{r,\,min}$ ***(x)***

The distribution of the ‘non-optimality’ of the observed routes is assumed to be representative of how (relatively) expensive paths have to be in order for the travellers not to consider and use them. We specified the threshold based on this: using a 95% interval induces a choice of τ=1.2 (i.e. the relative cost on 95% of all observed paths is within this threshold), which has then been used in the remainder of the paper.

5.1.2 Example of route exclusion, threshold condition

1,989 unique routes were removed by the threshold condition when using τ=1.2, d=4 and the MNL choice model with θ=0.2. Note, however, that the same unique path may have been generated and subsequently excluded several times during the iterations of the solution algorithm. This section presents an example of an OD movement (commercial business trip undertaken in van), for which a previously generated route was removed by the threshold condition at equilibrium.

Figure 4 illustrates the four unique routes generated (each of these has been the most attractive at some iteration), and Table 1 reports the corresponding equilibrium cost components, generalised cost and route flow share on each of these. All 4 routes were however not included in the equilibrated choice set, as flow was only distributed among paths 1, 2 and 4. Comparing the generalised costs, we see that Path 3 is considerably more expensive than the others. Accordingly, since this path is 32% more expensive than the cheapest path, the threshold condition removed it from the final choice set. We have verified that the flow distribution among the three remaining paths constitutes a MNL flow solution, and that the relative costs of these are below the threshold.

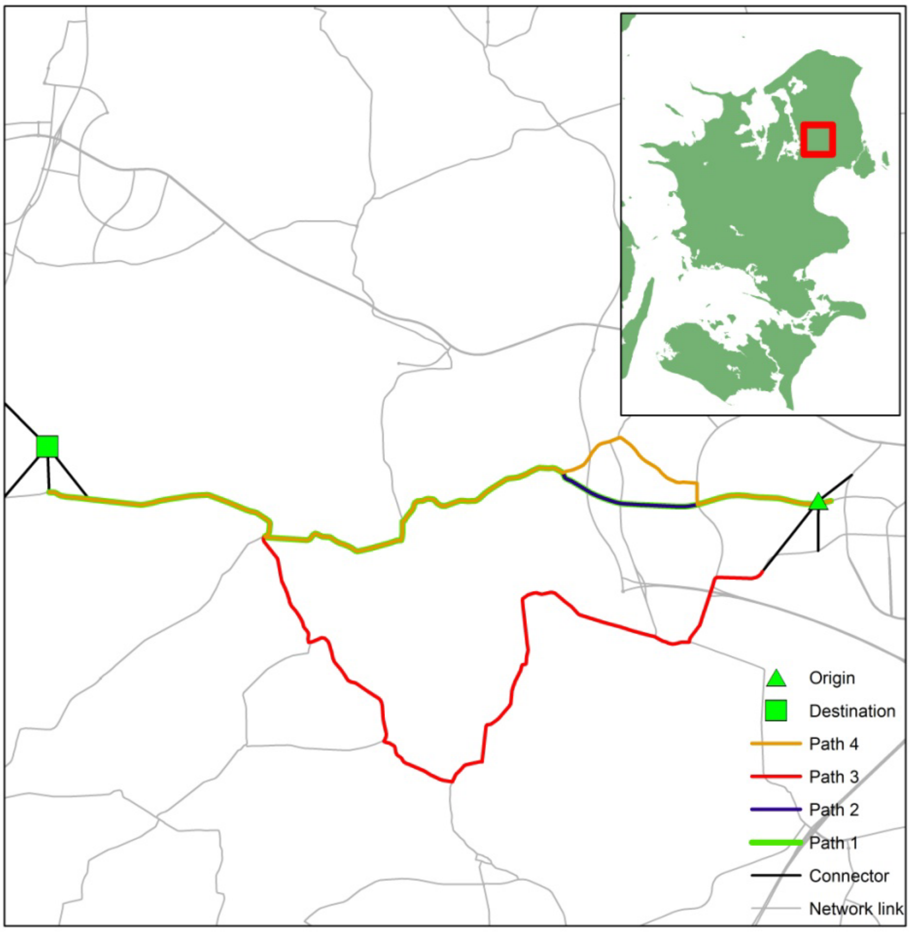

Figure 4 – Example of excluded route. 4 paths generated, but 3 utilised at convergence. MNL RSUET(min, 1.2·min), Zealand application

Table 1 – Specification of cost components, generalised costs, relative costs as well as flows at equilibrium. MNL RSUET(min, 1.2·min), Zealand application. l_{1r}, $t_{FreeTT,\ 1r}$ and $t_{CongTT,\ 1r}$ refer to the length, free-flow travel time and congested travel time of route r, respectively. $c_{1r}(\mathbf{x})$ and $c_{1,min}(\mathbf{x})$ refer to the cost on route r and the minimum cost across the used routes, respectively

Path	Category ID	l_{1r} [km]	$t_{FreeTT,\ 1r}(\mathbf{x})$ [min]	$t_{CongTT,\ 1r}(\mathbf{x})$ [min]	$c_{1r}(\mathbf{x})$	$c_{1r}(\mathbf{x})/c_{1,min}(\mathbf{x})$	Flow [%]
1	6	13.80	12.85	16.39	81.40	1.01	32.23
2	6	13.61	13.42	15.40	81.82	1.02	29.64
3	6	18.02	17.09	20.24	106.07	1.32	-
4	6	14.43	13.64	16.44	80.56	1.00	38.13

5.2 STEP-SIZE STRATEGY

The step-size parameter d specifies the ‘trust’ in the auxiliary solution and may thus influence the convergence speed (Rasmussen et al., 2014b). Posing a higher trust in the auxiliary solution may also lead to higher fluctuations in the path-flows between iterations, which may possibly cause additional/other paths to be attractive. The choice of d may thus influence not only the convergence speed, but also the solution in terms of the composition of the choice sets. The converged solutions should however all be RSUET solutions.

If the model parameters τ and θ are kept constant (τ=1.2, θ=0.2), the convergence measures can be directly compared across d-values for the RSUET and RSUE. Figure 5 and Figure 6 illustrate the convergence pattern of the MNL RSUET(min, 1.2·min) for different step-size strategies.

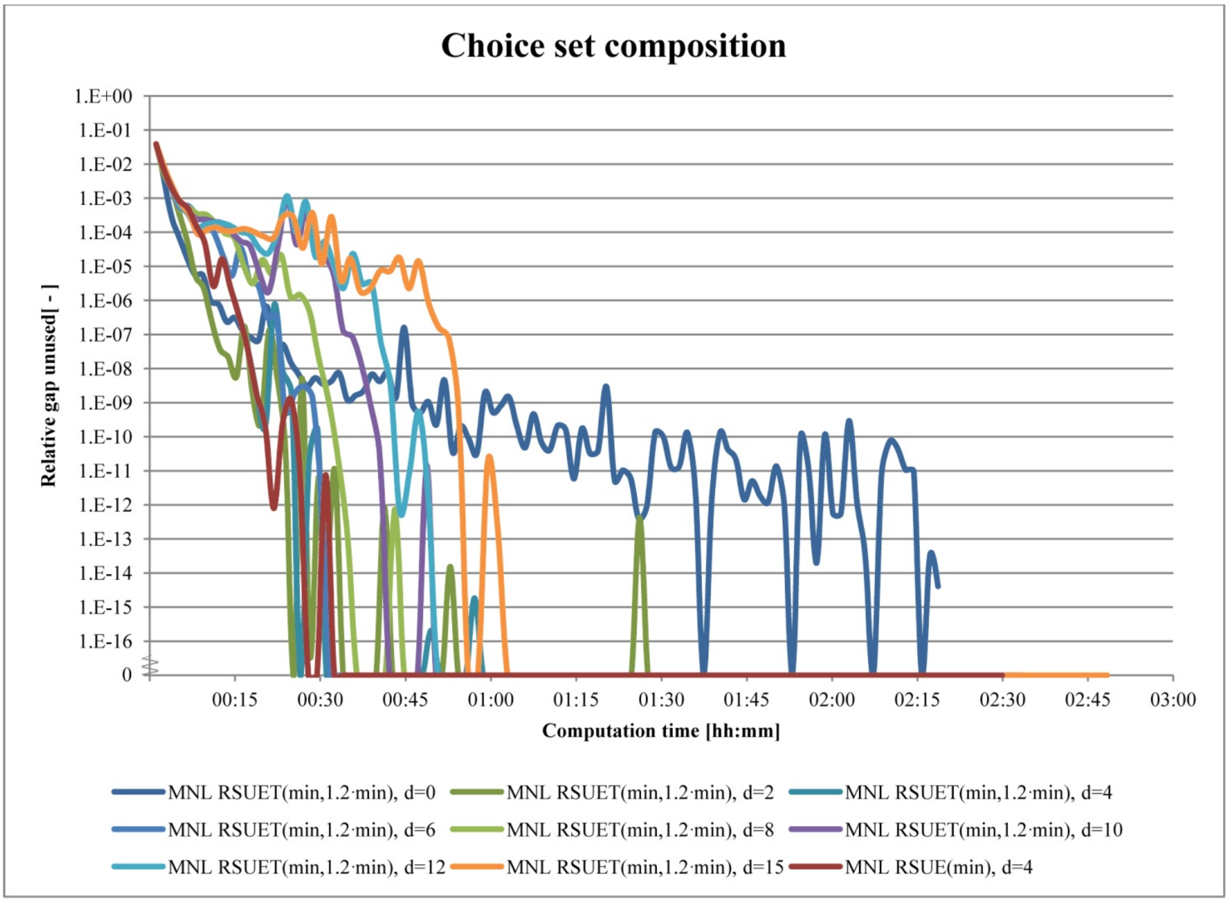

Figure 5 – Relative gap measure for convergence of choice set composition as function of computation time, Zealand application. MNL RSUET(min, 1.2·min) for various values of step-size parameter d as well as the MNL RSUE(min) with d=4. All with θ=0.2. Notice the log-scale on the vertical axis

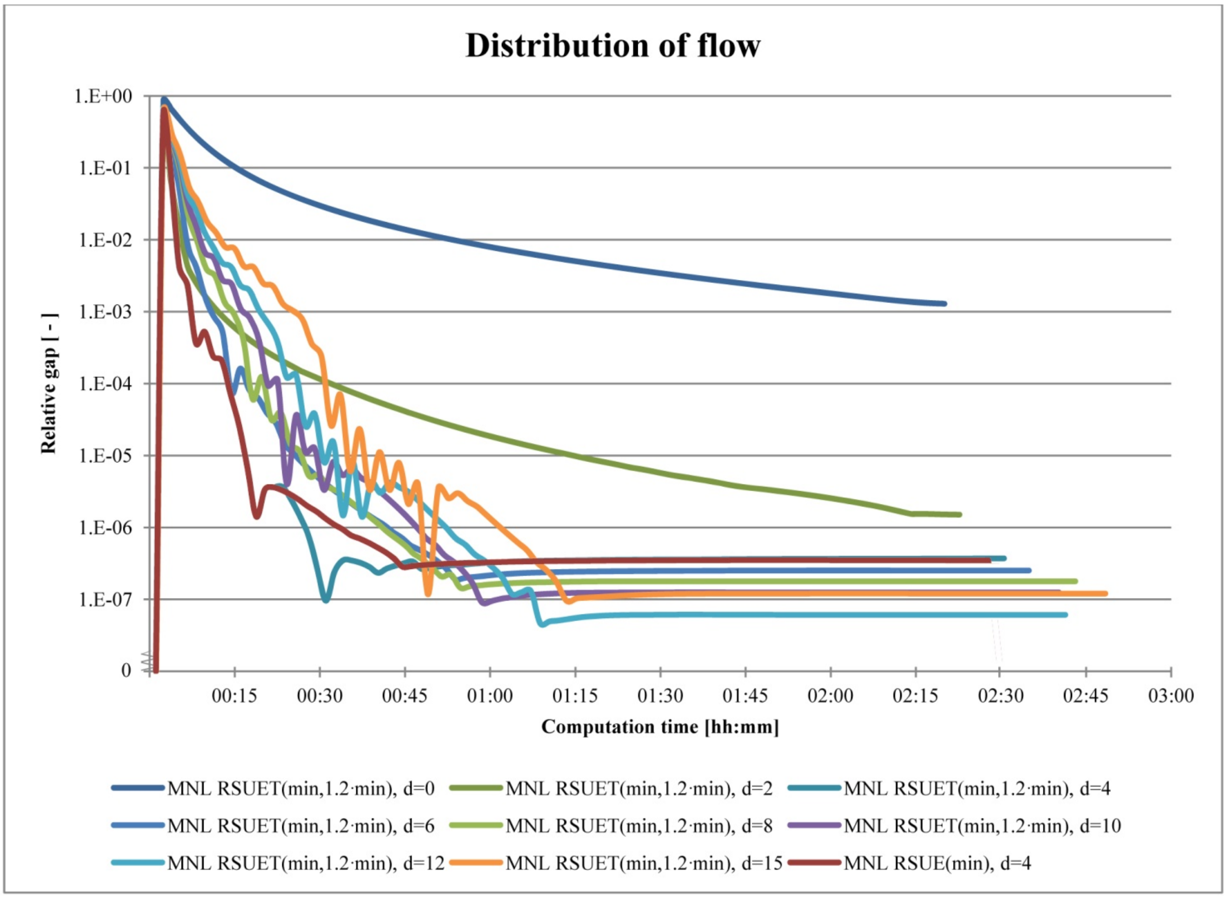

Figure 6 – Relative gap measure for convergence of flow distribution among routes in the choice set as a function of computation time, Zealand application. MNL RSUET(min, 1.2·min) for various values of step-size parameter d as well as the MNL RSUET(min) with d=4. All with θ=0.2. Notice the log-scale on the vertical axis

The choice set composition converged fast for all step-sizes, however with d=0 (MSA) being somewhat slower. Also the distribution of the flow among the paths in the choice set converged to a stable low level of approximately 1.0-3.5·10^{-7}, except for low values of d (d=0 and d=2) which were far from reaching this level at termination. Using d=4 caused the fastest convergence, as the final choice sets were generated within less than 30 minutes and the flow distribution converged within 35-40 minutes of calculation time. Consequently, the analyses presented in the remainder of the paper have been done using d=4.

We also evaluated the corresponding MNL RSUE(min) formulations. The convergence pattern of the RSUET(min, 1.2·min) was identical to that of the corresponding RSUE(min) application during the first 15 iterations. This seems reasonable since K_{min}=15. From iteration

15 onwards the convergence pattern was also very similar, converging to almost identical values of the relative gap measures. This is because only a very small share of the routes were removed by the threshold condition (e.g. 1,989 routes across 1,621,201 OD-pairs in the case of *d*=4). Consequently, in Figure 5 and Figure 6 we do not report the results for other applications of the RSUE(min) than the one using *d*=4.

The relative gap associated with the distribution of flow among paths did not seem to converge to zero, but rather stabilised at approximately $1\text{-}3.5\cdot10^{-7}$. This number is very low, and we do not see this stabilization to a non-zero value as an indication of the algorithm not converging, but rather an issue arising due to the limitations of the computer used; the relative gap is computed using exponential functions of the costs, which causes very small deviations to be amplified into large numbers. We performed a disaggregate analysis of the changes in flow and costs on routes between iterations when *d*=4. This showed that the average/maximum change in absolute cost and flow on the paths across all OD movements is a very low $2.9\cdot10^{-12}/2.3\cdot10^{-10}$ for cost and $6.2\cdot10^{-12}/1.0\cdot10^{-9}$ for flow. These numbers are at the limit of the C# software, and we expect the non-zero gap measure to be a consequence hereof.

5.3 SCALE PARAMETER

The scale parameter reflects the dispersion in the perception of costs among drivers. We note that a low value reflects large variation in the perception error of drivers (with complete 'random' allocation in the extreme case of $\theta \rightarrow 0$) and a high value reflects small variation in the perception error of drivers (with the limit of DUE when $\theta \rightarrow \infty$). Several different values of the scale parameter were tested, each application using the same value across all OD movements, i.e. $\theta_m=\theta$ for $m=1,2,\dots,M$. The relative gap measures were used to verify that all tests converged within reasonable computation time. The convergence measures can however not be compared across applications, as the scale parameter influences the relative gap measure. We therefore performed a series of alternative analyses to evaluate the performance of the solution algorithm for different values of the scale parameter. This also facilitated the comparison to the link-based MNP SUE and mixed MNP SUE solution methods.

1,169 observed link counts were available, and these were distributed throughout the case-study area. Figure 7 reports coefficient of determination (R^2) between the modelled and observed link counts. In general, very high correspondence was observed (all $R^2 \geq 0.942$), demonstrating that the RSUE/RSUET applications are successful in distributing the flow in a way that matches the observed counts. Only slight differences are seen between corresponding RSUE/RSUET applications, and the best performance was obtained when using θ=0.2. While the mixed MNP SUE performed better than the MNP SUE, it is prevailing that both MNP SUE applications performed considerably worse than all RSUE/RSUET applications in reproducing link counts.

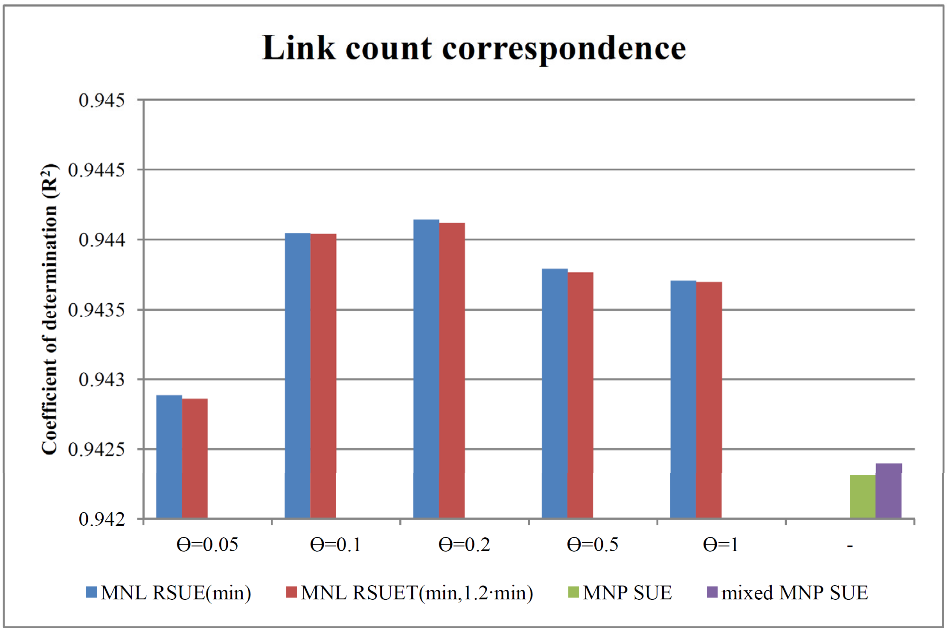

Figure 7 – Correspondence between modelled and observed link flows for various RSUE and RSUET configurations as well as the MNP SUE and mixed MNP SUE. Iteration 100, Zealand application

The analysis above showed good performance of the RSUE/RSUET on an aggregate level, by showing that these distribute flow in a way that reproduces link counts accurately. Moving to a disaggregate level, the solution algorithms should also be able to reproduce rational real-life route choices. We evaluated their ability to do so by using the 16,618 observed route choices collected via GPS, under the hypothesis that the observed routes should be represented in the corresponding choice sets generated. The coverage measure captures this, and Figure 8 reports the coverage measure as a function of the overlap threshold λ. We see decreasing coverage with increasing λ, as expected. Also, it can be seen that the 'relative' performance of the different θ values was somewhat the same across λ values.

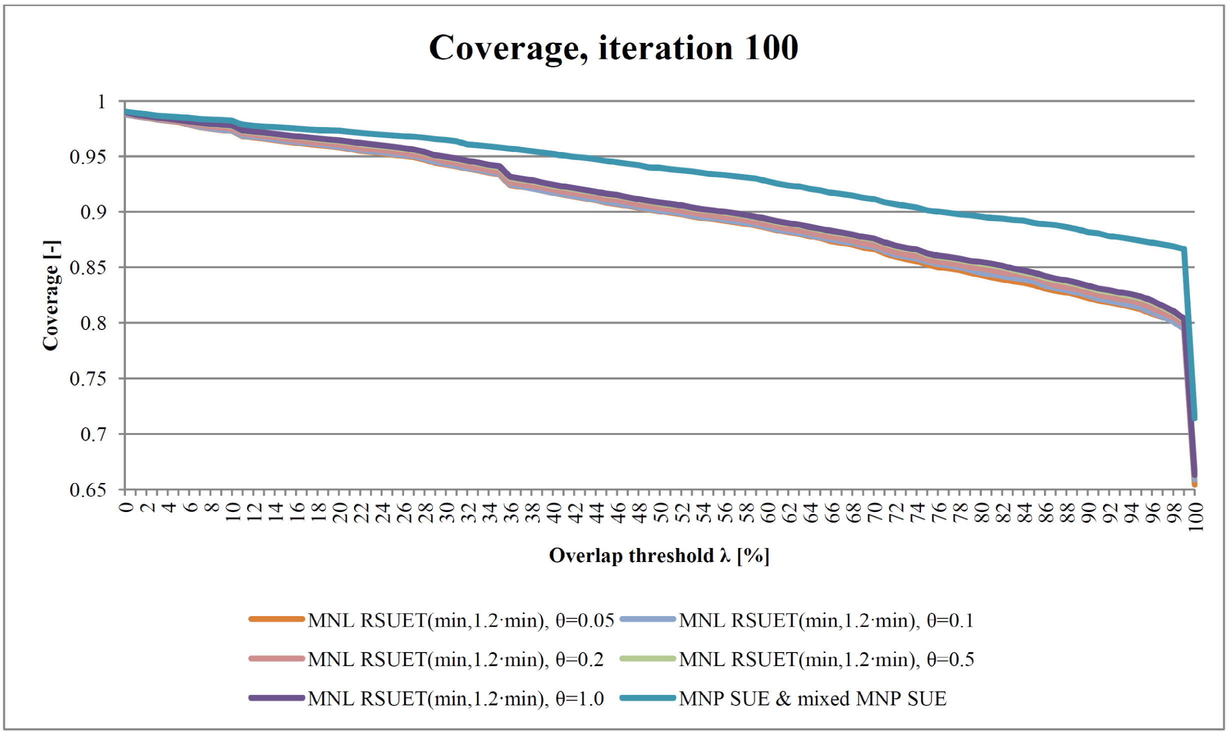

Figure 8 – Coverage as function of overlap threshold λ for various scale parameters in MNL RSUET(min, 1.2·min), iteration 100. Zealand application

Table 2 reports various characteristics of the solution generated, including the coverage obtained at iterations 25 and 100 when using a 80% overlap threshold. In general, high coverage levels were produced for all θ. It can be seen that adding the threshold on the relative costs does not seem to reduce the coverage for any of the chosen θ. This indicates that the paths removed by the threshold condition are in general non-relevant. Furthermore, the coverage seems to increase with increasing scale parameter. This increase is probably related to the larger fluctuations in flow in the initial iterations caused by the larger scale parameter; more weight is put on differences in costs (closer to DUE), leading to more 'extreme' auxiliary flows and thereby also larger fluctuations. These fluctuations cause more routes to be generated (seen through larger average choice set sizes) but also more routes to violate the threshold at equilibrium (and thus be removed, see Table 2). The number of paths removed were however at a very low level, considering that the network contains 1.6 million OD-pairs.

Table 2 – Coverage, choice set size, efficiency index and number of routes removed (when relevant) for various scale parameters in MNL RSUET(min, 1.2·min) and the MNL RSUE(min). The relevant measures are also reported for the MNP SUE and the mixed MNP SUE. Zealand application

		Coverage, λ=0.8		Choice set size			Efficiency index	Excluded paths
		Ite 25	Ite 100	Min.	Avg.	Max.		
θ=0.05	RSUE	0.8431	0.8431	1	2.364	10	0.9859	-
	RSUET	0.8431	0.8431	1	2.367	10	0.9859	1165
θ=0.1	RSUE	0.8452	0.8452	1	2.484	10	0.9733	-
	RSUET	0.8452	0.8452	1	2.484	10	0.9734	1180
θ=0.2	RSUE	0.8487	0.8487	1	2.696	13	0.9541	-
	RSUET	0.8487	0.8487	1	2.695	12	0.9543	1989
θ=0.5	RSUE	0.8535	0.8535	1	2.968	14	0.9335	-
	RSUET	0.8535	0.8535	1	2.967	13	0.9338	3784
θ=1.0	RSUE	0.8548	0.8548	1	3.059	13	0.9162	-
	RSUET	0.8548	0.8548	1	3.057	13	0.9165	4640
MNP SUE		0.8959	0.8959	1	14.894	100	0.6540	-
mixed MNP SUE		0.8959	0.8959	1	25.365	100	0.5460	-

The MNP SUE and mixed MNP SUE produced coverage levels which were considerably better than those of the RSUE and RSUET applications. This was however at the cost of generating large choice sets, which continued to grow without any clear tendency towards stabilisation. An average size of 37.0 routes was seen at iteration 200 for the mixed MNP SUE. The RSUE and RSUET on the other hand produce choice sets having a very computationally reasonable size, and which are equilibrated. The equilibrated choice sets were generated within a few iterations, which is also indicated by non-changing coverage from iteration 25 to iteration 100 (Table 2). The flow distribution also converged within a few iterations, highlighting that there is no need to perform many iterations to obtain an equilibrated RSUE/RSUET solution.

An efficient solution algorithm should produce a high coverage level while generating choice sets which are computationally attractive by containing only (a few) relevant routes. The efficiency index (14) captures this, and the RSUE/RSUET solution algorithms reached efficiency indexes ranging from 91.7% to 98.6%. The index for the RSUET is slightly better than the index generated by the corresponding RSUE formulations. This is due to the (slightly) smaller choice sets. The RSUE/RSUET solution algorithms generated significantly higher efficiency indeces than the MNP SUE and mixed MNP SUE. This highlights the weakness of the MNP SUE approaches, namely that they generated their high coverage levels at the cost of generating large choice sets.

The convergence pattern cannot be directly compared across θ-values, as mentioned earlier. In order to facilitate comparisons, the measures reported in Table 2 were supplemented by analyses of the link flow stability and the ability to reproduce observed link counts. This also facilitated the comparison to the MNP SUE and the mixed MNP SUE. It is however important to note that stability in link flows does not necessarily induce that an equilibrated solution has been found.

Figure 9 illustrates the link flow stability across iterations. We see very fast stabilisation in the link flows for all RSUE/RSUET applications. The effect of adding the threshold can clearly be seen, especially when $\theta \leq 0.2$, through a destabilisation of link flows at iteration 15 (~20min of computation time). Using $\theta=0.1$ or $\theta=0.2$ induces the best link flow stability. The stability of the MNP SUE and the mixed MNP SUE was considerably lower, indicating that convergence was not yet reached at iteration 100. This was also suggested by continuously increasing choice sets and is furthermore supported by a maximum relative deviation in link flow between iterations 99 and 100 of a very high 18.8%. This value was considerably lower for all the RSUET(min, 1.2·min) applications, e.g. $2.14 \cdot 10^{-5}$ % when $\theta=0.2$.

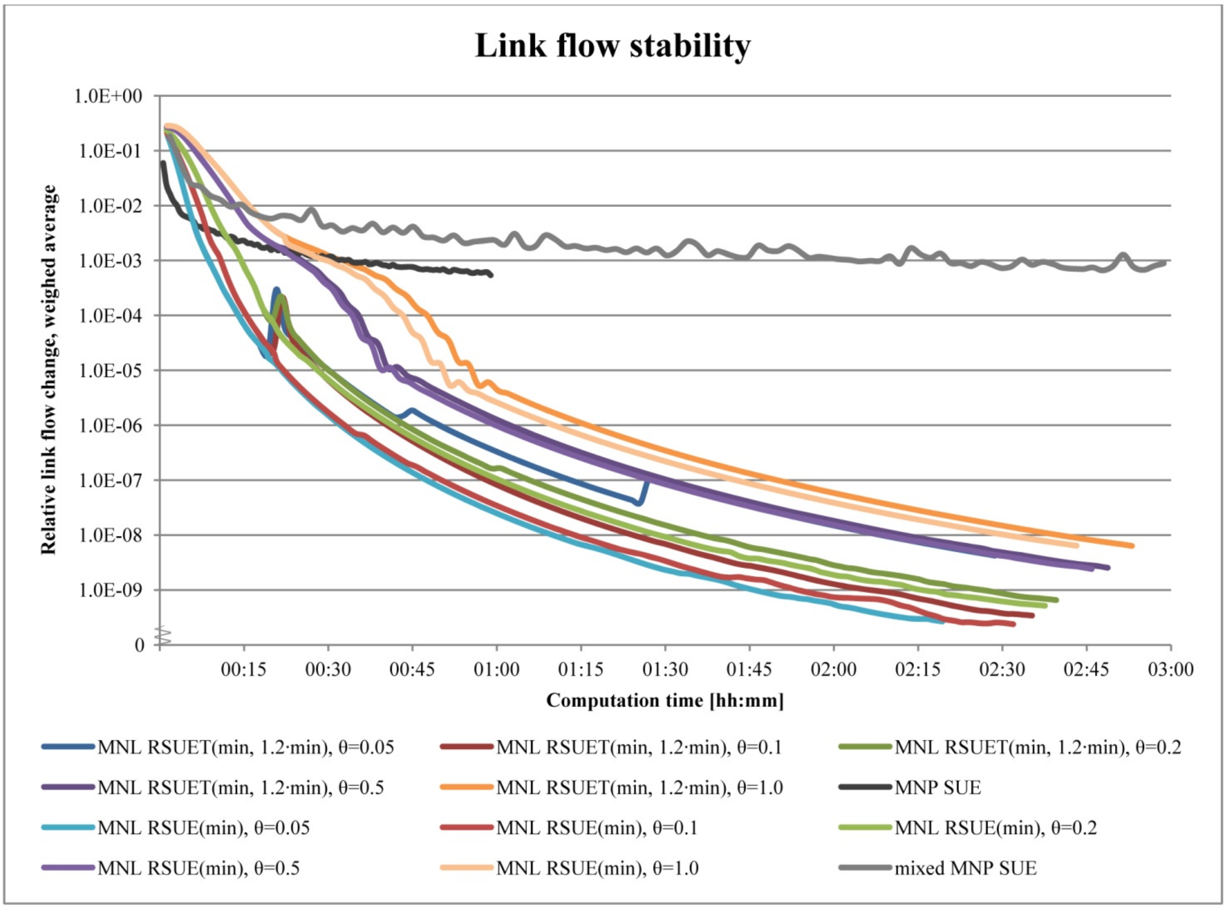

Figure 9 – Link flow stability across iterations, Zealand application. Notice the log-scale

Summarising, all tested values of θ produced good results for all evaluation criteria used. The best link count correspondence was however seen when using θ=0.2, and the analyses in the remainder of the paper have adopted this value.

We identified an issue related to the software implementation when performing the tests of alternative scale parameters on a previous version of the software. Using a large θ for OD-pairs with long costly routes, e.g. lorries travelling across the study area, caused the software to - erroneously - evaluate the associated exponential functions used in the Inner Logit flow allocation to a value of zero. I.e. the computer evaluates exp($-\theta \cdot c_{mr}$) as 0 if the product $\theta \cdot c_{mr}$ is large. Furthermore, the software evaluates the expression exp($\theta \cdot c_{mr}$) as *NaN* in the computation of the gap measures. In practice, we found this error to occur when $\theta \cdot c_{mr}$ was larger than approximately 750, indicating that this is the approximate limit of the C# software. In a new version of the software we resolved these issues by i) performing an all-or-nothing assignment to the shortest path in the initialisation of the algorithm (*Step 0*, iteration 1), and ii) subtracting a specific constant from the costs on all used paths when doing the Inner Logit assignment and the computation of the convergence measure. The all-or-nothing assignment avoids the evaluation of the exponential function. The subtraction 'moves' the costs to a range for which the exponential function can evaluate the values. The absolute differences in cost remain the same, thereby inducing no influence on neither the auxiliary solution nor the convergence measure. We defined the specific constant to subtract for each OD-pair to be the cost on the cheapest used path, enabling the computations to not fail before $\theta \cdot (c_{mr} - c_{m,\,min})$ increase above approximately 750. An alternative definition of the constant would be to define it as -750-$\theta \cdot c_{m,min}$, enabling the computations to not fail before $\theta \cdot (c_{mr} - c_{m,\,min})$ increase above approximately 1,500.

5.4 PATH OVERLAP CORRECTION

The MNL choice model fails to account for path overlapping. The study has also applied the PSL choice model to investigate the impact of accounting for this. This involved the specification of the parameter associated to the path-size correction factor. The identification of the optimal parameter value is not a one-dimensional problem, as e.g. the choice of a path-size parameter may influence the optimal value of θ and vice versa. The study did not seek to solve the resulting multidimensional optimisation problem. Rather, the PSL RSUET(min, 1.2·min) was applied for both d=0 and d=4, using θ=0.2 and $\beta_{PS,m} = \beta_{PS} = -3$. This parameter setting is assumed to be reasonable; section 5.3 found good performance when using θ=0.2 in the corresponding MNL RSUET and Rasmussen et al. (2014b) tested different values of β_{PS} for the PSL RSUE(min) on the same network (also using θ=0.2, but with d=2) and found best performance when $\beta_{PS} = -3$.

Equilibrated solutions were found, with convergence patterns almost identical to the pattern of the corresponding MNL application (and therefore not reported here). The same choice sets were generated across the choice models for almost all OD-pairs. This is supported by a difference in average choice set size of 0.001 and 0.002 routes when comparing corresponding applications across choice models for d=0 and d=4, respectively. The high similarity of choice sets seems reasonable, as the same path generation technique was used in the solution algorithm for the two choice models (deterministic shortest path search). The choice set composition however varied in a few cases. This was a consequence of the different flow distribution across the two choice models (due to the correction for path overlapping), which (in some cases) caused other routes to be attractive.

The similarity of the choice sets also led to almost identical coverage levels. Accounting for path overlapping does thus not improve coverage, but we however expect the distribution of flow among routes *in* the choice sets to be more behaviourally realistic for the PSL applications. Accordingly, while it is important to evaluate coverage (i.e. choice set composition), the evaluation of the distribution of flow among used paths is also important, as this may be more realistic for some choice models than others.

Figure 10 reports the computation time per iteration of the application of the MNL and PSL RSUET(min, 1.2·min) solution algorithms[3]. We see increasing computation time during the first iterations of the MNL RSUET(min, 1.2·min) applications. This seems reasonable and can be attributed to the path-based approach: The choice sets were generated within the first iterations, and storing an increasing number of paths in-memory and (re)distributing flow between these requires increasing memory and computational effort. The final choice sets were, generally, generated within the first 5-10 iterations when d=0 and d=4, and it is thus reasonable that we from this point on see stable computation times per iteration.

Quite different computation times in the initial iterations between the MNL and PSL applications were seen; the computation time of the MNL was strictly increasing until a certain level, whereas the computation time of the PSL increased rapidly in the initial iterations and then reduced to the level of the corresponding MNL application. This is directly linked to the computation of the path-size correction factors. Since these were based on overlap in length, they only need to be recomputed when a route is added to or removed from the choice set. The choice sets were formed in the initial iterations, and the path-size correction term thus had to be computed for many paths in these (the choice set changed for many OD movements and the correction terms had to be recomputed for all routes in each of these choice sets). This is

[3] The software allows writing intermediate output such as link and path flows/costs etc. per iteration. This is very time consuming, and to facilitate the best comparison possible we have disabled the writing of all output except the convergence log. The convergence log is not written while iterating but takes approximately ½ second to write upon termination (i.e. the convergence log is written once per application).

computationally expensive (especially as the number of routes in the choice sets grows) and explains the steep increase in computation time in the initial iterations. After a few iterations (iterations 4 and 6 for *d*=0 and *d*=4, respectively) new routes were generated for fewer OD movements, and fewer path-size correction terms thus had to be (re)computed. This reduced the computational effort. After the final choice sets were (more or less) generated at iteration 11, no further recalculation of path-size correction terms were needed. Therefore the computation time reduced to that of the corresponding MNL formulation.

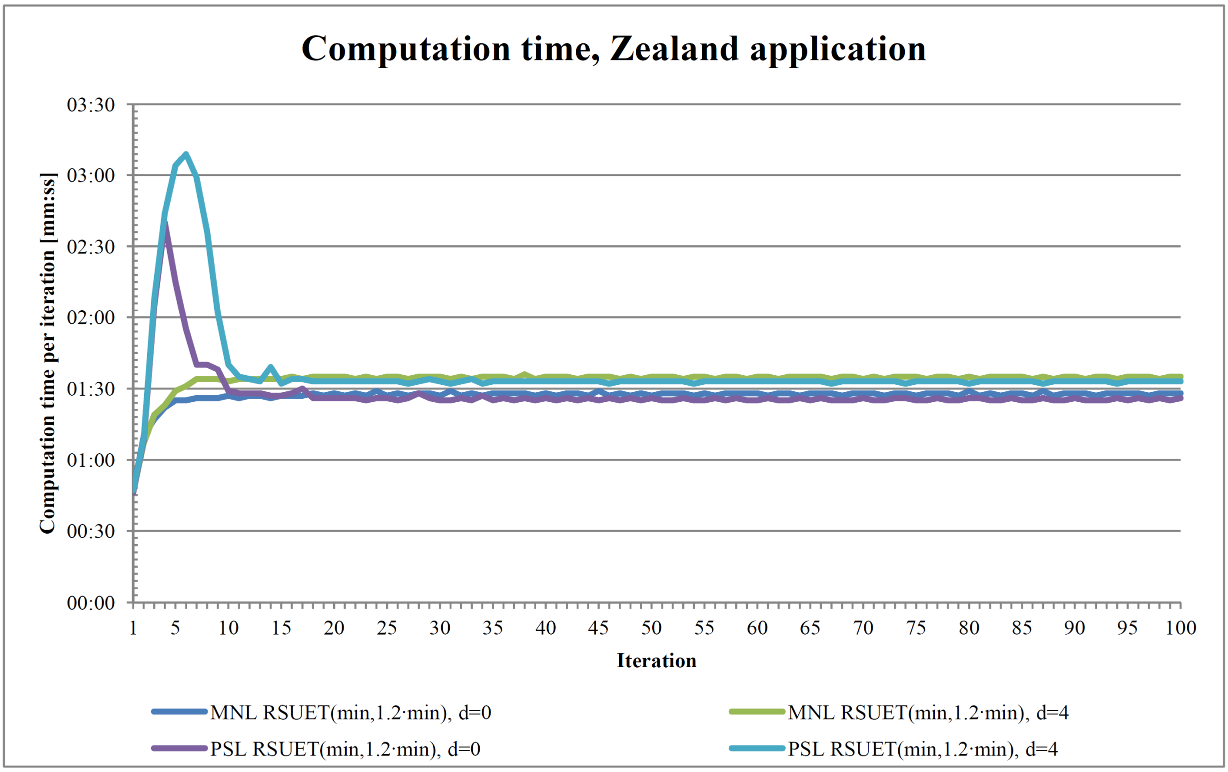

Figure 10 – Computation time per iterations for the MNL as well as PSL RSUET(min, 1.2·min) with d=0 and d=4. Zealand application

5.5 STABILITY TO CONGESTION LEVEL

The analyses above showed that the tested variants of the solution algorithm provide fast convergence to a stable solution which fulfils the RSUET(min, 1.2·min) conditions. However, good performance in the Zealand application does not guarantee good performance when applied to other case studies. One of the typical major challenges for solution algorithms is to also provide nice convergence patterns in high congestion real-life cases. We have applied the tested variant of the proposed solution algorithm with *d*=4 to a variety of scaled versions of the original demand matrices (the scale-factors tested are 1.25, 1.5, 1.75 and 2.0). This was done to test the robustness towards the general congestion level in the network. Figure 11 illustrates the volume-capacity ratio in the network links for the different demand levels.

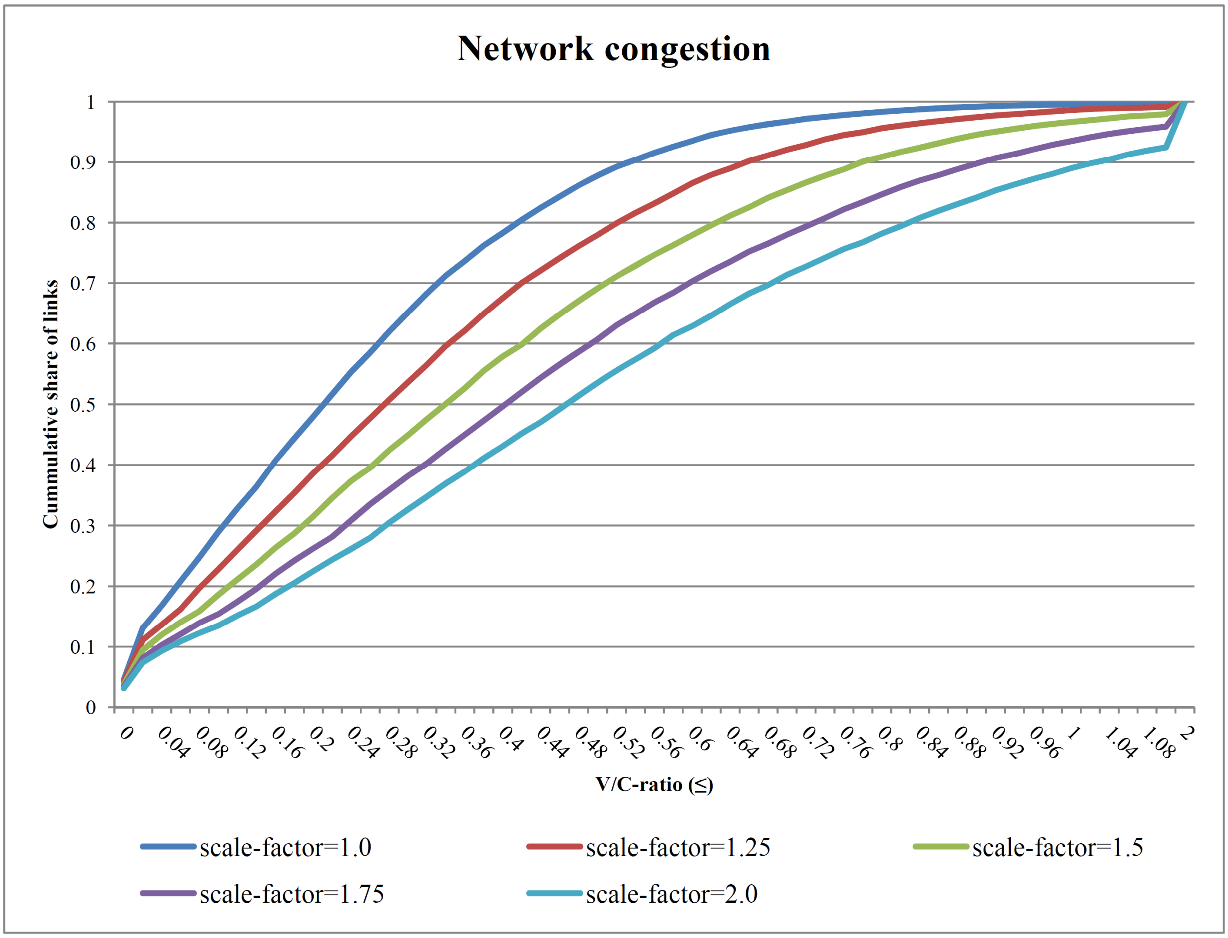

Figure 11 – Network congestion at various demand levels. Cumulative share of links as function of volume to capacity ratio, Zealand application

5.5.1 CONVERGENCE

Figure 12 and Figure 13 report the convergence measures for varying demand when performing 100 iterations. There was a clear tendency for slower convergence as the demand increased, both in terms of number of iterations needed as well as calculation time. However, a nice convergence pattern was seen for all the tested levels of demand. The travel times in the network fluctuated more in the initial iterations due to the larger demand which caused the choice set composition to require more iterations to converge and larger choice sets to be generated (as shall be highlighted in section 5.5.2). The higher fluctuations and travel time differences in the network also caused the distribution of flow among paths to require more iterations to converge for increasing demand levels, but even the highest congestion case (demand scale-factor 2.0) converged nicely once the final choice sets were generated. Longer calculation time to converge for increasing demand level is however not only due to the need for more iterations. The calculation time *per iteration* also increased, due to the larger choice sets and hence more paths to store in memory and assign traffic between. Consequently, the average calculation time per iteration was approximately 90/105/130/145/180 seconds for scale-parameter 1.0/1.25/1.5/1.75/2.0, respectively.

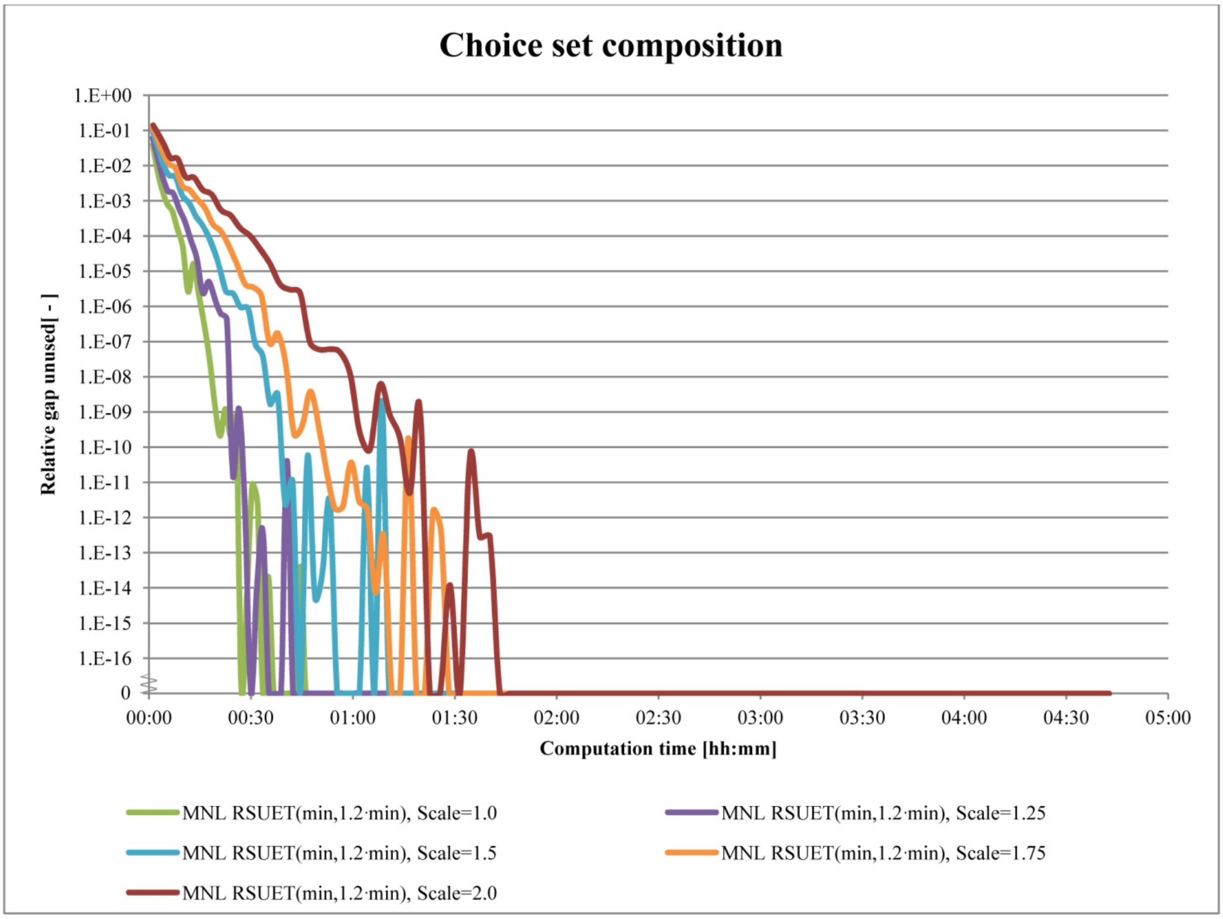

Figure 12 – Development of relative gap measuring convergence of the choice sets for various values of the factor scaling the demand, MNL RSUET(min, 1.2·min) with d=4, Zealand application

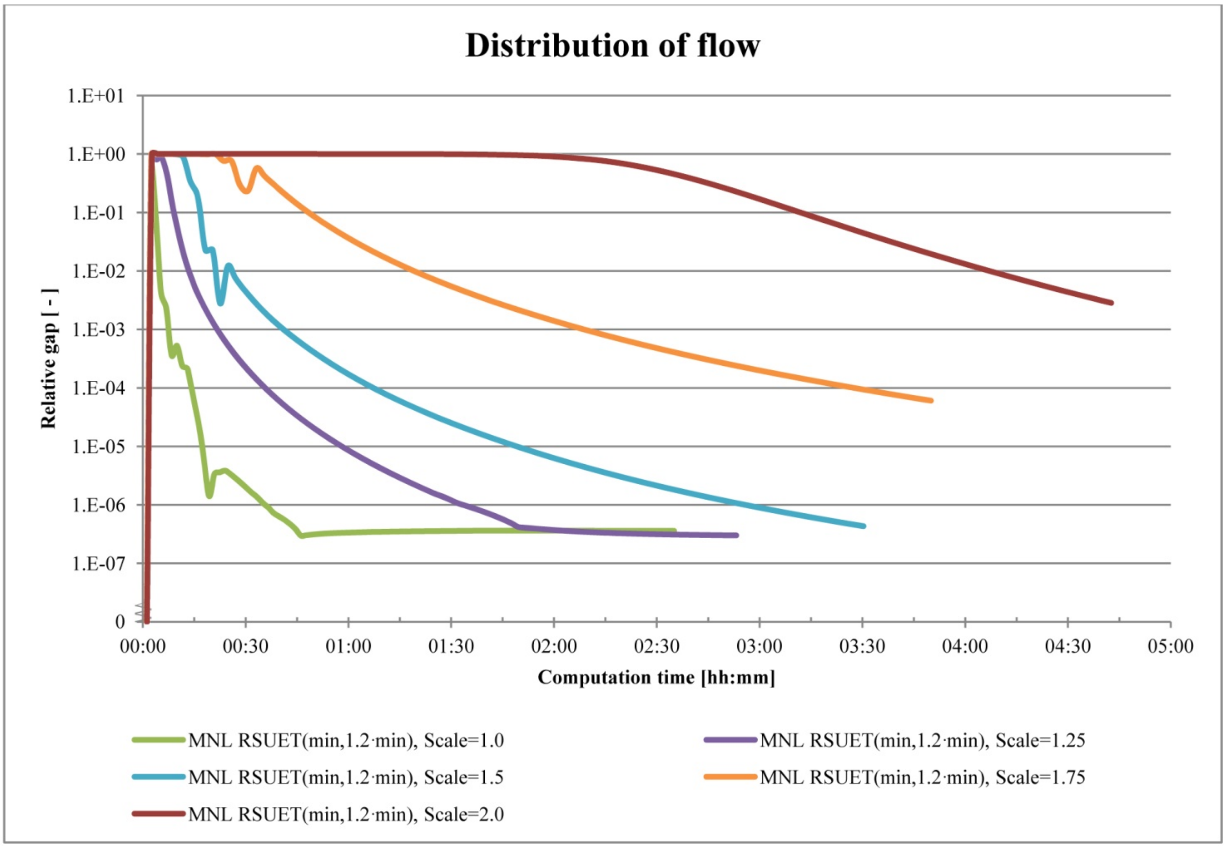

Figure 13 – Development of relative gap measuring convergence of the distribution of flow between paths for various 'scaled' demands, MNL RSUET(min, 1.2·min) with d=4, Zealand application

5.5.2 CHOICE SET SIZE, ROUTE EXCLUSION AND COST DISTRIBUTION

In the above, we found that more iterations were required for the choice set composition to converge when increasing the demand. This indicates that more routes – larger choice sets – were probably generated as the demand increased. Figure 14 verifies this. The average choice set size grew larger and required more iterations to stabilise when increasing the demand, but after iteration 13-30 (depending on demand level) no major changes of the average and maximum choice set size occurred. Furthermore, it can be seen that the choice sets had a very reasonable and computationally attractive size across all demand levels. For some movements only one route was generated, even for a very high demand (minimum choice set size was equal to 1 for all demand-levels, and is thus not reported in Figure 14). This also seems justifiable, since for some movements, such as e.g. neighbouring zones in rural areas, only one alternative may be reasonable, even at a high congestion level. We additionally note that even a doubling of the demand does not cause congestion on some (primarily rural) roads, as suggested by Figure 11.

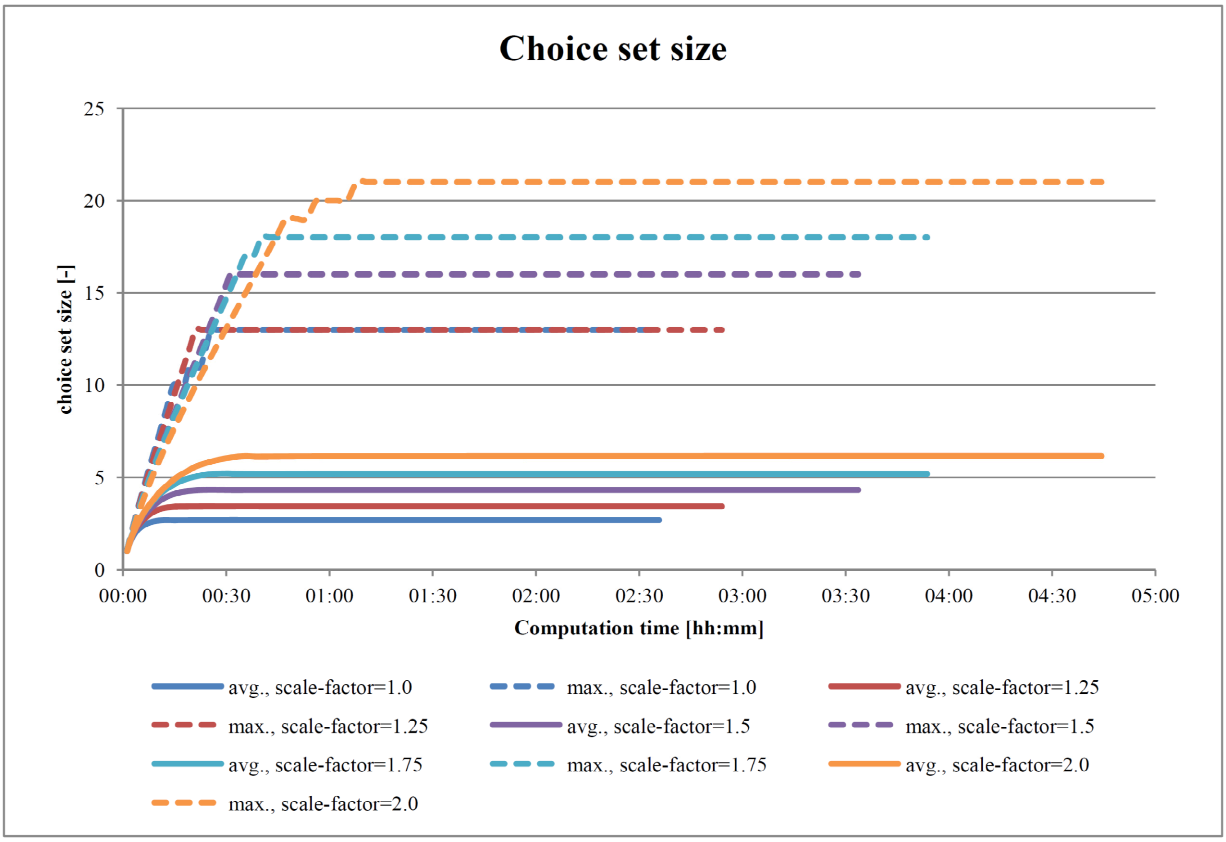

Figure 14 – Choice set characteristics for various values of the factor scaling the demand, MNL RSUET(min, 1.2·min) with d=4, Zealand application

Only a few routes violated the threshold condition by being more than 20% more costly than the cheapest path in the 'unscaled' Zealand application. *Step 5* of the solution algorithm did thus not remove many routes[4] - at termination only 1,989 unique routes had been generated and removed again from the choice sets[5]. The corresponding number was 10,744, 34,519, 85,478 and 160,192 routes when the demand scale factor was 1.25, 1.5, 1.75 and 2.0, respectively. The threshold condition thus removed more paths as the network congestion

[4] Note on implementation: Paths to be removed are not discarded/flushed from memory but rather flagged as 'inactive'. This is done because these might again become attractive in a later iteration, and 'reactivating' an inactive path requires far less computational effort than assigning a new route to memory.

[5] The same unique route may however have been introduced and subsequently removed again several times as the algorithm progressed.

increased, and one route was, on average, removed for each tenth OD movement when using a scale factor of 2.0. At this demand level the maximum number of unique paths removed for a single OD movement was 4 (this OD movement had 9 used paths in the resulting choice set at equilibrium). The increase in the number of paths removed for increasing demand seems reasonable, as link travel times fluctuate much more and thereby the route costs more 'easily' violate the threshold condition. The larger fluctuations in link travel times occur due to (i) the larger demand on OD-level, causing more flow to be reassigned in each iteration, and (ii) higher sensitivity to flow changes in the travel time functions when the general flow level is higher. We therefore also expect a larger variation on the relative costs among the routes left in the choice set at equilibrium. This is verified by Figure 15. From this we for example see that 7% of the routes were more than 4% more costly than the corresponding cheapest path in the 'unscaled' case, whereas it was 27% of the routes in the case where the scale-factor was equal to 2.0.

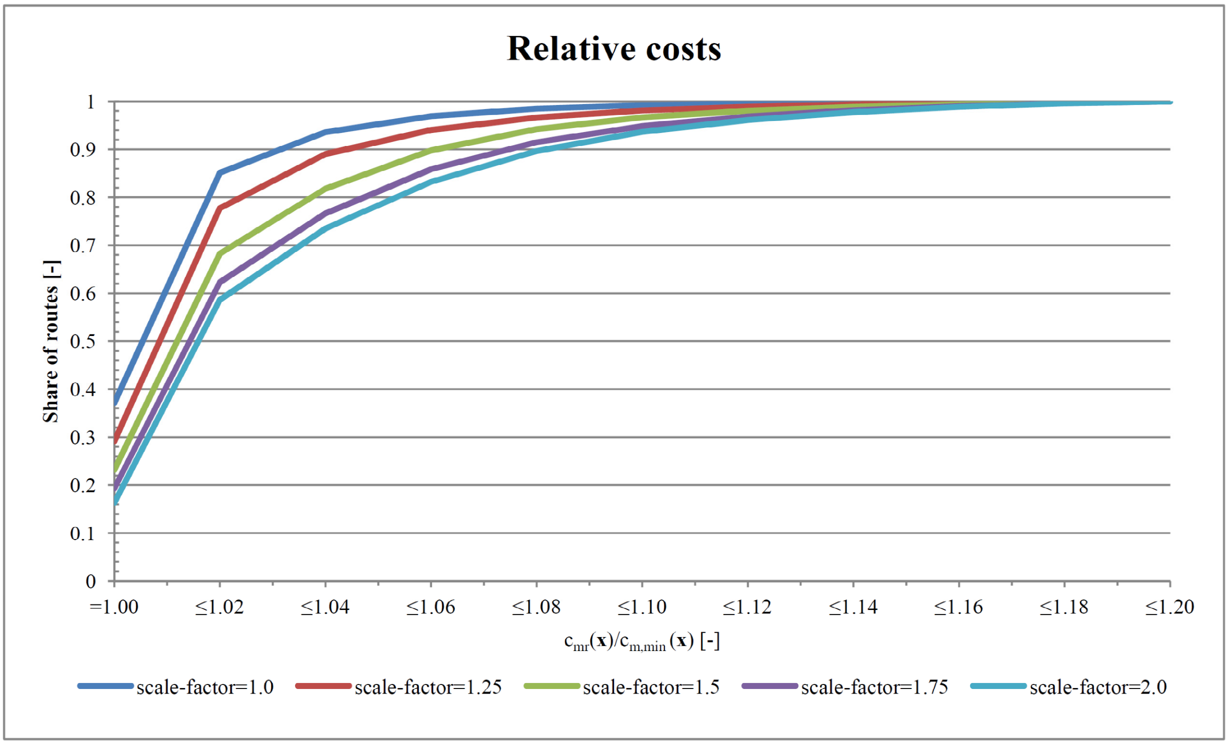

Figure 15 – Distribution of relative costs at convergence (iteration 100). Share of routes as a function of relative cost to the cheapest route in the corresponding choice set. MNL RSUET(min, 1.2·min) d=4 for varying values of the factor scaling the original demand, Zealand application

6 DISCUSSION AND CONCLUSIONS

The paper tackles the challenge of obtaining equilibrated RUM flow solutions among choice sets which do not leave attractive paths unused and contain only attractive paths. The RSUE only partially obtains this; no attractive paths are left unused, but some unattractive paths may be used at equilibrium. We overcome this problem by proposing the RSUET (RSUE with Threshold), as an extension to the RSUE. The extension adds a behaviourally realistic threshold condition that must be fulfilled by the costs on used routes. This ensures that only attractive paths fulfilling the cost threshold are kept in the choice set and thus are assigned traffic.

We have proposed a *corresponding RSUET solution algorithm* and validated several variants of this by application to the large-scale Zealand network. Well-behaved and extremely fast convergence patterns were seen to equilibrated solutions satisfying the underlying conditions (across different scale parameters, step-sizes, and congestion levels). Comparisons to observed routes and observed link flows verified that the composition of the choice sets and the distribution of flow are very reasonable. We investigated *the effect of adding the threshold* under different conditions and found that the threshold condition did not cause any of the observed paths to be removed, which seems reasonable. We also found that the importance of the threshold increased as congestion increased. A comparison to two commonly adopted simulation-based SUE algorithms clearly highlighted the benefits of the RSUE/RSUET by showing that the SUE algorithms (i) generated choice sets which continued to grow in size without showing signs of stabilisation, and (ii) did not stabilise in link flows nearly as fast as the RSUE/RSUET, indicating much slower convergence.

Numerous different *specifications of the threshold* can be formulated, but we focused on a formulation which specifies the threshold based on the cost of the least costly used route(s). The rationale is that there must be a limit to how large detours travellers find reasonable. The RSUET model thereby provides a very behaviourally realistic interpretation of the mechanism which distinguishes attractive and non-attractive paths. Many other models do not provide such a plausible interpretation, e.g. the models based on random walk with loops (e.g., Fosgerau et al., 2013) or simulation-based models, where the draws may induce the use of highly unattractive paths.

The application *focused on the RSUET(min, Ω)* and did not pursue to apply the RSUET(max, Ω). The addition of the threshold condition allows the user to ensure a reasonable distribution of the costs on routes that are *used*. However, the solution algorithm to the *max*-formulation still needs some additional development to enable large-scale applications. Specifically, if the threshold is not specified very 'tight', then the max-formulation will typically need to enumerate an unreasonably large number of *k*-shortest paths to fulfil the max-criterion. The difficulty arises, since numerous alternatives typically exist that are small local detours to the shortest path. All these have to be enumerated before any other relevant and distinct alternative is identified. This issue is highlighted by a search for the 10,000 shortest paths between Rome and Copenhagen on the Transtools network (Rich et al., 2009). These all turn out to be very minor deviations (detours, less than 2% deviation in cost) to the single shortest path, whereas no distinct alternatives are identified. On the other hand, the RSUET(min,Ω) solution algorithm might have overlooked feasible and reasonable routes. These are routes which have not been the minimum cost route at any iteration during the calculation, but are less costly than some used path at equilibrium. To solve this issue, one could seek to propose a RSUET(Φ, Ω) formulation which mediates between the RSUET(min,Ω) and the RSUET(max,Ω). This should be accompanied by a corresponding heuristic principle for choice set generation that fulfil the underlying conditions, while avoiding the use of a 'greedy' *k*-shortest path search routine.

We believe that the RSUET model framework and solution methods fit especially well in combination with *disaggregate activity-based models*. The activity-based models operate at an individual level, and the utility functions become individual-based. In the right integration, this removes the need to account for taste heterogeneity in the assignment model and, thereby, enables the application of the proposed RSUET solution methods. Not only this allows a rich and consistent specification of the utility function that can improve the behavioural realism significantly, but also the extremely fast convergence of the RSUET solution algorithm allows for low computation times in the integrated model framework. An additional benefit is the absence of stochasticity in the output of the model (which may have a major implication for project appraisals) as simulation is avoided. We note that while the solution algorithm fits particularly well with individual-based approaches, they can also be used to approximate mixed logit models and, thereby, represent taste heterogeneity. This can be done by generating quantiles of the distribution of the preferences – e.g. of the value-of-time – and then consider each of these as separate user classes in the solution algorithm (parameters specified as mean value for the corresponding quantile).

In summary, we have proposed a model generic to the RSUE and have demonstrated that this modification supports an improvement of the behavioural realism in disaggregate large-scale applications, especially for high-congestion cases. We proposed a corresponding generic solution algorithm and verified several variants of this in different parameter settings in a highly complex network. The algorithm converged extremely fast to an equilibrated solution fulfilling the underlying conditions, even in large-scale case studies and for high-demand cases.

ACKNOWLEDGEMENTS

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7 APPENDIX I – ROUTE CHOICE PARAMETERS

7.1 CATEGORIES

Category ID	Name	Trip length	Per car unit / type	Max speed [km/h]
1	Commute, education	Short	1 / Car	130
2	Business, business night	Short	1 / Car	130
3	Shopping	Short	1 / Car	130
4	Escort, leisure, other	Short	1 / Car	130
5	Vacation	Short	1 / Car	130
6	Business, commercial	Short	1 / Van	130
7	Other	Short	1 / Van	130
8	Truck	Short	2 / Truck below 12 tonnes	80
9	Truck +12 tonnes	Short/Long	2 / Truck above 12 tonnes	80
10	Truck with trailer	Short/Long	2 / Truck with trailer	80
18	Truck, gigaliner	Short/Long	2 / Truck gigaliner	80
11	Commute, education	Long	1 / Car	130
12	Business, business night	Long	1 / Car	130
14	Escort, shopping, leisure,other	Long	1 / Car	130
15	Vacation	Long	1 / Car	130
16	Business, commercial	Long	1 / Van	130
17	Other	Long	1 / Van	130
19	Freight	Short	1 / Van	130
20	Freight	Long	1 / Van	130

Category ID	Application	$\beta_{FreeTT,m}$		$\beta_{CongTT,m}$		$\beta_{l,m}$	$\beta_{m\varepsilon}$
		μ	σ^2	μ	σ^2	μ	
1	MNL/PSL RSUET(min,1.2·min)	1.692		1.947		0.870	
	MNP SUE	1.692		1.947		0.870	0.010
	mixed MNP SUE	1.692	0.655	1.947	0.670	0.870	0.010
2	MNL/PSL RSUET(min,1.2·min)	2.587		4.028		0.900	
	MNP SUE	2.587		4.028		0.900	0.010
	mixed MNP SUE	2.587	0.582	4.028	0.638	0.900	0.010
3	MNL/PSL RSUET(min,1.2·min)	0.978		1.693		0.870	
	MNP SUE	0.978		1.693		0.870	0.010
	mixed MNP SUE	0.978	0.423	1.693	0.496	0.870	0.010
4	MNL/PSL RSUET(min,1.2·min)	1.087		1.970		0.870	
	MNP SUE	1.087		1.970		0.870	0.010
	mixed MNP SUE	1.087	0.470	1.970	0.551	0.870	0.010
5	MNL/PSL RSUET(min,1.2·min)	1.293		2.544		0.870	
	MNP SUE	1.293		2.544		0.870	0.010
	mixed MNP SUE	1.293	0.559	2.544	0.655	0.870	0.010
6	MNL/PSL RSUET(min,1.2·min)	3.270		5.573		1.090	
	MNP SUE	3.270		5.573		1.090	0.010

	mixed MNP SUE	3.270	0.654	5.573	0.724	1.090	0.010
7	MNL/PSL RSUET(min,1.2·min)	1.087		1.970		1.090	
	MNP SUE	1.087		1.970		1.090	0.010
	mixed MNP SUE	1.087	0.470	1.970	0.551	1.090	0.010
8	MNL/PSL RSUET(min,1.2·min)	3.300		3.630		2.900	
	MNP SUE	3.300		3.630		2.900	0.010
	mixed MNP SUE	3.300	1.300	3.630	1.309	2.900	0.010
9	MNL/PSL RSUET(min,1.2·min)	3.380		3.758		3.150	
	MNP SUE	3.380		3.758		3.150	0.010
	mixed MNP SUE	3.380	1.352	3.758	1.363	3.150	0.010
10	MNL/PSL RSUET(min,1.2·min)	3.500		3.955		3.180	
	MNP SUE	3.500		3.955		3.180	0.010
	mixed MNP SUE	3.500	1.400	3.955	1.413	3.180	0.010
11	MNL/PSL RSUET(min,1.2·min)	2.200		2.531		0.870	
	MNP SUE	2.200		2.531		0.870	0.010
	mixed MNP SUE	2.200	0.851	2.531	0.866	0.870	0.010
12	MNL/PSL RSUET(min,1.2·min)	3.881		6.043		0.900	
	MNP SUE	3.881		6.043		0.900	0.010
	mixed MNP SUE	3.881	0.776	6.043	0.832	0.900	0.010
14	MNL/PSL RSUET(min,1.2·min)	1.739		3.152		0.870	
	MNP SUE	1.739		3.152		0.870	0.010
	mixed MNP SUE	1.739	0.751	3.152	0.833	0.870	0.010
15	MNL/PSL RSUET(min,1.2·min)	1.185		2.234		0.870	
	MNP SUE	1.185		2.234		0.870	0.010
	mixed MNP SUE	1.185	0.819	2.234	0.907	0.870	0.010
16	MNL/PSL RSUET(min,1.2·min)	4.905		8.360		1.090	
	MNP SUE	4.905		8.360		1.090	0.010
	mixed MNP SUE	4.905	0.981	8.360	1.051	1.090	0.010
17	MNL/PSL RSUET(min,1.2·min)	1.739		3.152		1.090	
	MNP SUE	1.739		3.152		1.090	0.010
	mixed MNP SUE	1.739	0.500	3.152	0.581	1.090	0.010
18	MNL/PSL RSUET(min,1.2·min)	3.600		4.068		3.400	
	MNP SUE	3.600		4.068		3.400	0.010
	mixed MNP SUE	3.600	1.440	4.068	1.453	3.400	0.010
19	MNL/PSL RSUET(min,1.2·min)	3.270		5.573		1.090	
	MNP SUE	3.270		5.573		1.090	0.010
	mixed MNP SUE	3.270	0.654	5.573	0.724	1.090	0.010
20	MNL/PSL RSUET(min,1.2·min)	4.905		8.360		1.090	
	MNP SUE	4.905		8.360		1.090	0.010
	mixed MNP SUE	4.905	0.981	8.360	1.051	1.090	0.010

APPENDIX 7: CASE STUDIES

A7.1 NETWORK AND DEMAND

All large-scale applications are based on data from the LTM (Rich et al., 2010). Rasmussen et al. (2014a) utilise the schedule-based digital representation of the public transport network, whereas Prato et al. (2014) and Rasmussen et al. (2014cd) utilise the road network of the LTM. Furthermore, Rasmussen et al. (2014cd) also use the zone-structure and demand matrixes developed and estimated for the LTM.

A7.1.1 RASMUSSEN ET AL. (2014A)

The public transport network used in the LTM is timetable-based and covers all public transport (including ferries, but excluding air transport) in Denmark as well as the relevant international lines in the surrounding countries. The extent of the network can be seen in Figure 1. The travel diary data used however only contain trips conducted within the Greater Copenhagen area. To facilitate faster computation times in the analysis, the geographical extent of the network was therefore reduced. The resulting network includes all public transport conducted within, in to, out from and across the Greater Copenhagen area. Figure 1 illustrates the extent of the full LTM network and the network used in the analysis, and Table 1 lists key characteristics of the two.

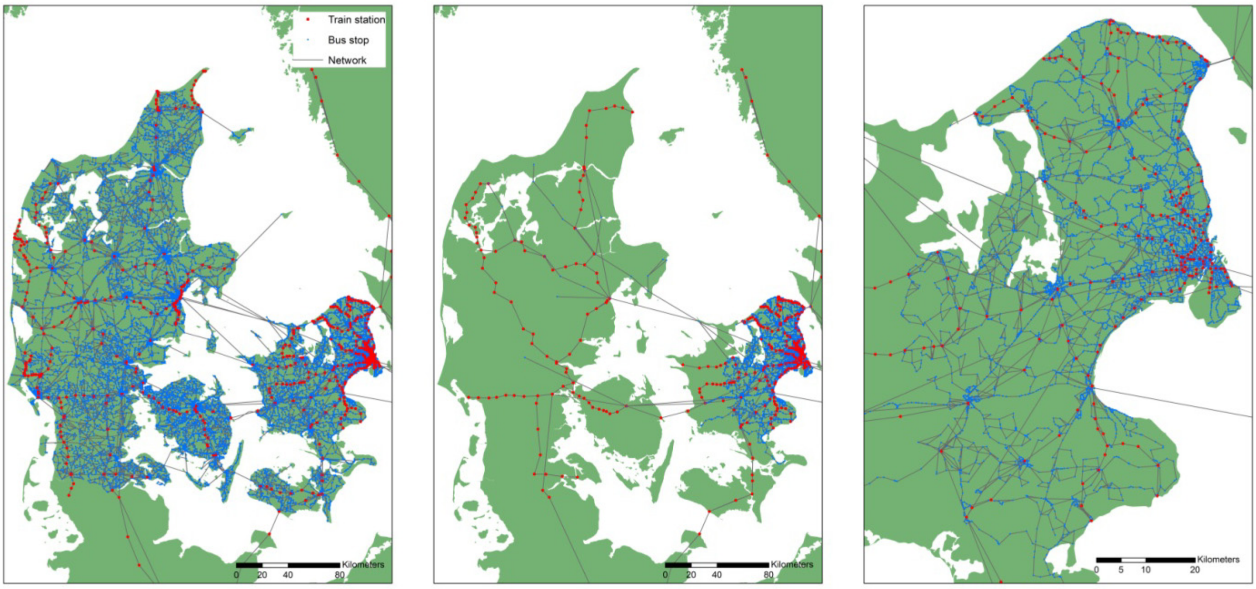

Figure 1 – LTM public transport network. Left: Full LTM network. Middle: Reduced LTM network used in Rasmussen et al. (2014a). Right: Reduced LTM network used in Rasmussen et al. (2014a), zoom

Table 1 – Characteristics of the full LTM network and the reduced LTM network used in Rasmussen et al. (2014a)

	Full network	Greater Copenhagen area
Lines	1,607	479
Line variants	7,804	1,677
Changes	5,566	560
Stops	21,736	5,652
Stop departures	1,258,729	635,027

The network is multi-modal and represents all public transport in the area. The departure-frequency and stopping pattern of each line typically varies during the day. Each line can be classified into one of eight service types by its service level and characteristics. The eight service types and the corresponding typical headway between departures can be seen in Table 2.

Table 2 – Frequency of public transport service types in the Greater Copenhagen area (Anderson, 2013)

Service type	Headway
A-bus	Approximately 3 min. in day hours
E-bus	Approximately 10 min. in peak hours
Bus	10-60 min.
S-train	10 min.
Metro	2-4 min. in day hours, 6-8 min outside day hours
Regional and IC-train	20-120 min. (some lines only one departure per day)
Local train	30 min.
Harbour bus	30 min.

A7.1.2 PRATO ET AL. (2014)

The LTM road network is used in Prato et al. (2014) and this consists of a total of 35,251 possibly bidirectional links. The extent of the network can be seen in Figure 2. The observations used in the analysis are however concentrated primarily in the eastern part of Denmark (see section A7.2.2.1).

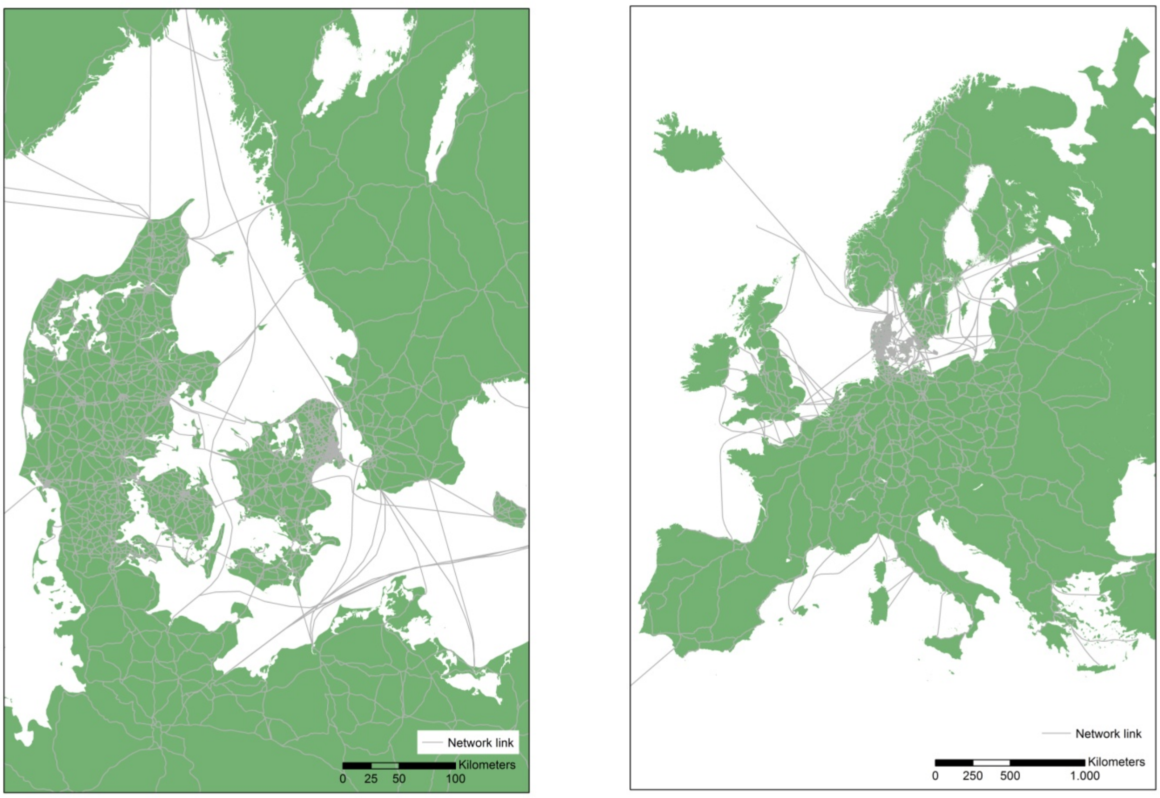

Figure 2 – LTM road network (including ferries). Left: Zoom Denmark, Right: Zoom full network

The primary focus of the paper is the investigation of the value of different travel time components (travel time reliability, free-flow and congested travel time). The analysis however also evaluates whether the number of turns (distinguishing between right and left turns) influences the route choice of travellers. The 'standard' LTM network however does not contain information about turning movements between links, and this attribute thus had to be identified as a part of the study. The PhD study developed a script for the identification of this attribute, and this has subsequently been used in various other studies at DTU Transport (e.g., Sølvason, 2012). Using ArcGIS modules, the script iterates through links and identifies angles between connected link-pairs in the network. These angles are used to identify turning movements between links (straight, right, left), and this information can be accumulated to identify the number of right and left turns of a given route. A right/left turn is identified if the change in direction is greater than ±50 degrees. Note that this method only identifies turns when these happen at link ends (intersection), and not if the direction changes rapidly on a link (e.g. winding road). Figure 3 and Table 3 shows an example of the identification the number of right and left turns. The trip traverses 12 links and makes two left turns and one right turn at intersections (see Figure 3).

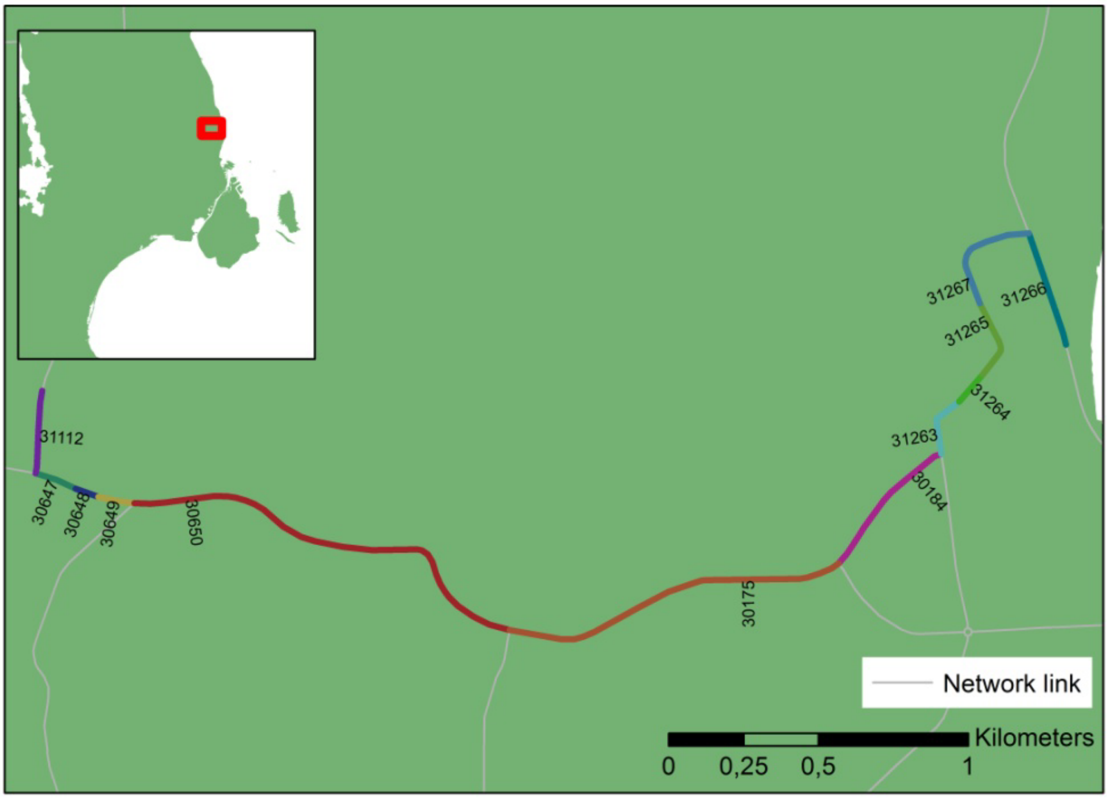

Figure 3 – Each link used on the route is highlighted by a unique colour and is labelled by the associated linkID. The trip traverses 12 links and makes two left turns and one right turn at intersections between links (see Table 3)

Table 3 – Example of determination of turns at link ends. The table is associated to Figure 3

Route	from linkID	to linkID	Angle	Right turn	Left turn
1	31112	30647	-84.7	0	1
1	30647	30648	-5.7	0	0
1	30648	30649	-8.0	0	0
1	30649	30650	-5.4	0	0
1	30650	30175	-5.5	0	0
1	30175	30184	-15.3	0	0
1	30184	31263	-79.5	0	1
1	31263	31264	-11.8	0	0
1	31264	31265	-2.9	0	0
1	31265	31267	5.3	0	0
1	31267	31266	75.9	1	0
			Sum	1	2

A7.1.3 RASMUSSEN ET AL. (2014CD)

The study applied the solution algorithms proposed in Rasmussen et al. (2014cd) to a road network covering the eastern part of Denmark (Zealand). This area has approximately 2.5 million inhabitants, and the network used consists of 12,451 possibly bidirectional links. This network is a geographically limited subnetwork of the network used in the LTM (see Figure 4).

Figure 4 – Case-study network Rasmussen et al. (2014cd). Zealand area

The demand matrices used was also developed for the LTM. This is a zone-based system covering all of Denmark and the most important international destinations/origins. The study area are however smaller than this, and there is therefore the need to adapt the 'standard' LTM matrices. Limiting the matrices to contain only the relations with both origin and destination within the Zealand area (389 of a total of 907 zone centroids) would cause an unrealistic representation of congestion; congestion in the network is affected by e.g. trucks passing Zealand from Jutland to Sweden using the motorways within the study area. In order to try to capture this, port-zones were added to the case-study network on the locations where a link crosses the border of the case-study area. A total of 12 port-zones were added, and demand into/out of these represent (i) traffic going to/from a zone within the study area, and (ii) traffic crossing the study area. The total demand into/out and across the case-study area could be obtained from the original LTM zones. This however had to be split in a realistic manner between the port-zones. Imagine e.g. a trip from Northern Zealand to Germany – in this case some would possibly choose the route across the Great Belt Bridge and others would choose the route using the ferry to Germany, and the split between these should be realistic. In order to compute the split in a realistic manner, a full LTM assignment was conducted using the link-based solution algorithm used in the LTM (Rich et al., 2010). Filters were used, which allowed the identification of the origin and the destination of all traffic passing the 'port-zone' links. This allowed a consistent aggregation of the demand to the port-zones.

The study adopted the connecters of the LTM network within the case-study area, whereas the port-zones were connected to the network link crossing the border of the study area.

A7.2 REVEALED PREFERENCE DATA

A7.2.1 PUBLIC TRANSPORT ROUTE CHOICE TRAVEL SURVEY

The data on route choice of public transport users stem from the Danish Travel survey (TU). The TU is an on-going data collected among a representative sample of the Danish population. The survey collects travel diaries of trips conducted on the day before the interview as well as corresponding respondent and household socio-demographic data (income, gender, age etc.) (Christiansen, 2012). Since 2009, the TU has also collected detailed information on the trips conducted by public transport (see Anderson (2010) for more details). This extension of the TU was developed with the aim to be detailed enough to allow realistic and disaggregate representation on a digital version of the network, while still being easy to fill in by the respondents. Many observations have been collected due to the affiliation with the traditional

TU survey, and new data are continuously being collected. Information about addresses and purposes at start points, change points and end points of each trip as well as detailed information about the modes used *en route* are collected:

- *Walk, bike, car, airplane, etc.*
    - Length and travel time
- *Bus*
    - Waiting time, bus line, length and travel time
- *Suburban train (S-train)*
    - Waiting time, boarding station, S-train line, alighting station, length and travel time
- *Train, Metro*
    - Waiting time, boarding station, alighting station, length and travel time

Figure 5 shows an example of the data collected for one trip consisting of 7 trip parts.

Transport modes in correct order:

	Transport mode		Line	Length		Time		
1.	Walking			0.5	km	8	min	
				Waiting time:		3	min	
2.	Bus		701	2	km	5	min	
3.	Walking			0.2	km	2	min	
	From-station:	Nykøbing F		Waiting time:		7	min	We calculated the distance to 146 km from the stated stations
4.	Other train			146	km	100	min	
	Transfer at:	København H		Waiting time:		8	min	
5.	S-train		E	13.9	km	20	min	We calculated the distance to 13.9 km from the stated stations
	To-station:	Kgs. Lyngby		Waiting time:		5	min	
6.	Bus		300s	5	km	10	min	
7.	Walking			0.5	km	5	min	
			Sum:	168.1	km	173	min	

Figure 5 – Example of description of trip, public transport route choice survey (Anderson and Rasmussen, 2010)

The collected data had to be map matched to the network used for the choice set generation in Rasmussen et al. (2014a). A method to conduct this map matching was developed in Rasmussen (2010), and this has been used to map match the data used in Rasmussen et al. (2014a). The method is complex by using advanced mechanisms to e.g. determine the location of boarding and alighting when doing bus trips etc. The method will not be described in detail here, but refer to Anderson and Rasmussen (2010) for a description in English.

A7.2.2 GPS DATA

The analysis in Prato et al. (2014) and Rasmussen et al. (2014d) utilise information collected by GPS units. The dataset consists of data from two data sources, both collected by DTU Transport. All data have been collected in 2011 and with a logging interval of 1-second. One part of the dataset was collected by in-vehicle units installed in vehicles of employees of the Municipality of Copenhagen. The other part of the dataset was collected as part of the ACTUM project. The PhD study developed the setup and handled various practicalities associated to the ACTUM data collection. The data was collected as individual-based rather than vehicle-based, among members of 50 families living in the Greater Copenhagen area. The analyses conducted in the two papers focus on data associated to car trips, and the individual-based data has thus been thoroughly analysed to identify trips, trip legs and the most probable mode of transport. Trips using other modes of transport than car has been removed from the dataset, see more in Rasmussen et al. (2014b). The data collected by in-vehicle mounted units were cleaned by assuming that a new trip starts after a long pause between logs (the GPS-unit turns off if the car is stationary for a while or if the ignition of the car is turned off).

The dataset consisted of at total of $56.3 \cdot 10^6$ GPS logs. A part of this was ‘scatter’ observations, which may arise due to numerous different reasons. One possibility could be that scatter logs may have been generated by travellers wearing the GPS-unit when moving around within their house (relevant only for the individual-based survey). Another possibility could be that the built-in motion-sensor (which reacts to vibrations) may have caused the units to erroneously turn on when the car was stationary, e.g. due to vibration caused by a passing truck (only relevant for vehicle-based survey). Additional possible reasons for generation of scatter surely exist. However, most scatter observations were removed from the datasets by requiring that a trip should consist of more than 60 observations. Applying this rule to the vehicle-based dataset and the rules explained in Rasmussen et al. (2014b) to the individual-based dataset results in the identification of a total of 46,000 GPS trips.

After the initial removal of scatter observations, the next step was to map match the 46,000 GPS trips. However, before this, the trips which started and/or ended outside the study area were removed from the dataset (Prato et al. (2014) used Denmark as the study area, whereas it was the Zealand Area for Rasmussen et al. (2014d)). The map matching was done using a software-implementation of the algorithm presented by Nielsen and Jørgensen (2004). Rather than adopting e.g. a closest link approach, the algorithm utilises a branch and bound approach to obtain coherent and correctly map matched routes. The algorithm will not be presented in detail here, refer to Nielsen and Jørgensen (2004). Among the output of the map matching software is a table consisting of one row for each link that the GPS trace has been matched to. Each row is associated with several attributes such as e.g. the time at which the vehicle entered the link. 'Matched' is another attribute, which takes the value -1, 0 or 1. 'Matched'=1 indicated that actual GPS logs have been associated with the link, whereas 'matched'=0 indicates that the link is associated to the path as part of the shortest path between links where points have been associated to (i.e. having 'matched'=1). This e.g. arises when the traveller has been driving, for sections of the trip, on small local roads not represented in the somewhat aggregate LTM network. An example of such a trip can be seen in Figure 7. Figure 6 illustrates an example of a trip where the whole route has been successfully map matched (i.e. 'matched'=1 for all links on the route).

A row with 'matched'=-1 does not have an associated linkID, as this indicates a section where no GPS points can be associated *and* no shortest path between two adjacent used links (i.e. having 'matched'=1) can be found. This can e.g. happen as a consequence of an unconnected network. All traces containing a part with 'matched'=-1 is initially removed from the dataset.

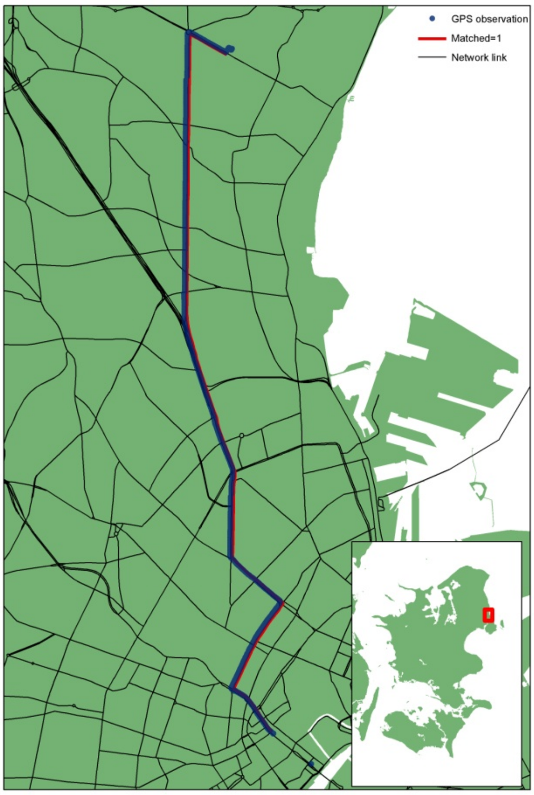

Figure 6 – Illustration of GPS trace and matched route

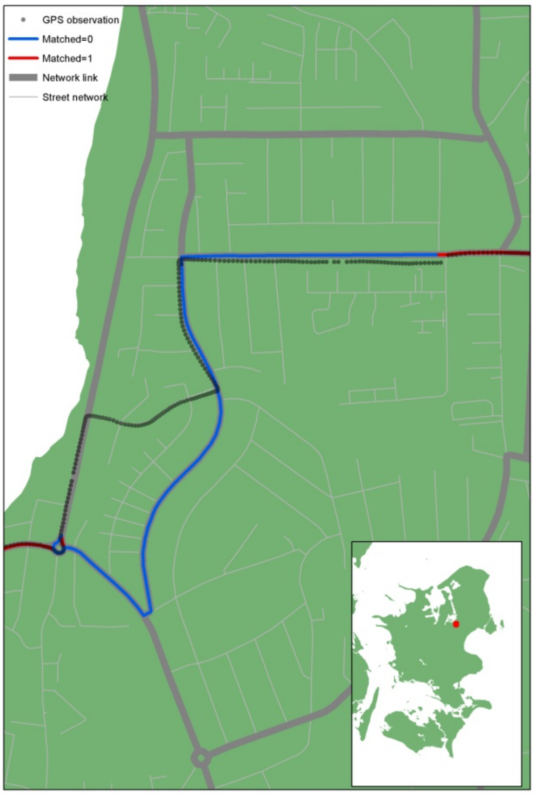

Figure 7 – Illustration of GPS trace and matched route. Network aggregation causes wrong matching on section

A7.2.2.1 Prato et al. (2014)

The paper performed a two-part utilisation of the GPS data, each part setting different criteria for the ‘successfulness’ of the map matching; the estimation of link travel time reliability used the travel times on link level, and did not require the whole path to be matched fully. This part of the analysis therefore considered all links to which data have been matched with ‘matched’=1 (but did not require all links of paths from which a link is used to be ‘matched’=1). A subset of the dataset was also used for the choice set generation and model estimation. This part of the analysis used the map matched routes on ‘path level’, and did thus require for the whole paths to be correctly represented to allow consistent comparisons to generated routes. Consequently, only observations for which the path only consists of links with ‘matched’=1 were included in this part of the analysis.

The paper splits the day into the ten time periods also used in the LTM. Table 4 lists these, along with the distribution of the observations used for the choice set generation. Note that the model estimation was based on fewer observations, as some observations were removed because only one route was generated in the choice set generation.

Table 4 – Definition of time periods and distribution of two datasets to be joined

Time period	Time span	Trips Municipality of Copenhagen	Trips ACTUM
1	21-05	927	20
2	05-06	26	3
3	06-07	187	13
4	07-08	745	67
5	08-09	1,137	60
6	09-15	6,691	137
7	15-16	1,652	48
8	16-17	1,587	65
9	17-18	1,425	52
10	18-21	2,184	89

The travel time reliability on each link (for each direction and time period) was identified for links with more than 10 observations (for the corresponding direction and time period). The travel time reliability was defined as the difference between the 90th and the 50th percentiles of the travel time distribution. The travel time distribution was estimated using SAS, identifying the mean value μ_a and variance σ_a^2 of the travel time on link a. From these parameters, the 90th percentile of the travel time distribution ($TT_{90,a}$) could be found using the following (formula to determine confidence interval):

$$TT_{90,a} = \mu_a + t_{90/2} \cdot \sqrt{\sigma_a^2} = \mu_a + 1.645 \cdot \sqrt{\sigma_a^2} \qquad \text{(A7.1)}$$

For links where there were less than 10 observations, an ‘average’ travel time reliability was used, dependent on location, link type and congestion level (on the computation of this, see Appendix 8).

A7.2.2.2 Rasmussen et al. (2014d)

Rasmussen et al. (2014d) used map matched routes on 'path level'. The whole path should be correctly represented to allow consistent comparisons to the corresponding routes generated. Consequently, only paths/observations for which all links had the attribute 'matched'=1 were included in the GPS-dataset used in Rasmussen et al. (2014d). Overall, three criteria were specified for the exclusion of observations; (i) the trip was less than 1 km long, (ii) the trip started and/or ended outside the study area, and (iii) one or several parts of the GPS trip could not be fully matched by GPS observations. If one of these criteria were fulfilled, the observation was removed from the resulting dataset. This induced the dataset to include a total of 16,618 map matched GPS traces, which is considerably less than the approximately 46,000 trips in the full dataset. Several of the trip characteristics varied considerably across observations, and the remainder of this section illustrates the spatial and temporal distribution as well as the distribution of trip lengths.

The spatial distribution of the observed paths is illustrated in Figure 8.

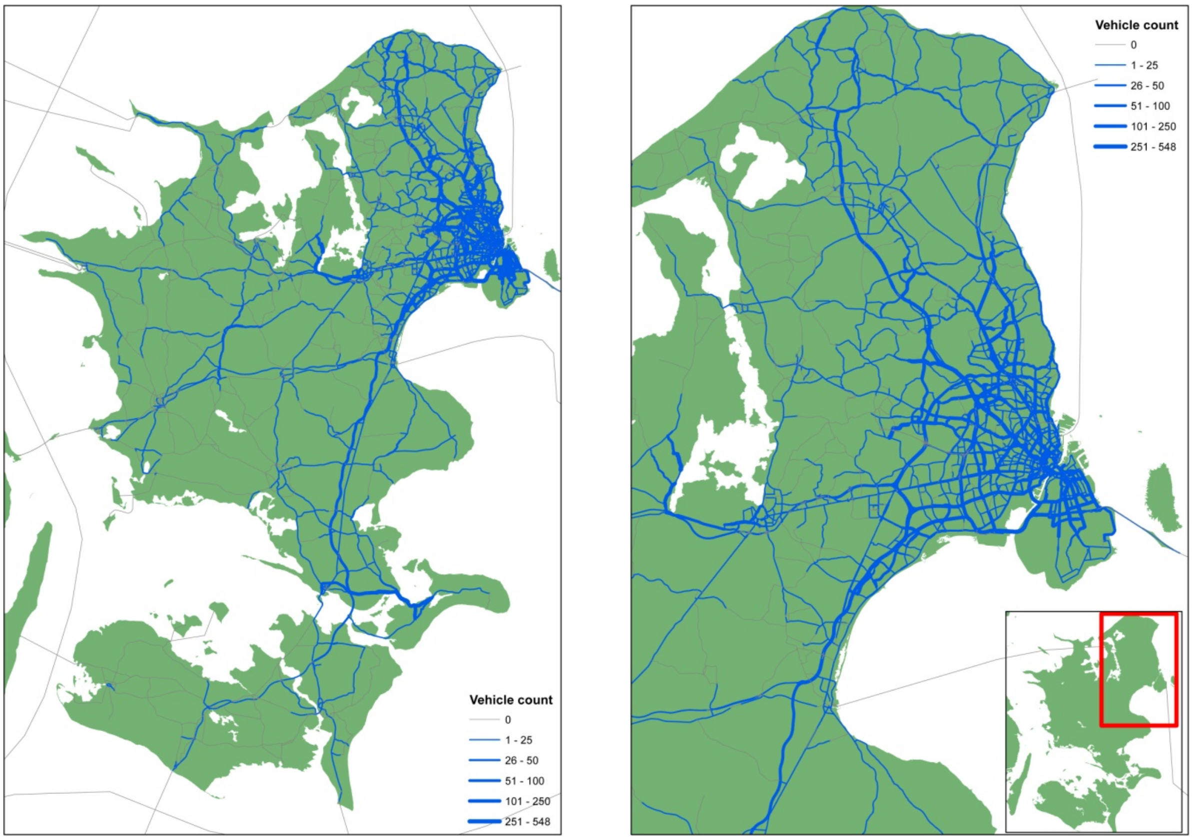

Figure 8 – Network coverage, observation count. Left: Zoom case study area, Right: Zoom Greater Copenhagen area

As can be seen, the trips covered most parts of the study area. There were observations in urban as well as rural areas, however with most trips concentrated in the Greater Copenhagen area.

The distribution of the length of the observed routes is illustrated in Figure 9. Most trips were less than 15 kilometres long. This can partly be explained by many of the trips being performed within the Greater Copenhagen area, and partly by the trip identification algorithm which in some cases split trips wrongly, e.g. when long waiting times occur at intersections (Rasmussen et al. (2014b)). However, we did expect an overrepresentation of short trips, as trips which start and/or end outside the study area were removed from the dataset.

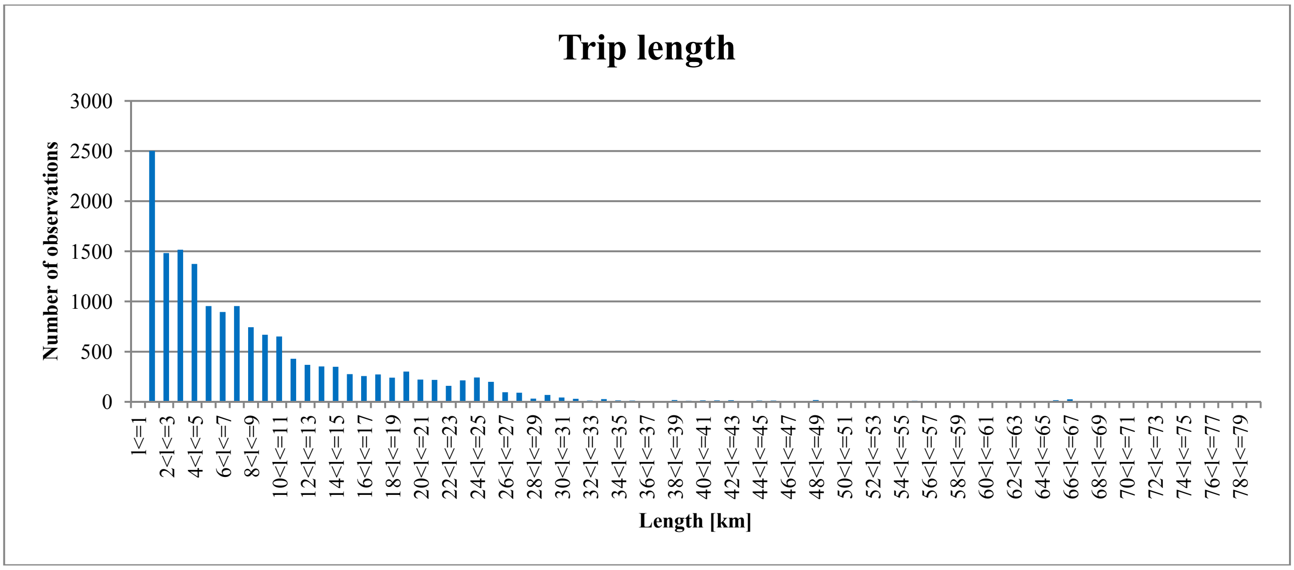

Figure 9 – Number of observations as function of trip length

The temporal distribution of the observed trip legs is reported in Figure 10. As can be seen, most trips were conducted within the time span from 06 to 23, whereas only a few trips were conducted during night. The demand peaked during afternoon peak hours and partly during morning peak hours, but also many observations were obtained between these two peaks. One would expect the two peak periods to be more 'distinct', with a larger drop in trips in the period between these (as reported in the Danish National Travel Survey, Christiansen (2012)). However, many of the vehicles equipped with the GPS units were not only used for private transportation, but also for business purposes such as e.g. home care personnel travelling between resident houses. These trips are typically conducted within working-hours, and may explain the fairly large share of trips being conducted between peak-hours. This might also partly explain why the dataset contains a large share of short trips.

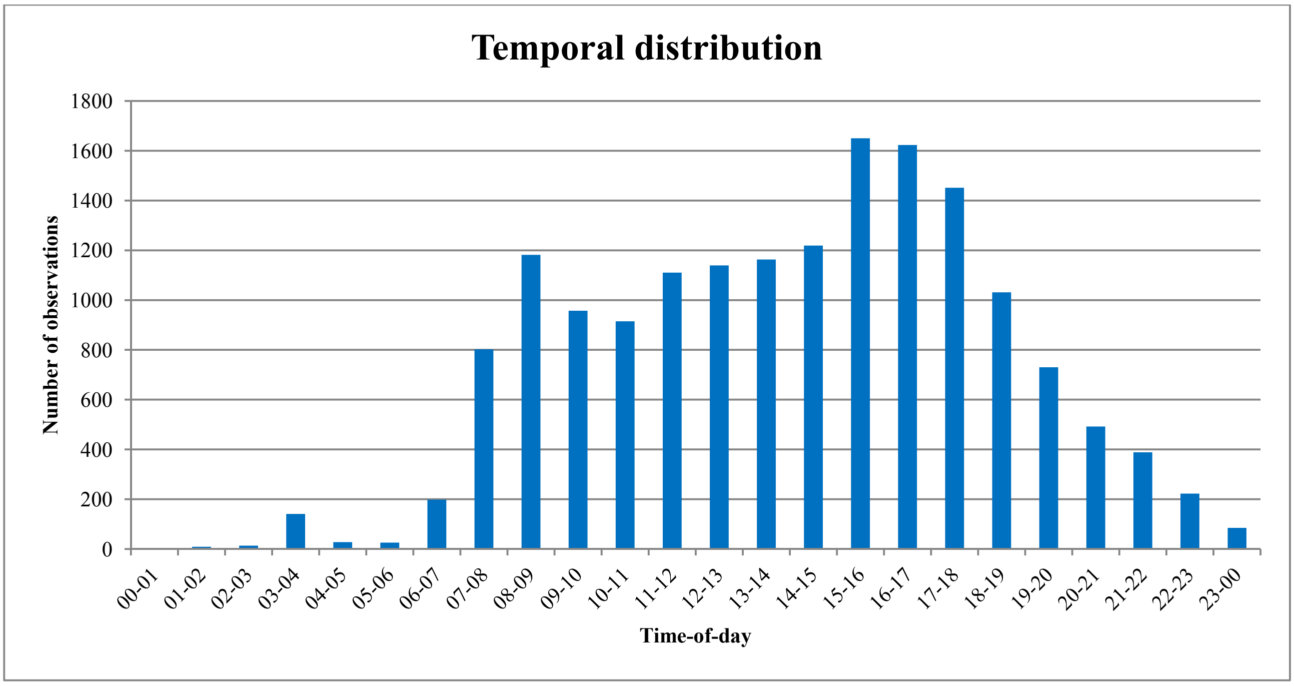

Figure 10 – Number of observations as function of time-of-day

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APPENDIX 8: PRATO ET AL. (2014) DATA PREPARATION

This appendix supplements the description in Appendix 7. Section A8.1 sets of by giving a description of how the travel time components were determined. The section also provides details about the computation of the travel time reliability when a few or no observations were available. Section A8.2 provides some details about the choice set generation method and the generated choice sets. The section also gives a disaggregate example comparing the choice set generated to a corresponding observed path.

A8.1 LINK TRAVEL TIMES AND LINK TRAVEL TIME RELIABILITY

The model estimation included various variables, including route free-flow travel time, route congested travel time and route travel time reliability. The route free-flow travel time was obtained as the sum of the free-flow travel time on the links used by the route. The route congested travel time was obtained as the sum of the difference between the average link travel time and the free-flow travel time on the links used by the route. The route travel time reliability was obtained as the sum of link travel time reliability on links used by the route. The travel time reliability was defined as the difference between the 90th and the 50th percentile of the travel time distribution (Appendix 7).

The average link travel time was defined as the modelled average speed. This speed was obtained from an assignment of the LTM demand matrices (split in the 10 time periods listed in Table 4 of Appendix 7) onto the network. Each link was thus associated with 10 different average travel times – one for each time period. The assignment model was the doubly stochastic link-based mixed probit assignment model applied in the LTM, and the assignment was assumed to give a realistic representation of the average congestion on the network links.

The link travel time reliability was computed using the available GPS data. The observed link travel time reliability per direction and time period was calculated at disaggregate level where possible, as explained in Appendix 7. However, not all network links had 10 or more observations per direction in each time period. The travel time reliability was computed in the following manner for cases where a link with less than 10 observations had one or more adjacent links with 10 or more observations (for the same time period and direction):

$$TT_{90,a} = TT_{90,k} \cdot \frac{l_a}{l_k}$$

where $TT_{90,a}$ and l_a are the travel time reliability and length on link a (with less than 10 observations), whereas $TT_{90,\mathrm{k}}$ and l_k are the travel time reliability and length on link k (adjacent link with 10 or more observations).

If a link and none of its adjacent links had 10 or more observations, then the travel time reliability were determined by assuming that the reliability on the links depend on link type and congestion level. Moreover, a congestion level was defined on nearby links with 10 or more observations, and an average travel time reliability per length was then computed for each combination of link type and congestion level. This average travel time reliability per length was then used to calculate the link travel time reliability on the remaining uncovered links. The congestion level was specified using the relative speed (V_{rel}), defined as the ratio between the congested (modelled) speed and the free-flow speed. Three congestion levels were defined; 1: $V_{rel} \leq 0.45$, 2: $0.45< V_{rel} \leq 0.85$, 3: $V_{rel} >0.85$. This definition is based on Brems and Nielsen (2012).

A8.2 CHOICE SET GENERATION AND CHOICE SET CHARACTERISTICS

To facilitate the model estimation, corresponding choice sets has to be generated for each of the observed routes. The analysis applied a doubly stochastic choice set generation method based on simulation. Rasmussen et al. (2014a) used the same approach for choice set generation in a public transport network. For each observation, 100 corresponding routes were generated by performing a shortest path search under consideration of preferences drawn for the parameters and simulated link impedances. Some of the routes generated were non-unique, and Figure 1 shows the average number of *unique* routes generated as a function of the number of routes generated.

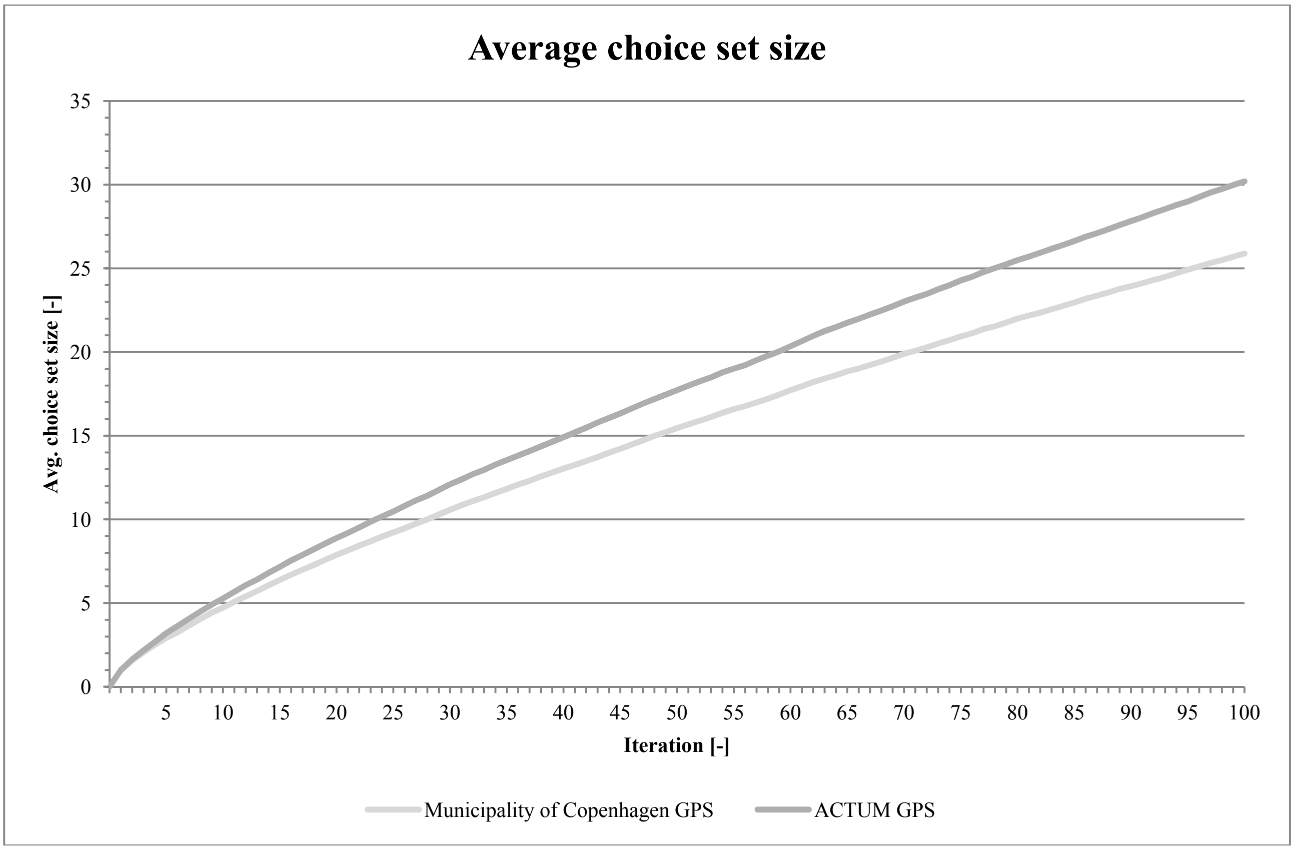

Figure 1 – Average number of unique routes in choice sets as function of iteration number (number of routes generated)

It is important that the choice sets generated contain all relevant alternatives (including the observed route) while leaving out non-sensible routes (as discussed in e.g. Rasmussen et al. (2014d)). Using a 90% overlap threshold, Figure 2 shows the coverage (share of observations for which the observed route is represented in the choice set) as a function of the number of routes generated. Figure 1 in Prato et al. (2014) shows the relation between the coverage obtained at iteration 100 and the overlap threshold.

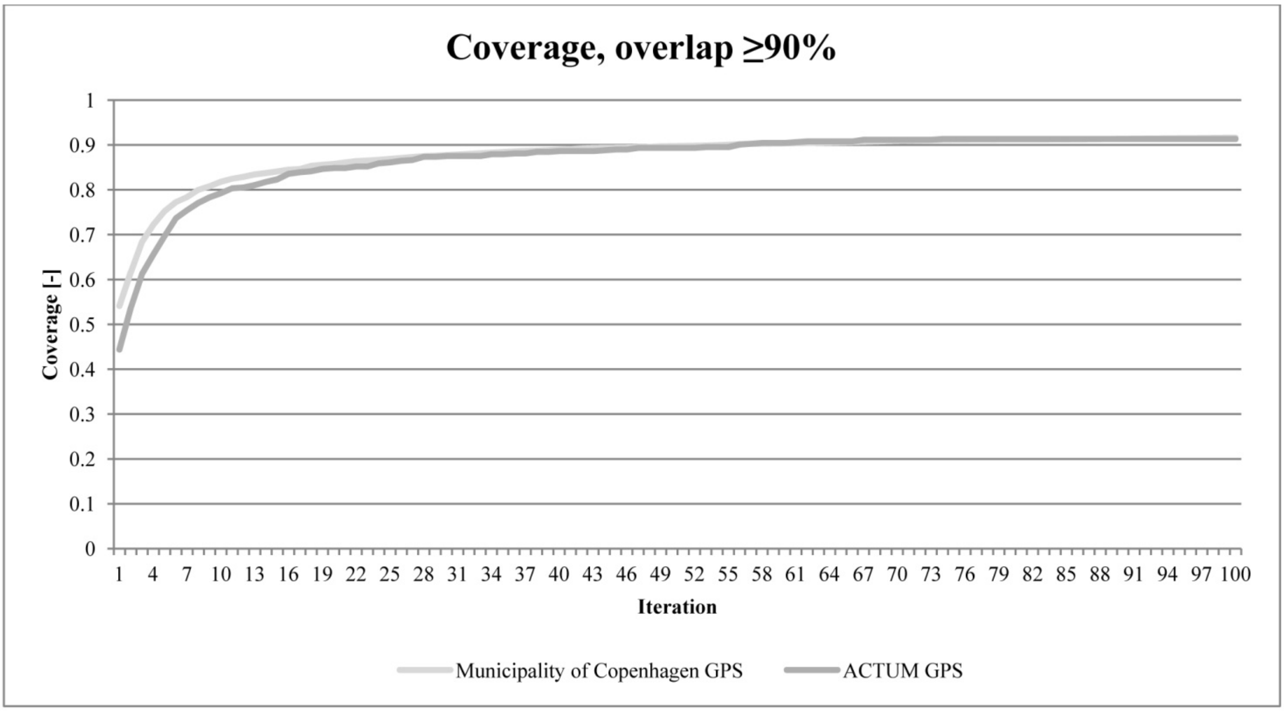

Figure 2 – Coverage as function of iteration number, 90% overlap threshold

The remainder of this section is dedicated to an example. The example illustrates the unique alternatives generated for one corresponding observed path and lists attributes of these. A total of 24 unique routes were generated within the 100 iterations. Figure 3 illustrates the observed route as well as the links used by one or several of the generated routes.

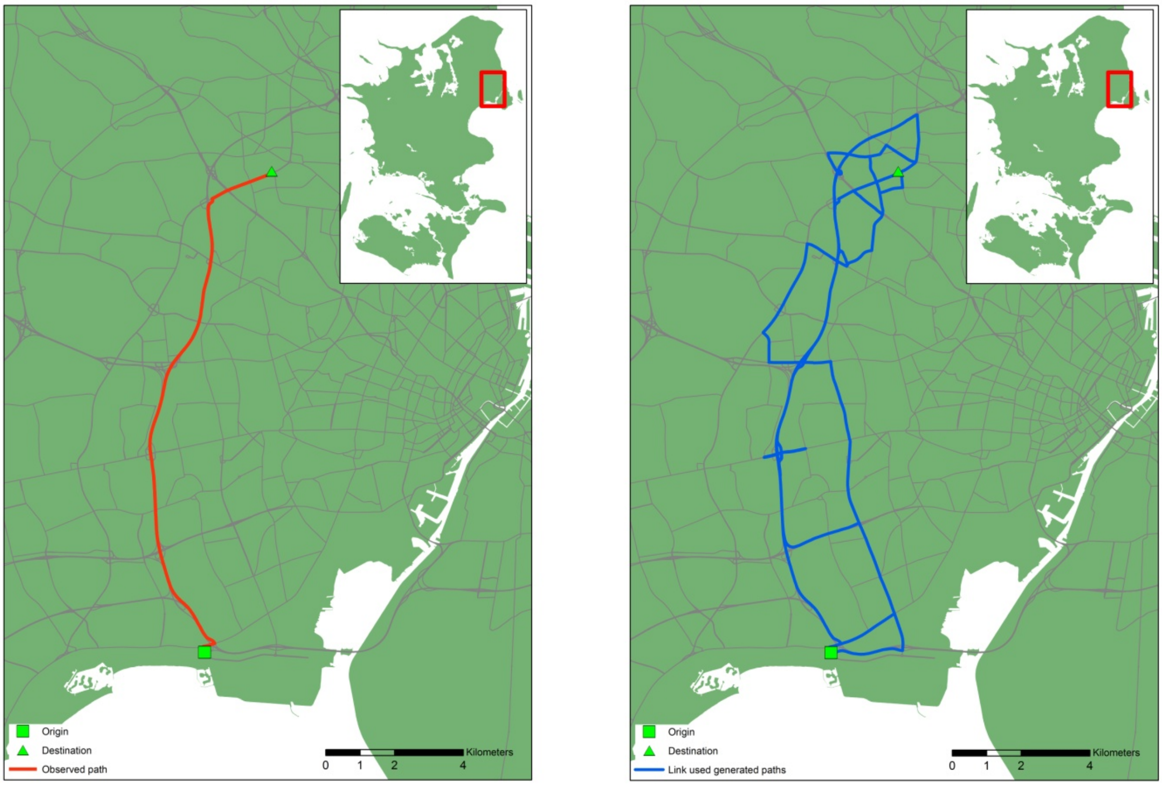

Figure 3 – Left: Observed route. Right: links used by generated routes

Various attributes of the 24 generated routes are listed in Table 1.

Table 1 – *Free-flow travel time, congested travel time, length, reliability and overlap with observed route. 24 generated paths*

Alternative	Iteration	Free-flow travel time [sec]	Total travel time [sec]	Length [km]	Common length [km]	Overlap [-]	Reliability [sec]
1	1	630.1	641.7	16.5	15.4	0.94	147.5
2	2	962.5	979.7	19.3	5.9	0.36	159.6
3	3	621.6	633.3	16.4	16.4	1.00	151.2
4	4	694.9	707.6	17.8	15.7	0.96	140.4
5	5	711.9	727.2	17.9	15.4	0.94	155.0
6	6	635.4	647.3	16.7	16.0	0.97	151.3
7	8	825.3	837.2	20.7	11.9	0.72	170.3
8	13	710.3	728.4	17.9	15.1	0.92	156.4
9	15	743.5	757.7	17.1	12.7	0.77	100.0
10	20	643.8	655.8	16.7	15.0	0.91	147.5
11	23	695.0	707.7	17.4	15.3	0.93	153.8
12	25	733.0	746.1	20.1	14.1	0.86	116.9
13	29	706.3	717.9	17.8	15.0	0.91	129.7
14	30	951.1	970.6	21.4	13.9	0.85	199.7
15	31	708.7	721.6	18.0	15.3	0.93	140.4
16	38	681.2	693.7	17.2	15.7	0.95	153.8
17	40	920.4	946.2	19.4	9.8	0.59	168.1
18	41	760.8	774.4	20.2	13.3	0.81	111.9
19	46	781.7	796.0	19.2	14.4	0.88	113.2
20	50	689.7	702.1	17.2	14.6	0.89	150.1
21	58	732.2	747.4	17.1	13.5	0.82	113.5
22	62	740.6	755.9	17.1	12.5	0.76	108.4
23	84	714.8	726.3	17.8	13.9	0.85	125.3
24	85	879.0	896.5	18.6	6.4	0.39	176.4

Figure 4 illustrates the total travel time as well as the travel time reliability of the 24 routes.

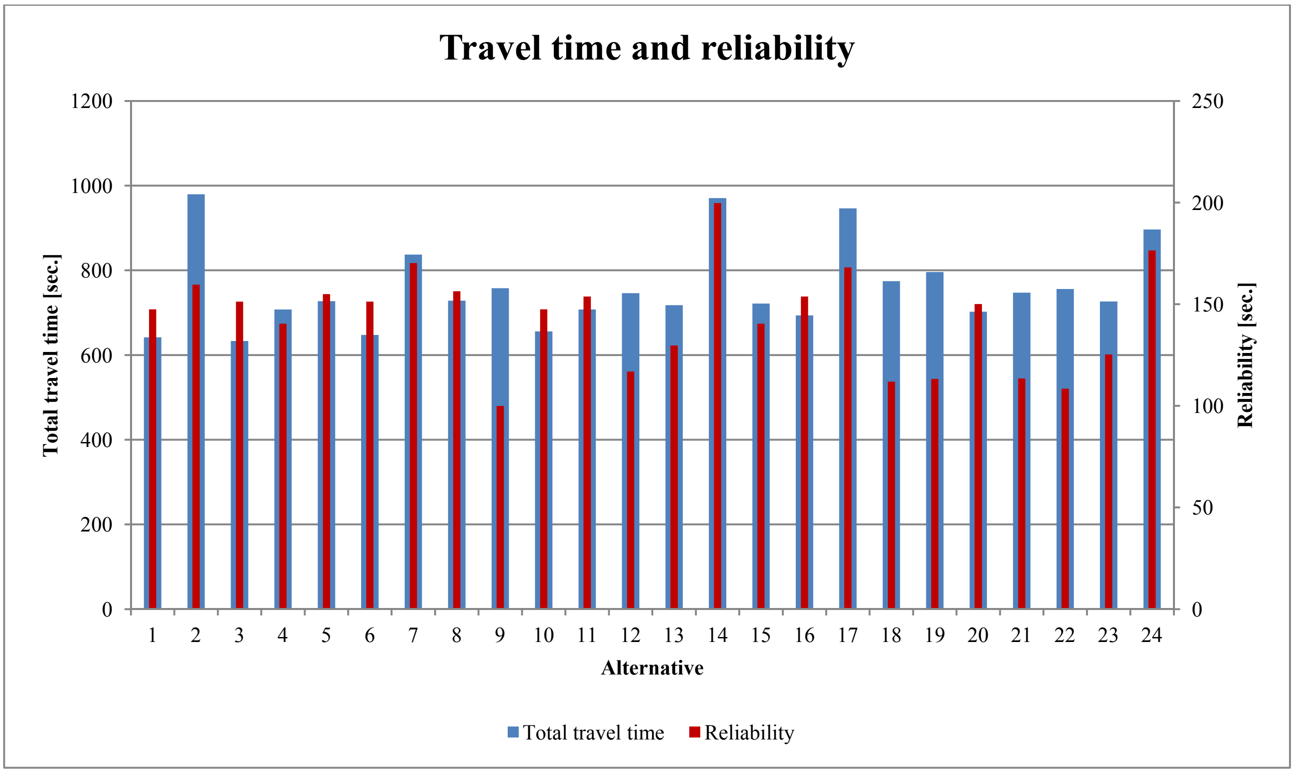

Figure 4 – Total travel time and reliability, 24 alternatives

Figure 5 illustrates the 90 % confidence interval for the total travel time.

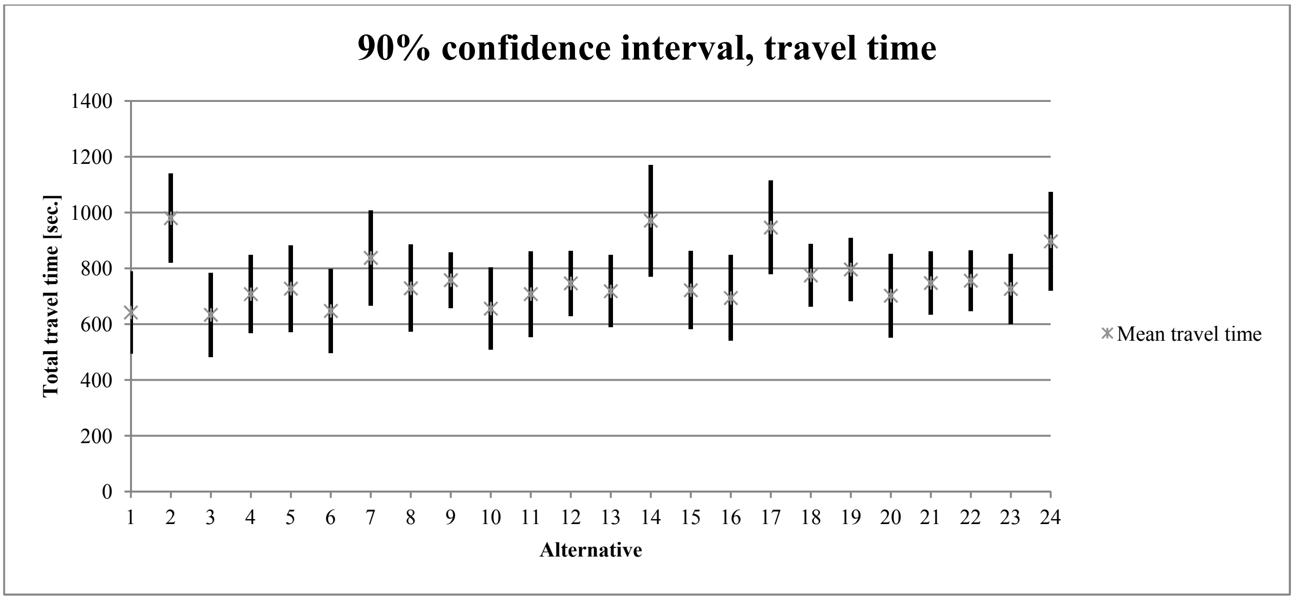

Figure 5 – 90% confidence interval, total travel time

Alternative 3 is the chosen alternative. This has the shortest total travel time, and also a reasonably low reliability-value (indicating high reliability in the travel time). Other alternatives with lower reliability-value are available (e.g. alternative 9), but these also have longer travel times – apparently the lower 'risk' of delay is not worth the longer travel time. By visual inspection it was found that a certain stretch in the network contributes very much to the variation (reliability). Alternatives which avoids this stretch (e.g. alternative 9, 18 and 19) has a lot less variability. The stretch is the northern part of Ring 3 (highlighted by red in Figure 6), which in general is known to very unreliable.

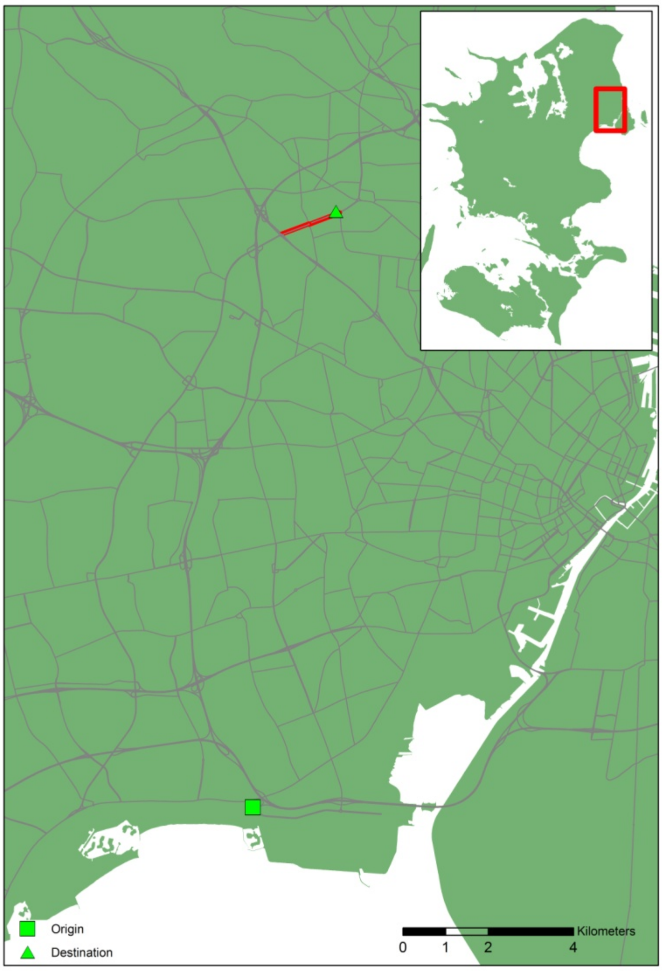

Figure 6 – Stretch of Ring 3 with high variability of travel time (highlighted in red)

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